#22

Examples
Friction

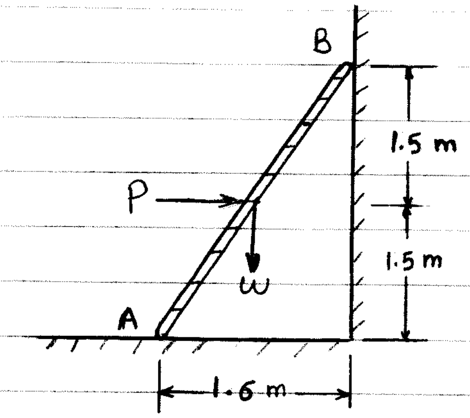
Example 1:

Given :

The uniform ladder shown

$W = 100$ N

$\mu_s = 0.20$ @ A and B



Req'd. :

minimum P for equilibrium

Sol.n. :

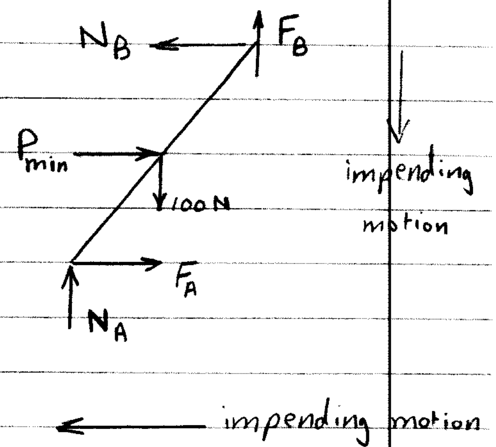
In the FBD shown,

$F_A = \mu_s^A N_A = 0.2\ N_A$

$F_B = \mu_s^B N_B = 0.2\ N_B$

(Note that $F = \mu N$ because of $P_{min} \Rightarrow$ impending motion)

Notice the direction of F because of the direction of impending motion.



$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$P_{min} + F_A - N_B = 0$

$P_{min} + 0.2\ N_A - N_B = 0$ ①

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$N_A + F_B - 100 = 0$

$N_A + 0.2\ N_B - 100 = 0$ ②

$\circlearrowleft + \Sigma M_B = 0 \Rightarrow$

$1.5\ P_{min} + 100(0.8) - 1.6\ N_A + 3\ F_A = 0$

$1.5\ P_{min} - 1.6\ N_A + 0.6\ N_A + 80 = 0$

$1.5\ P_{min} - N_A + 80 = 0$ ③

From ③, $N_A = 1.5\ P_{min} + 80$ ④

From ①, $N_B = P_{min} + 0.2\ N_A = P_{min} + 0.2(1.5\ P_{min} + 80)$ ⑤

From ④ and ⑤ into ② $\Rightarrow$

$(1.5\ P_{min} + 80) + 0.2(P_{min} + 0.3\ P_{min} + 16) - 100 = 0 \Rightarrow$ $\boxed{P_{min} = 9.55 \text{ N} \rightarrow}$

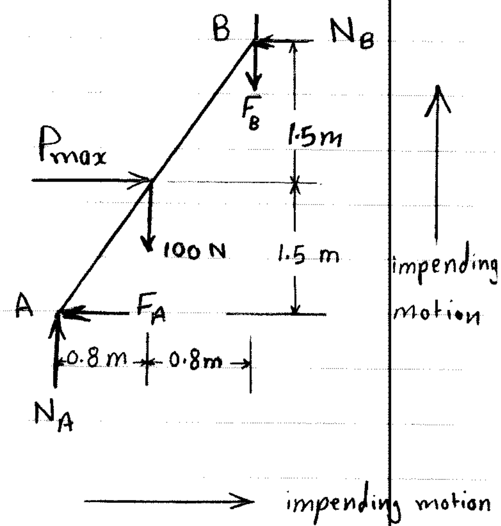
Example 2 :

Given :

As in Example 1 above

Reqd. :

maximum force P_{max} which can be applied with equilibrium still maintained.



Soln. :

The "major" difference between this problem and the previous on is that impending motion will be in the opposite direction because of max. P. ⇒ Friction forces are in the opposite directions as shown in the FBD.

