#4

Examples

Force Vector Directed Along a Line

Example 1 :

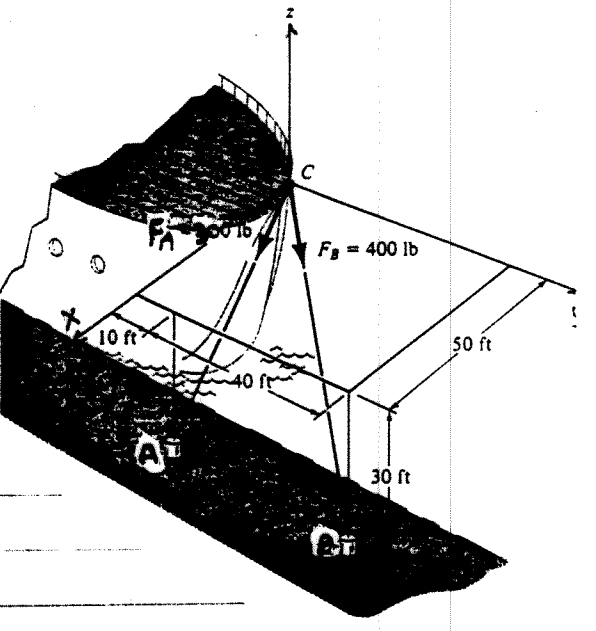
Given :

The forces in the figure shown

Req.d. :

a) The forces in Cartesian vectors

b) The magnitude and direction of the resultant



Solu. :

A (50, 10, -30)

B (50, 50, -30)

C (0, 0, 0)

$\vec{CA} = 50\vec{i} + 10\vec{j} - 30\vec{k}$ (ft) $\Rightarrow$ CA = 59.16 ft

$\vec{CB} = 50\vec{i} + 50\vec{j} - 30\vec{k}$ (ft) $\Rightarrow$ CB = 76.81 ft

a) $\vec{F}_A = F_A \vec{u}_{CA} = F_A \frac{\vec{CA}}{CA}$

$\Rightarrow \vec{F}_A = \frac{300}{59.16}(50\vec{i} + 10\vec{j} - 30\vec{k}) \Rightarrow$ **$\vec{F}_A = 254\vec{i} + 50.7\vec{j} - 152\vec{k}$ (lb)**

$\vec{F}_B = \frac{F_B}{CB}\vec{CB} = \frac{400}{76.81}(50\vec{i} + 50\vec{j} - 30\vec{k}) \Rightarrow$ **$\vec{F}_B = 260\vec{i} + 260\vec{j} - 156\vec{k}$ (lb)**

b) $\vec{R} = \Sigma\vec{F} = \vec{F}_A + \vec{F}_B = (254 + 260)\vec{i} + (50.7 + 260)\vec{j} + (-152 - 156)\vec{k}$

$= 514\vec{i} + 311\vec{j} - 308\vec{k}$ (lb)

Steps :

① Coordinates

② position vec.

③ unit vec.

④ force vec.

⑤ $\Sigma\vec{F} = \vec{R}$

$R = \sqrt{(514)^2 + (311)^2 + (-308)^2} \Rightarrow$ **R = 675 lb**

$\cos\theta_x = \frac{R_x}{R} = \frac{514}{675} \Rightarrow$ **$\theta_x = 40.4°$**

$\cos\theta_y = \frac{R_y}{R} = \frac{311}{675} \Rightarrow$ **$\theta_y = 62.6°$**

$\cos\theta_z = \frac{R_z}{R} = \frac{-308}{675} \Rightarrow$ **$\theta_z = 117°$**

Check

$\cos^2\theta_x + \ldots = 1$

?

Note that when you form the vector $\vec{CA}$, you must take A(x,y,z) − C(x,y,z) not C − A. Also, $\vec{CB}$ = B(x,y,z) − C(x,y,z)

$\vec{DF}$ = F(x,y,z) − D(x,y,z) ..... etc

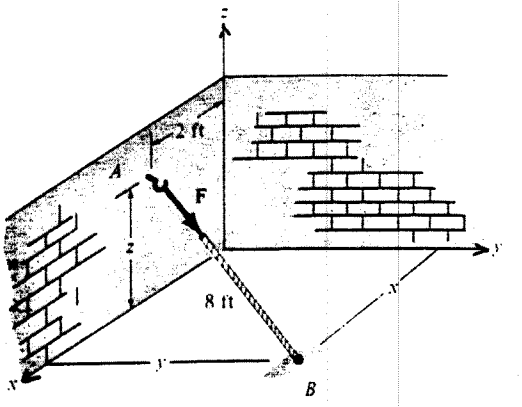
#4

Example 2 :

Given :

The figure shown

$\vec{F} = 12\,\vec{i} + 9\vec{j} - 8\,\vec{k}$ (lb)



Req'd. :

The coordinate x , y , and z

Soln. :

A (2, 0, z)

B (x, y, 0)

Start the solution by making "educational guessing" for the answers !

$$\vec{AB} = (x-2)\,\vec{i} + (y-0)\,\vec{j} + (0-z)\,\vec{k}$$
$$= (x-2)\,\vec{i} + y\,\vec{j} - z\,\vec{k} \quad \text{(ft)}$$

$$\vec{F} = 12\,\vec{i} + 9\vec{j} - 8\,\vec{k} \quad \text{(lb)}$$

$$\Rightarrow F = \sqrt{(12)^2 + 9^2 + (-8)^2} = 17 \text{ lb}$$

$$\vec{u}_F = \frac{\vec{F}}{F} = \frac{12}{17}\,\vec{i} + \frac{9}{17}\,\vec{j} - \frac{8}{17}\,\vec{k} = \vec{u}_{AB} \quad \text{(same line F // AB)}$$

$$\vec{u}_{AB} = \frac{\vec{AB}}{AB} \Rightarrow \vec{AB} = AB\,\vec{u}_{AB} \Rightarrow$$

$$(x-2)\,\vec{i} + y\,\vec{j} - z\,\vec{k} = 8\left(\frac{12}{17}\,\vec{i} + \frac{9}{17}\,\vec{j} - \frac{8}{17}\,\vec{k}\right)$$
$$= 5.647\,\vec{i} + 4.235\,\vec{j} - 3.7647\,\vec{k}$$

$\Rightarrow \quad x - 2 = 5.647 \Rightarrow$ **$x = 7.65$ ft**

$y = 4.235 \Rightarrow$ **$y = 4.24$ ft**

$-z = -3.7647 \Rightarrow$ **$z = 3.76$ ft**

Are the answers "reasonable" ? Compare with your guessing !