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Examples

Shear and Moment Equations and Diagrams

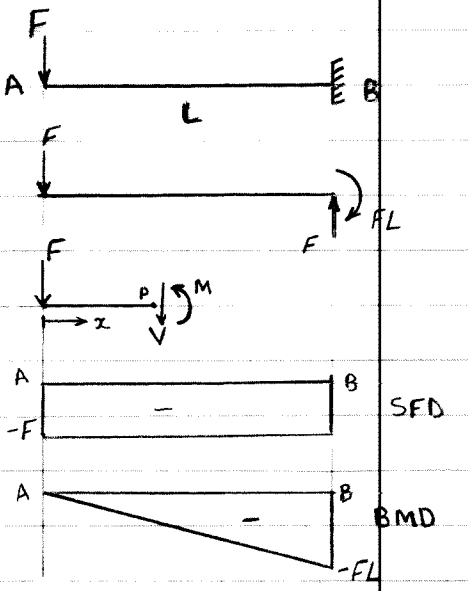
Example 1:

Given:

The beam shown

Req'd.:

The shear force and bending moment equations and diagrams.



Soln.:

The beam can be taken as one segment ⇒ one section.

$0 \le x \le L$

In the FBD shown, $+\uparrow \Sigma F_y = 0 \Rightarrow \boxed{V = -F}$

$+\circlearrowleft \Sigma M_P = 0 \Rightarrow$

$M + Fx = 0 \Rightarrow \boxed{M = -Fx}$

The shear force diagram (SFD) and the bending moment diagram (BMD) are shown in the figure above.

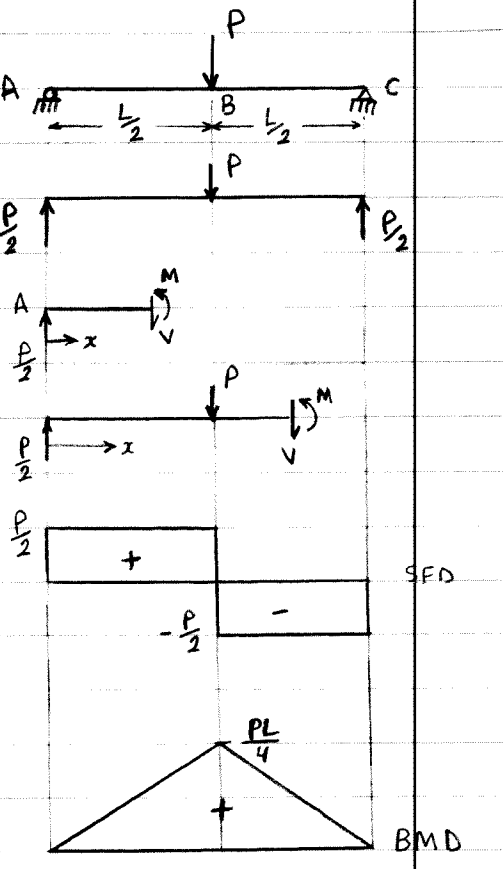
Example 2:

Given:

The beam shown

Req'd.:

SFD & BMD



Soln.:

The reactions are shown.

We need two sections.

AB $(0 \le x \le \frac{L}{2})$

In the FBD shown, $+\uparrow \Sigma F_y = 0 \Rightarrow V = \frac{P}{2}$

$+\circlearrowleft \Sigma M = 0 \Rightarrow M = \frac{P}{2} x$

BC $(\frac{L}{2} \le x \le L)$

In the FBD shown, $+\uparrow \Sigma F_y = 0 \Rightarrow V = -\frac{P}{2}$

$+\circlearrowleft \Sigma M = 0 \Rightarrow M = \frac{P}{2} x - P(x - \frac{L}{2}) = \frac{P}{2}(L - x)$

The SFD & BMD are shown.