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$P_{max} - F_A - N_B = 0$

$P_{max} - 0.2\, N_A - N_B = 0$ ①

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$N_A - F_B - 100 = 0$

$N_A - 0.2\, N_B - 100 = 0$ ②

$+\circlearrowleft \Sigma M_B = 0 \Rightarrow$

$1.5\, P_{max} - 1.6\, N_A - 3\, F_A + 100(0.8) = 0$

$1.5\, P_{max} - 1.6\, N_A - 0.6\, N_A + 80 = 0$

$1.5\, P_{max} - 2.2\, N_A + 80 = 0$ ③

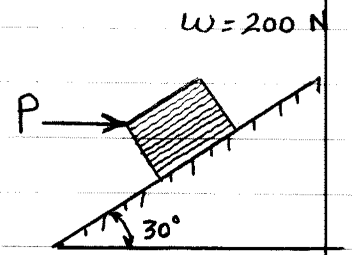
Solving the three eqs. above gives $\boxed{P_{max} = 122.2 \text{ N} \rightarrow}$

⇒ From Examples 1 and 2 , $9.55 \leq P \leq 122.2$ N

What can you conclude if $P_{min} \leq 0$?

What is P_{max} (theoratically) if end A or B is hinged ?

Example 3:



Given:

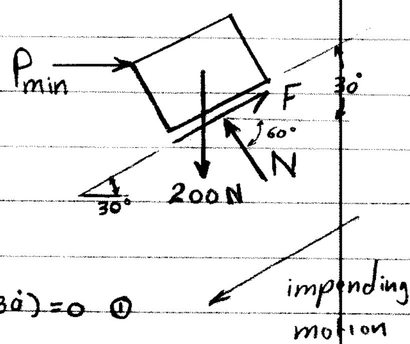
The crate resting on the incline shown

$\mu_s = 0.3$ between the crate and incline

Reqd.:

The range of P for which the crate will be in statical equilibrium.

Soln.:



First, calculate P_{min} to prevent the crate from sliding down. $\Rightarrow$ FBD ①

$\xrightarrow{+} \Sigma F_x = 0$ (x may be chosen along the incline)

$P_{min} - N \cos 60° + F \cos 30° = P_{min} + N(-\cos 60° + 0.3 \cos 30°) = 0$ ①

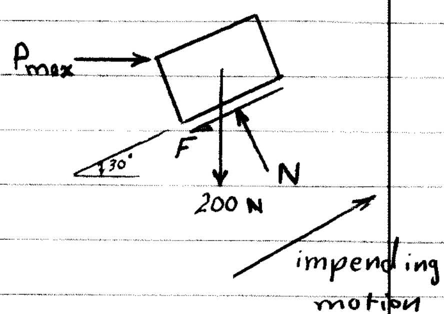
$+\uparrow \Sigma F_y = 0 \Rightarrow$

$-200 + N \sin 60° + F \sin 30° = 0$

$-200 + N \sin 60° + 0.3 N \sin 30° = 0$ ②

$\Rightarrow N = 200/(\sin 60 + 0.3 \sin 30°) = 196.8$ N

$\Rightarrow$ From ①, $P_{min} = -196.8(-\cos 60° + 0.3 \cos 30°) = 47.3$ N



Second, determine P_{max} which is the force needed to just start the crate to move up.

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$P_{max} - N \cos 60° - F \cos 30° = 0$

$P_{max} + N(-\cos 60° - 0.3 \cos 30°) = 0$ ③

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$-200 + N \sin 60° - F \sin 30° = 0$

$-200 + N(\sin 60° - 0.3 \sin 30°) = 0$ ④

$\Rightarrow N = 200/(\sin 60° - 0.3 \sin 30°) = 279.3$ N

$\Rightarrow$ From ③,

$P_{max} = -279.3(-\cos 60° - 0.3 \cos 30°) = 212$ N

Thus, the crate will be in stable statical equilibrium for values of P satisfying the following

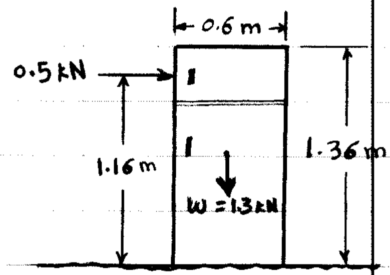
$$\boxed{47.3 < P < 212 \text{ N}}$$

Example 4 :

Given :

The load applied to the refrigerator shown

$\mu_s = 0.4$



Req'd. :

Can the refrigerator be in statical equilibrium?

