#3

Examples

Cartesian Vectors in 3-D

Example 1:

Given:

The force vector $\vec{F} = 700\,\vec{i} - 820\,\vec{j} + 960\,\vec{k}$ (N)

Req'd.:

The magnitude and direction cosines of $\vec{F}$

Soln.:

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(700)^2 + (-820)^2 + (960)^2} \Rightarrow \boxed{F = 1444 \text{ N}}$$

$$\cos\theta_x = \cos\alpha = \frac{F_x}{F} = \frac{700}{1444} \Rightarrow \boxed{\theta_x = 60.1^\circ}$$

$$\cos\theta_y = \cos\beta = \frac{F_y}{F} = \frac{-820}{1444} \Rightarrow \boxed{\theta_y = 124.1^\circ}$$

$$\cos\theta_z = \cos\gamma = \frac{F_z}{F} = \frac{960}{1444} \Rightarrow \boxed{\theta_z = 48.3^\circ}$$

Check the answers!

Example 2:

Given:

a) $\vec{F}$ forms equal angles with $+x$, $+y$, $+z$

b) $\vec{F}$ forms equal angles with $+y$, $+z$, and 45° with $+x$

Req'd.:

θ_y if each case a and b

Soln.:

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

a) Since angles are equal, then $\theta_x = \theta_y = \theta_z \Rightarrow$

$$3\cos^2\theta_y = 1$$

$$\Rightarrow \cos\theta_y = \pm\sqrt{1/3} \Rightarrow \boxed{\theta_y = 54.7^\circ \text{ or } 125.3^\circ}$$

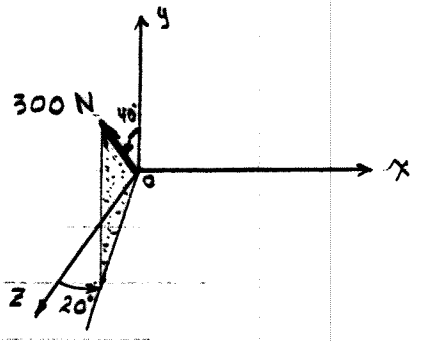
b) $\cos^2(45^\circ) + \cos^2\theta_y + \cos^2\theta_z = 1$

But $\theta_y = \theta_z \Rightarrow$

$$2\cos^2\theta_y = 1 - \cos^2 45^\circ \Rightarrow \boxed{\theta_y = 60^\circ \text{ or } 120^\circ}$$

#3

Example 3:



Given:

The figure shown

Req'd.:

a) The x, y, and z components of the force

b) The angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes

Solu.:

$F_x = (300 \sin 40^\circ) \sin 20^\circ \Rightarrow$ **$F_x = 66.0$ N**

$F_y = 300 \cos 40^\circ \Rightarrow$ **$F_y = 230$ N**

$F_z = (300 \sin 40^\circ) \cos 20^\circ \Rightarrow$ **$F_z = 181$ N**

$\cos \theta_x = \frac{F_x}{F} = \frac{66.0}{300} \Rightarrow$ **$\theta_x = 77.3^\circ$**

$\cos \theta_y = \frac{F_y}{F} = \frac{230}{300} \Rightarrow$ **$\theta_y = 40^\circ$** (given)

$\cos \theta_z = \frac{F_z}{F} = \frac{181}{300} \Rightarrow$ **$\theta_z = 52.9^\circ$**

Note that all the angles are positive and less than 90° because the force is in the first octant (all positive).

Check the angles:

$\cos^2 77.3^\circ + \cos^2 40^\circ + \cos^2 52.9^\circ = 1$ ok

Are the force components reasonable? Explain!