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Example 3 :

Given :

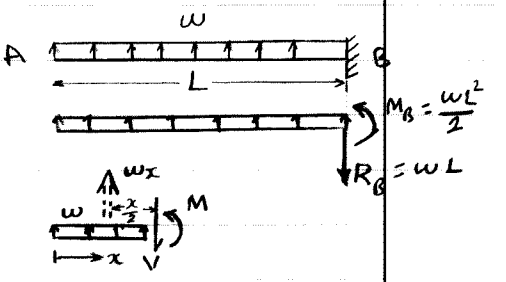
The beam shown

Reqd. :

SFD & BMD

Soln. :

Only one section is needed. Reactions are shown

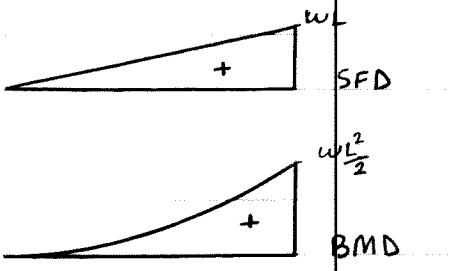


AB $(0 \le x \le L)$

In the FBD shown, $+\uparrow \Sigma F_y = 0 \Rightarrow$ $V = wx$

$+\circlearrowleft \Sigma M = 0 \Rightarrow$ $M = wx(\frac{x}{2}) = \frac{wx^2}{2}$

SFD & BMD are shown.



Example 4 :

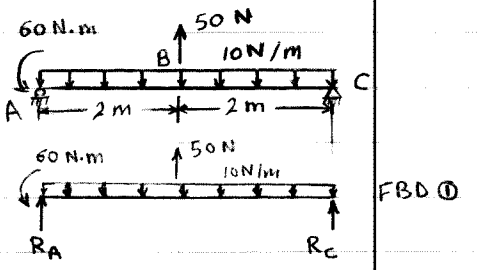
Given :

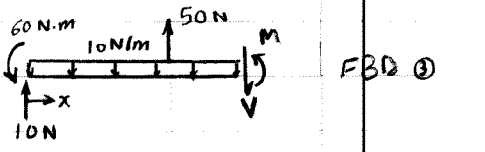
The beam shown

Reqd. :

SFD & BMD

Soln. :





The reactions are first calculated from FBD ①

$+\circlearrowleft \Sigma M_C = 0 = 60 - 50(2) + 10(4)(2) - 4R_A \Rightarrow R_A = 10\ N \uparrow$

$+\uparrow \Sigma F_y = 0 = R_C + 50 - 10(4) + 10 \Rightarrow R_C = -20\ N = 20\ N \downarrow$

Two sections are needed :

AB $(0 \le x \le 2m)$

In FBD ②, $+\uparrow \Sigma F_y = 0 \Rightarrow$

$10 - 10x - V = 0 \Rightarrow$ $V = 10 - 10x = 10(1-x)$

$+\circlearrowleft \Sigma M = 0 = M - 10x + 10x(\frac{x}{2}) + 60 \Rightarrow$ $M = -60 + 10x - 5x^2$

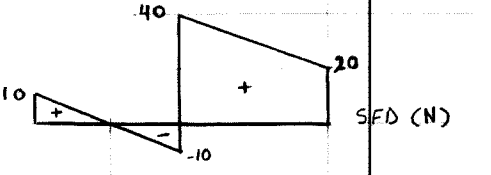
BC $(2 \le x \le 4\ m)$

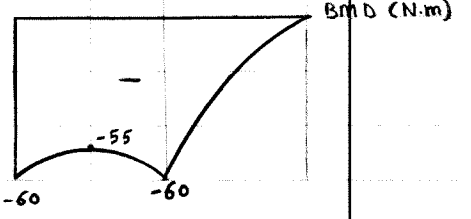
In FBD ③, $+\uparrow \Sigma F_y = 0 \Rightarrow$

$10 - 10x + 50 - V = 0 \Rightarrow$ $V = 60 - 10x = 10(6-x)$

$+\circlearrowleft \Sigma M = 0 \Rightarrow$

$M + 60 - 10x - 50(x-2) + 10x(\frac{x}{2}) = 0 \Rightarrow$ $M = -160 + 60x - 5x^2$





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Example 5 :

Given :

The beam shown

Reqd. :

SFD & BMD

Soln. :

First, reactions are needed.

