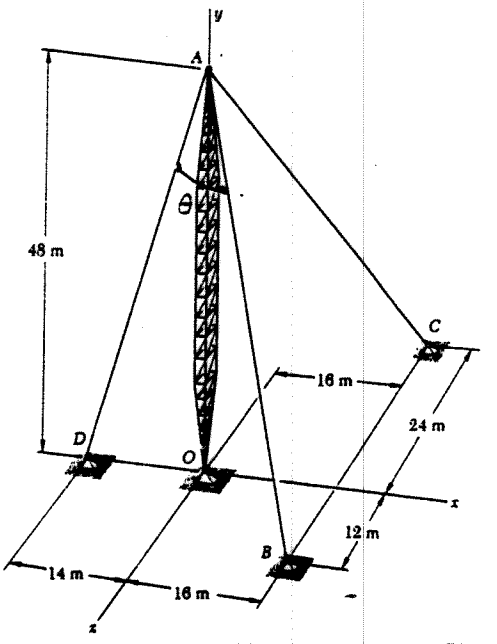
#5

Examples
Dot Product

Example 1:



Given:

The tower and cable: shown

Req'd.:

The angle formed by cable AB and AD

Soln.:

A (0, 48, 0)

B (16, 0, 12)

D (-14, 0, 0)

$\vec{AB} = (16-0)\vec{i} + (0-48)\vec{j} + (12-0)\vec{k}$

$= 16\vec{i} - 48\vec{j} + 12\vec{k}$ (m)

$AB = \sqrt{(16)^2 + (-48)^2 + (12)^2} = 52$ m

$\vec{AD} = (-14-0)\vec{i} + (0-48)\vec{j} + (0-0)\vec{k} = -14\vec{i} - 48\vec{j}$ (m)

$AD = \sqrt{(-14)^2 + (-48)^2} = 50$ m

$$\cos\theta = \frac{\vec{AB} \cdot \vec{AD}}{(AB)(AD)}$$

$$= \frac{(16\vec{i} - 48\vec{j} + 12\vec{k}) \cdot (-14\vec{i} - 48\vec{j} + 0\vec{k})}{(52)(50)}$$

$$= \frac{16(-14) + (-48)(-48) + 12(0)}{52(50)}$$

$$= 0.8$$

$\Rightarrow$ $\boxed{\theta = 36.87^\circ}$

Is it the "expected" answer? Explain!

#5

Example 2:

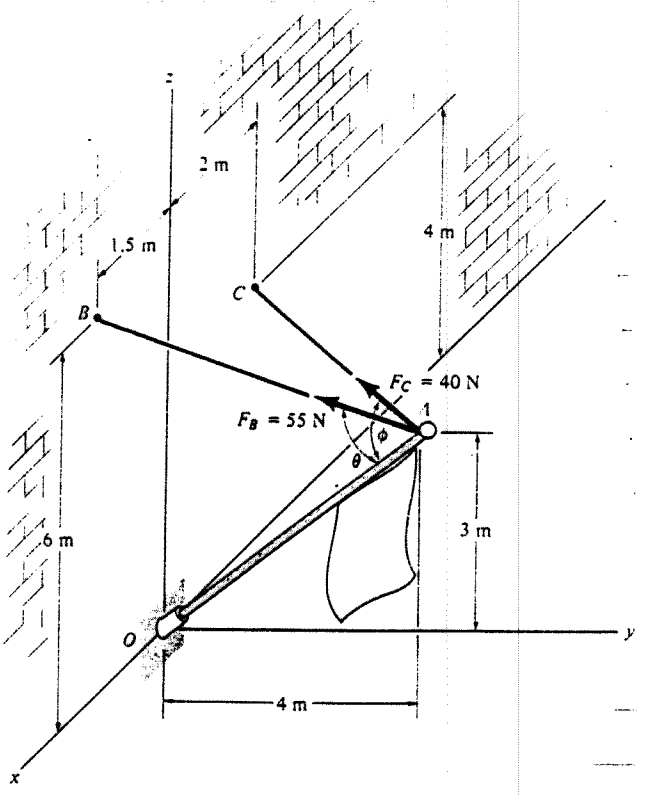
Given:

The figure shown

Req'd.:

a) The angles θ and Φ

b) The components of F_B and F_C on axis AO



Soln.:

A(0, 4, 3), B(1.5, 0, 6), C(-2, 0, 4)

$\vec{AO} = -4\vec{j} - 3\vec{k} \Rightarrow AO = 5$ m

$\vec{AB} = 1.5\vec{i} - 4\vec{j} + 3\vec{k} \Rightarrow AB = \sqrt{27.25}$

$\vec{AC} = -2\vec{i} - 4\vec{j} + \vec{k} \Rightarrow AC = \sqrt{21}$

a) $\cos\theta = \dfrac{\vec{AO}\cdot\vec{AB}}{(AO)(AB)}$

$= \dfrac{(0)(1.5)+(-4)(-4)+(-3)(3)}{5\sqrt{27.25}} = 0.2682 \Rightarrow \boxed{\theta = 74.4^\circ}$

$\cos\Phi = \dfrac{\vec{AO}\cdot\vec{AC}}{(AO)(AC)} = \dfrac{(-4)(-4)+(-3)(1)}{5\sqrt{21}} = 0.5674 \Rightarrow \boxed{\Phi = 55.4^\circ}$

b) $(F_{AB})_{AO} = F_{AB}\cos\theta$

$= 55\,(0.2682) \Rightarrow \boxed{(F_{AB})_{AO} = 14.8 \text{ N}}$

$(F_{AC})_{AO} = F_{AC}\cos\Phi$

$= 40(0.5674) \Rightarrow \boxed{(F_{AC})_{AO} = 22.7 \text{ N}}$

Note that if the angles were not calculated in part (a), then the projections (components) can be calculated by the formulas

$(F_{AB})_{AO} = \vec{F}_{AB}\cdot\vec{u}_{AO}$ and $(F_{AC})_{AO} = \vec{F}_{AC}\cdot\vec{u}_{AO}$

e.g.

$\vec{u}_{AO} = \dfrac{\vec{AO}}{AO} = -0.8\vec{j} - 0.6\vec{k}$; $\vec{F}_{AB} = \dfrac{55}{\sqrt{27.25}}(1.5\vec{i} - 4\vec{j} + 3\vec{k}) = 15.80\vec{i} - 42.14\vec{j} + 31.61\vec{k}$

$(F_{AB})_{AO} = (15.80\vec{i} - 42.14\vec{j} + 31.61\vec{k})\cdot(-0.8\vec{j} - 0.6\vec{k}) = -42.14(-0.8) + 31.61(-0.6) = 14.8$ N as above