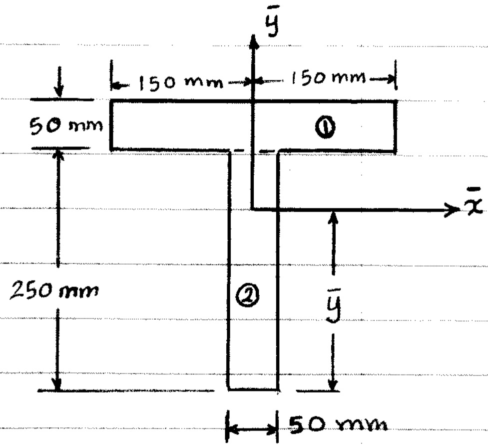
#26

Examples
Moments of Inertia
Composite Areas Method

Example 1:



Given:

The T-section shown

Req.d.:

$\bar{I}_{\bar{x}}$ and $\bar{I}_{\bar{y}}$

Sol.n.:

First, $\bar{y}$ is needed. ⇒

$(300\times 50 + 250\times 50)\bar{y} = 300(50)(275) + 250(50)(125)$

$\Rightarrow \bar{y} = 206.8$ mm

$\bar{I}_{\bar{x}①} = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(300)(50)^3 + 300(50)(275-206.8)^2 = 7.289(10)^7$ mm^4

$\bar{I}_{\bar{x}②} = \frac{1}{12}(50)(250)^3 + 50(250)(125-206.8)^2 = 1.487(10)^8$ mm^4

$\bar{I}_{\bar{x}} = \bar{I}_{\bar{x}①} + \bar{I}_{\bar{x}②}$

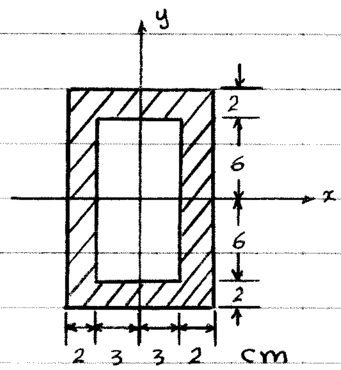
$= 7.289(10)^7 + 1.487(10)^8 \Rightarrow$ $\boxed{\bar{I}_{\bar{x}} = 2.22(10)^8 \text{ mm}^4}$

$\bar{I}_{\bar{y}} = \bar{I}_{\bar{y}①} + \bar{I}_{\bar{y}②}$

$= \frac{1}{12}(50)(300)^3 + \frac{1}{12}(250)(50)^3 \Rightarrow$ $\boxed{\bar{I}_{\bar{y}} = 1.15(10)^8 \text{ mm}^4}$

no (Ad^2); why?!

Example 2:



Given:

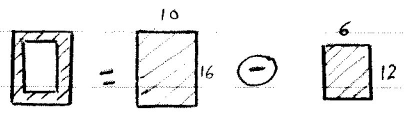
The shaded area shown

Req.d.:

I_x and I_y

Sol.n.:

Method ①

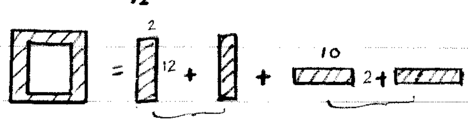


$I_x = \frac{1}{12}(10)(16)^3 - \frac{1}{12}(6)(12)^3 \Rightarrow$ $\boxed{I_x = 2550 \text{ cm}^4}$

$I_y = \frac{1}{12}(16)(10)^3 - \frac{1}{12}(12)(6)^3 \Rightarrow$ $\boxed{I_y = 1120 \text{ cm}^4}$

Note that x and y are centroidal axes (double symmetry)

Method ②

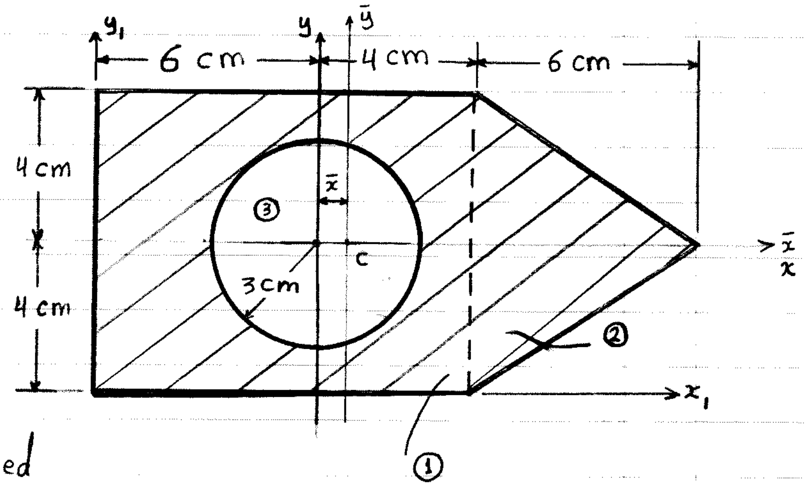


$I_x = 2\left[\frac{1}{12}(2)(12)^3\right] + 2\left[\frac{1}{12}(10)(2)^3 + 2(10)(7)^2\right] = 2550$ cm^4 as above

$I_y = 2\left[\frac{1}{12}(12)(2)^3 + 2(12)(4)^2\right] + 2\left[\frac{1}{12}(2)(10)^3\right] = 1120$ cm^4 as above

Example 3:

Given :
The shaded area shown



Req.d. :
$\bar{I}_{\bar{y}}$

Soln. :
First, the centroid is needed.
Using the method discussed earlier, the centroid can be found. $\Rightarrow$ $\bar{x} = 0.84$ cm , $\bar{y} = 0$

Since the area is composed of several segments, it is easier to use a table for the solution as shown below.

Note that $\bar{I}_{\bar{y}} = \bar{I}_{\bar{y}_i} + A_i d_{x_i}^2$ for each segment

* important: d is the distance between the centroidal axis of the segment and the axis of interest (which might be the centroidal axis of the composite area).

Component	$\bar{I}_{\bar{y}_i}$ (cm^4)	A (cm^2)	$\lvert d_x \rvert$ (cm)	Ad_x^2 (cm^4)
①	$\frac{1}{12}(8)(10)^3 = 667$	80	1.84	271
②	$\frac{1}{36}(8)(6)^3 = 48$	24	5.16	639
③	$-\frac{1}{4}\pi(3)^4 = -63.6$	-9π	0.84	-20
Σ	651.4			890

$$\bar{I}_{\bar{y}} = \sum_{i=1}^{n}\left[\bar{I}_{\bar{y}_i} + (Ad^2)_i\right] = 651. + 890 \Rightarrow \boxed{\bar{I}_{\bar{y}} = 1541 \text{ cm}^4}$$

Note that area ③ is negative
Recall that for a

rectangle : $\bar{I} = \frac{1}{12} bh^3$

triangle : $\bar{I} = \frac{1}{36} bh^3$

circle : $\bar{I} = \frac{\pi}{4} r^4$

Imp. note : If I is needed for any axis other than the centroidal axis, then no need to locate the centroid for the composite area. Try I_{x_1} and I_{y_1} above.

These formulas are for the moments of inertia of each about their centroidal axes (for each).