In FBD ①, $+\circlearrowleft \Sigma M_c = 0 = 75(5) - 3R_B + 90(1)$

$\Rightarrow R_B = -95\ N = 95\ N \downarrow$

In FBD ②, $+\uparrow \Sigma F_y = 0 \Rightarrow R_E = 110\ N \uparrow$

$+\circlearrowleft \Sigma M_E = 0 = -75(10) + 95(8) + 90(6) - 200 + M_E$

$\Rightarrow M_E = 350\ N\cdot m \circlearrowleft$

AB ($0 \le x \le 2\ m$) : FBD ③

$+\uparrow \Sigma F_y = 0 \Rightarrow \underline{V = 75\ N}$

$+\circlearrowleft \Sigma M = 0 \Rightarrow \underline{M = 75x}$

BC ($2 \le x \le 5\ m$) : FBD ④

Note that if there is a distributed load on the segment, then one should first "cut"; after that an equivalent concentrated load can be placed at the centroid.

$+\uparrow \Sigma F_y = 0 = 75 - 95 - 20(x-2)\left(\frac{x-2}{2}\right) - V$

$\Rightarrow \underline{V = -20 - 10(x-2)^2}$

$+\circlearrowleft \Sigma M = 0 = -75x + 95(x-2) + 20(x-2)\left(\frac{x-2}{2}\right)\left(\frac{x-2}{3}\right) + M$

$\Rightarrow \underline{M = 190 - 20x - \frac{10}{3}(x-2)^3}$

CD ($5 \le x \le 9\ m$) : FBD ⑤

$+\uparrow \Sigma F_y = 0 \Rightarrow \underline{V = -110\ N}$

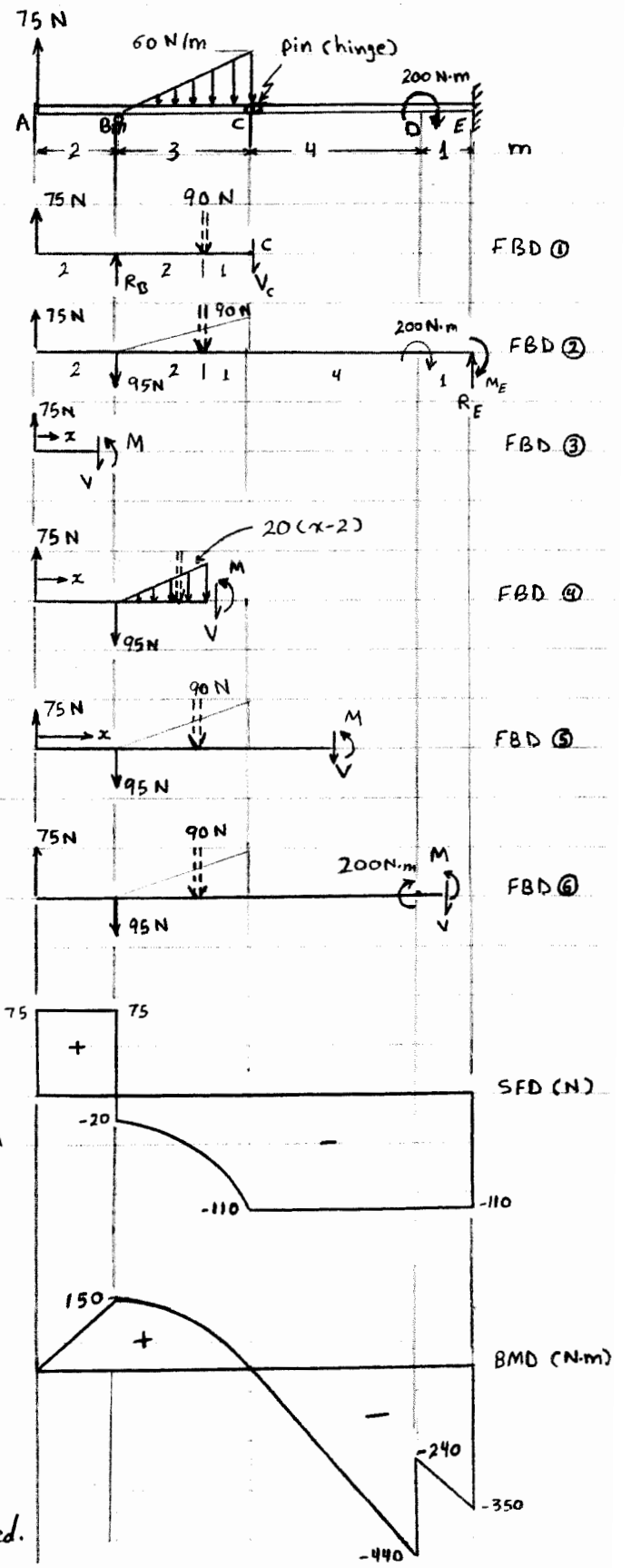
$+\circlearrowleft \Sigma M = 0 \Rightarrow \underline{M = 550 - 110x}$

