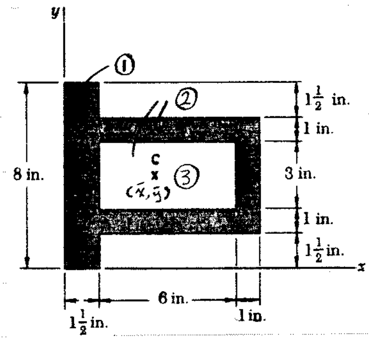
\# 24

Examples
Centroid
Composite Body Method

Example 1:



Given:
The area shown

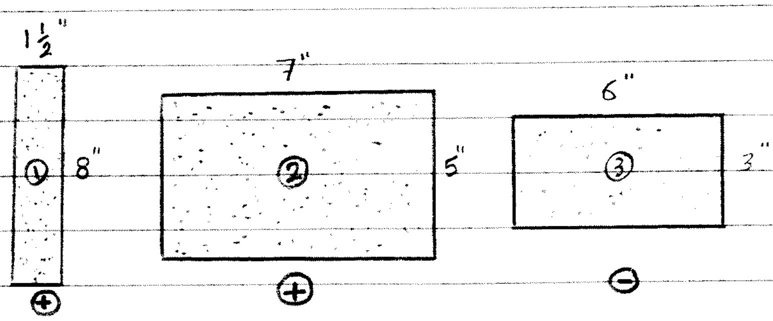
Reqd:
The centroid

Soln.: « Guess the final answers! »

From symmetry about a line parallel to the x-axis,

$$\boxed{\bar{y} = 4 \text{ in}}$$

The area is divided into 3 segments as shown. Note that area ③ is negative.



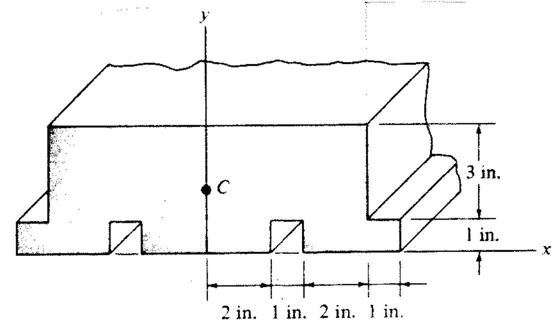
Area #	A (in²)	$\bar{x}$ (in)	$\bar{x}A$ (in³)
①	12	0.75	9
②	35	5	175
③	− 18	4.5	− 81
Σ	29		103

$$\bar{X} = \frac{\sum_{i=1}^{n} \bar{x}_i A_i}{\sum_{i=1}^{n} A_i} = \frac{103}{29} \Rightarrow \boxed{\bar{X} = 3.55 \text{ in}} \Leftarrow$$

Reasonable answer?!

Imp.: $\bar{x}_i$ is the distance between the centroid of segment (area) i and the reference axis

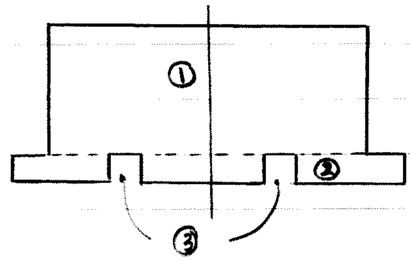
Example 2 :



Given :
The figure shown

Req.d. :
The centroid

Soln. :
From symmetry, $\boxed{\bar{x} = 0}$



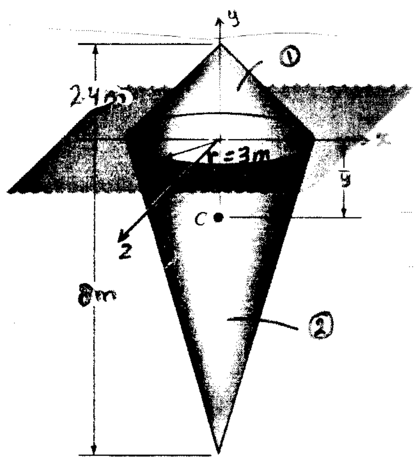
Area #	A^* (in^2)	$\bar{y}$ (in)	$\bar{y}A$ (in^3)
①	30	2.5	75
②	12	0.5	6
③	-2	0.5	-1
Σ	40		80

* You may work with half of area (sym.)

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{80}{40} \Rightarrow \boxed{\bar{y} = 2 \text{ in}}$$

⇐ As expected !!

Example 3 :



Given :
The volume shown composed of two cones having a common radius of 3 m

Req.d :
The centroid

Soln. :
From symmetry, $\boxed{\bar{x} = \bar{z} = 0}$

$$\bar{y} = \frac{\sum_{i=1}^{2} \bar{y}_i V_i}{\sum_{i=1}^{2} V_i}$$

$$= \frac{\left[\frac{2.4}{4}\left(\frac{\pi}{3}\right)3^2(2.4)\right] + \left[\frac{-8}{4}\left(\frac{\pi}{3}\right)3^2(8)\right]}{\left[\left(\frac{\pi}{3}\right)3^2(2.4)\right] + \left[\left(\frac{\pi}{3}\right)3^2(8)\right]} \Rightarrow \boxed{\bar{y} = -1.4 \text{ m}}$$

⇐ Seems OK ?

Note that no need for a table since we have two segments only

Example 4 :

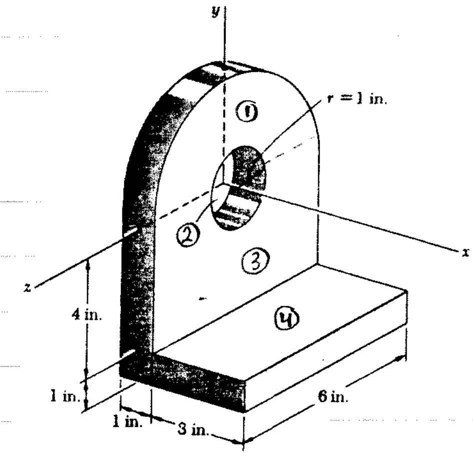
Given :

The figure shown

Req'd. :

The centroid

Sol'n :



xy plane is a plane of symmetry

$\Rightarrow \boxed{\bar{z} = 0}$

The volume will be divided into 4 segments as shown.
Note that segment ① is a half cylinder. ② is negative

Segment	Volume (in^3)	$\bar{x}$ (in)	$\bar{y}$ (in)	$\bar{x}V$ (in^4)	$\bar{y}V$ (in^4)
①	$\frac{1}{2}(\pi)(3)^2(1) = 14.14$	0	$\frac{4(3)}{3\pi} = 1.273$	0	18
②	$-\pi(1)^2 1 = -3.14$	0	0	0	0
③	$4(6)(1) = 24$	0	-2	0	-48
④	$6(4)(1) = 24$	1.5	-4.5	36	-108
Σ	59			36	-138

$$\bar{x} = \frac{\sum_{i=1}^{4} \bar{x}_i V_i}{\sum_{i=1}^{4} V_i} = \frac{36}{59} \Rightarrow \boxed{\bar{x} = 0.610 \text{ in}}$$

$$\bar{y} = \frac{\sum_{i=1}^{4} \bar{y}_i V_i}{\sum_{i=1}^{4} V_i} = \frac{-138}{59} \Rightarrow \boxed{\bar{y} = -2.34 \text{ in}}$$

Are the answers "reasonable" ? Why ? Explain !