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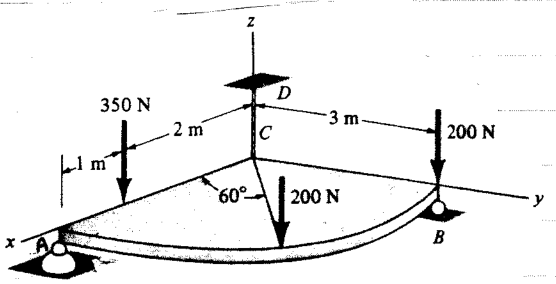
Examples
Equilibrium of Rigid Bodies in 3-D

Example 1:



Given:

The quarter circular plate with the loading and support conditions shown

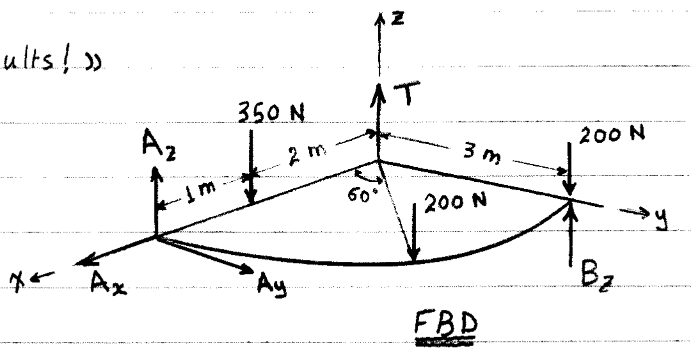
Req.d.:

The reactions

Sol.n.: « Think about the expected results! »

I) Vector Analysis:

From the FBD shown, $\Sigma \vec{F} = \vec{0}$



$\Sigma F_x = 0 \Rightarrow \boxed{A_x = 0}$

$\Sigma F_y = 0 \Rightarrow \boxed{A_y = 0}$

$$\Sigma \vec{M}_B = \vec{0} \Rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -3 & 0 \\ 0 & 0 & T \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -3 & 0 \\ 0 & 0 & A_z \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 0 \\ 0 & 0 & -350 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & -0.4019 & 0 \\ 0 & 0 & -200 \end{vmatrix} = \vec{0}$$

$\Rightarrow -3T\vec{i} - 3A_z\vec{i} - 3A_z\vec{j} + 1050\vec{i} + 700\vec{j} + 80.38\vec{i} + 300\vec{j} = \vec{0}$

$\Rightarrow (-3T - 3A_z + 1050 + 80.38)\vec{i} + (-3A_z + 700 + 300)\vec{j} + 0\vec{k} = \vec{0}$

$\Rightarrow -3T - 3A_z + 1130.38 = 0$

$-3A_z + 1000 = 0 \Rightarrow \boxed{A_z = 333\ N} \Rightarrow \boxed{T = 43.5\ N}$

$\Sigma F_z = 0 \Rightarrow$

$43.46 - 350 - 200 - 200 + 333.3 + B_z = 0 \Rightarrow \boxed{B_z = 373\ N}$

{{ Answers ok ?! }}

⊗ Note that $\Sigma M_z = 0$ is trivial $\Rightarrow$

⊗ only $\underline{\underline{5}}$ unknowns (A_x, A_y, A_z, B_z, T) and $\underline{\underline{5}}$ "usable" eqs. ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$, $\Sigma M_x = 0$, $\Sigma M_y = 0$).

II) Scalar Analysis:

$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma F_y = 0 \Rightarrow A_y = 0$

$\circlearrowleft \Sigma M_{x\text{-axis}} = 0 \Rightarrow -200(3 \sin 60°) - 200(3) + 3B_z = 0 \Rightarrow B_z = 373\ N$

$\circlearrowleft \Sigma M_{y\text{-axis}} = 0 \Rightarrow 350(2) + 200(3 \cos 60°) - 3A_z = 0 \Rightarrow A_z = 333\ N$

$\Sigma F_z = 0 \Rightarrow 373 - 350 - 200 - 200 + 333.3 + T = 0 \Rightarrow T = 43.5\ N$

Note that in this problem it is easier to use scalar anlaysis; however, in most 3-D problems it is harder to determine the scalar quanities (moment arms, signs, ... etc.).

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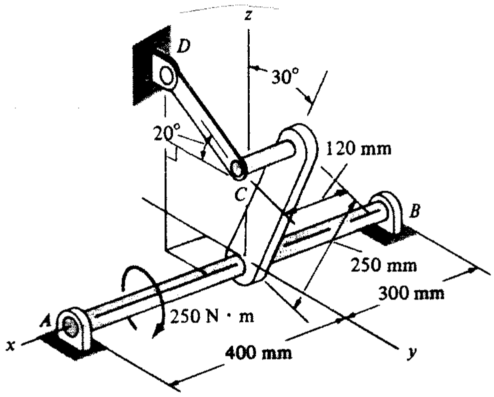
Example 2 :

Given :

The shaft assembly shown

Smooth journal bearings at A and B which are properly aligned

The short link DC lines in a plane parallel to the y-z plane.