DE ($9 \le x \le 10\ m$) : FBD ⑥

$+\uparrow \Sigma F_y = 0 \Rightarrow \underline{V = -110\ N}$

$+\circlearrowleft \Sigma M = 0 \Rightarrow \underline{M = 750 - 110x}$

Note that in this problem 4 sections are needed.



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Example 6 :

Given :

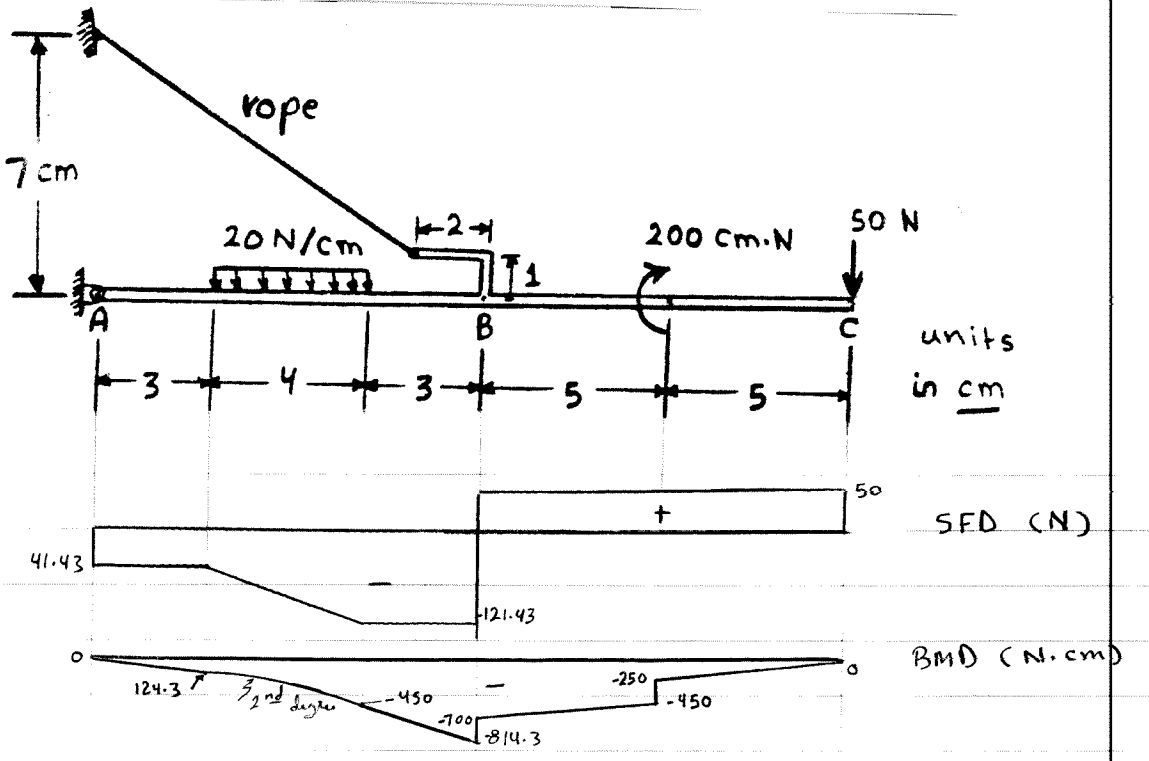
The beam shown below

Req.d. :

SFD and BMD

Soln. :

The diagrams are drawn below. Verify that by drawing FBD's and writting eqs.



Example 7 :

As in example 6