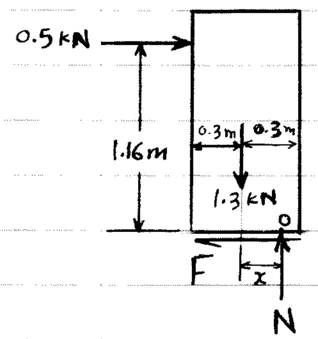
Soln. :

Two "motions" are possible :

sliding (slipping) and tipping.

First, check sliding :

In the FBD shown,



$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$0.5 - F = 0 \Rightarrow F = 0.5 \text{ kN}$

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$N - 1.3 = 0 \Rightarrow N = 1.3 \text{ kN}$

maximum friction force $= F_m$

$F_m = \mu_s N = 0.4(1.3) = 0.52 \text{ kN}$

Since $F = 0.5 \text{ kN} < F_m = 0.52$,

the refrigerator will not slide

Second, check tipping :

$\circlearrowleft + \Sigma M_o = 0$

(o is the point where the resultant of the normal force N applies.)

$\Rightarrow 1.3x - 1.16(0.5) = 0$

$\Rightarrow x = 0.446 \text{ m} > 0.3 = \frac{b}{2}$

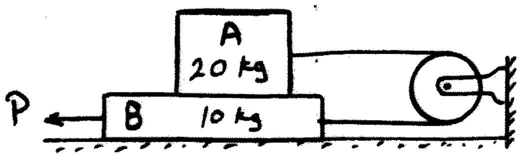
$\Rightarrow$ impossible

Since $x = 0.446 \text{ m} > 0.3 \text{ m}$, the refrigerator will tip.

Thus,

$\boxed{\text{The refrigerator can not be in statical equilibrium as it will tip.}}$

Example 5 :



Given :

The figure shown ; $\mu = 0.20$ at all surfaces of contact

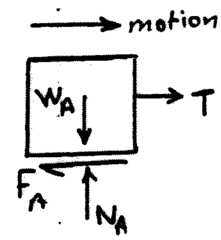
Req'd. :

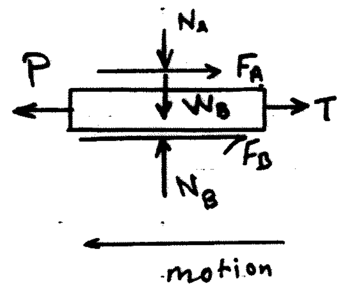
The fore P required to move block B to the left

Sol'n. :

The pulley is assumed "smooth" (frictionless).

Draw FBD's for A and B separately. (Why?)





Block A

$+\uparrow \Sigma F_y = 0 \Rightarrow \quad N_A = 20\,(9.81)$

$= 196.2 \text{ N}$

Since we are looking for P required to move B to the left, A must also move to the right. $\Rightarrow$ "impending" motion

$\Rightarrow \quad F_A = \mu N_A = 0.2\,(196.2)$

$= 39.24 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow \quad T = 39.24 \text{ N}$

Block B

In the FBD, N_A and F_A are equal and opposite of those on A

$+\uparrow \Sigma F_y = 0 \Rightarrow \quad N_B - N_A - W_B = 0$

$\Rightarrow \quad N_B = 196.2 + 10\,(9.81)$

$= 294.3 \text{ N}$

"impending" motion $\Rightarrow \quad F_B = \mu N_B$

$= 0.2\,(294.3) = 58.86 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow$

$T + F_A + F_B - P = 0$

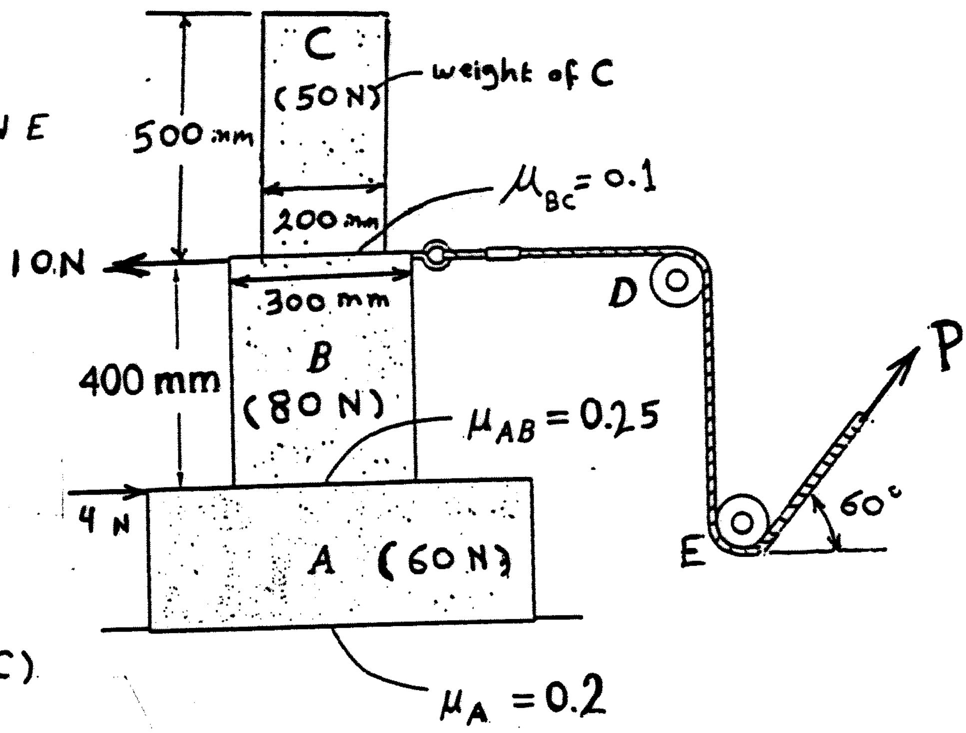
$\Rightarrow P = 39.24 + 39.24 + 58.86$

$\Rightarrow \boxed{P = 137.34 \text{ N}}$

⊛ Rework the problem but without the cable and pulley.

$\Rightarrow$??

Example 6 :



Given:

The figure shown

Smooth pulleys at D and E

Req.d :

The largest force P which can be applied without causing any kind of motion

Soln. :

1. C will not slide or tip alone. (Why ?)

2. Sliding of B (with C)

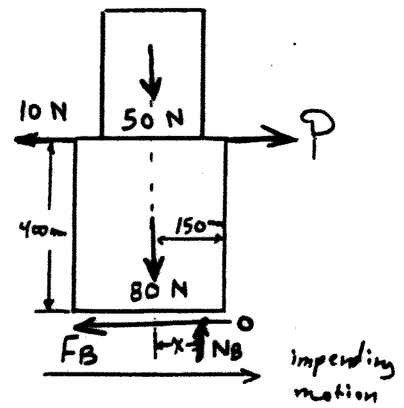
FBD ① is drawn.

What is the effect of the 60°-angle ?!

$+\uparrow \Sigma F_y = 0 \Rightarrow N_B = 50 + 80 = 130 \text{ N}$

$F_B \equiv F_{max} = \mu N_B = 0.25\,(130) = 32.5 \text{ N}$ « Why ? »

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow P_① - 10 - 32.5 = 0 \Rightarrow \underline{P_① = 42.5 \text{ N}}$



FBD ①

3. Tipping of B (with C) : FBD ①

$x \equiv 150 \text{ mm}$ « Why ? »

$\circlearrowleft \Sigma M_o = 0 \Rightarrow 10(400) - 400\,P_② + 130\,(150)$

$\Rightarrow \underline{P_② = 58.75 \text{ N}}$

4. Sliding of A (with B and C)

FBD ② is drawn.

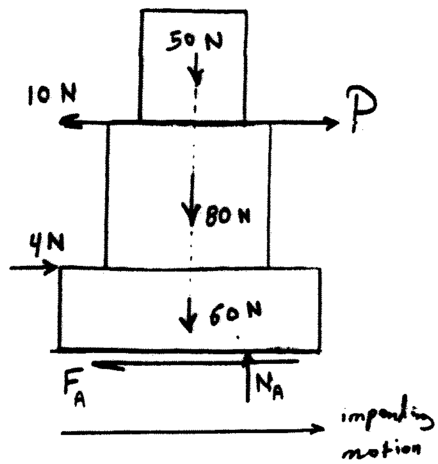
$+\uparrow \Sigma F_y = 0 \Rightarrow N_A = 50 + 80 + 60 = 190 \text{ N}$

$F_A \equiv F_{max} = \mu N_A = 0.2\,(190) = 38 \text{ N}$ « Why ? »

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow P_③ - 10 - 38 + 4 = 0$

$\Rightarrow \underline{P_③ = 44 \text{ N}}$

Tipping of A is not considered. (Why ?!)



* Note that other motions (e.g., moving to the left ←) are not possible. Why ??!!

$\Rightarrow P_{max} = \min(P_①, P_②, P_③)$ {Why ?!!} $\Rightarrow \boxed{P_{max} = 42.5 \text{ N}}$