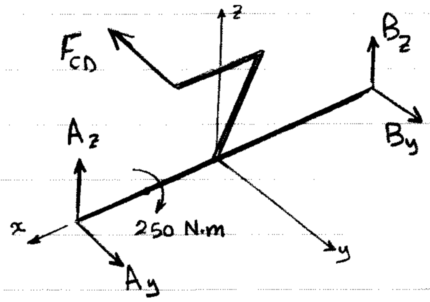
Reqd. :

The reactions

Soln. :

From the FBD shown, there are $\underline{5}$ unknowns (A_y, A_z, B_y, B_z, F_{CD}) and $\underline{5}$ eq. ($\Sigma F_x = 0$ is useless here).

FBD



Dimensions as above

$$\Sigma \vec{M}_B = \vec{0} \Rightarrow -250\,\vec{i} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.7 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.42 & 0.125 & 0.2165 \\ 0 & -F_{CD}\cos 20^\circ & F_{CD}\sin 20^\circ \end{vmatrix} = \vec{0}$$

$$-250\vec{i} + (-0.7A_z\vec{j} + 0.7A_y\vec{k}) + (0.125\,F_{CD}\sin 20^\circ\,\vec{i} + 0.2165\,F_{CD}\cos 20^\circ\,\vec{i} - 0.42\,F_{CD}\sin 20^\circ\,\vec{j} - 0.42\,F_{CD}\cos 20^\circ\,\vec{k}) = \vec{0}$$

$$\Rightarrow \Sigma M_x = 0 \Rightarrow -250 + 0.125\,F_{CD}\sin 20^\circ + 0.2165\,F_{CD}\cos 20^\circ = 0$$

$$\Rightarrow \boxed{F_{CD} = R_D = 1015\text{ N} \simeq 1.02\text{ kN}}$$

$\Sigma M_y = 0 \Rightarrow$

$$-0.7A_z - 0.42(1015\sin 20^\circ) = 0 \Rightarrow \boxed{A_z = -208\text{ N} = 208\text{ N} \downarrow}$$

$\Sigma M_z = 0 \Rightarrow$

$$0.7A_y - 0.42(1015\cos 20^\circ) = 0 \Rightarrow \boxed{A_y = 572\text{ N as shown}}$$

$\Sigma F_y = 0 \Rightarrow$

$$572 - 1015\cos 20^\circ + B_y = 0 \Rightarrow \boxed{B_y = 382\text{ N as shown}}$$

$\Sigma F_z = 0 \Rightarrow$

$$-208 + 1015\sin 20^\circ + B_z = 0 \Rightarrow \boxed{B_z = -139\text{ N} = 139\text{ N} \downarrow}$$

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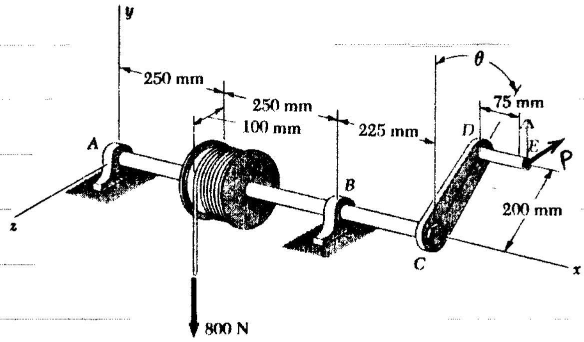
Example 3 :

Given :

The figure shown , $\theta = 60°$.

$\vec{P}$ is in a direction $\perp$ plane CDE .

no axial thrust in the bearing at B



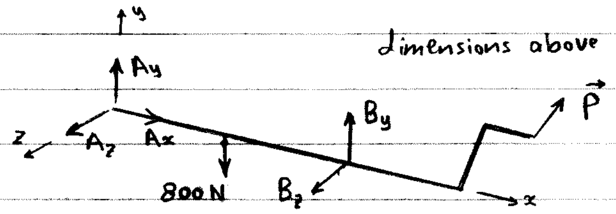
Req.d. :

The magnitude of P and the reactions

dimensions above

Soln. :

The FBD is drawn.



FBD

((Properly aligned bearings are assumed))

$\vec{P} = 0\,\vec{i} + P\cos 30°\,\vec{j} + P\cos 60°\,\vec{k}$

$\Sigma F_x = 0$

$\Rightarrow \boxed{A_x = 0}$

$\Sigma \vec{M}_A = 0 \Rightarrow$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 250 & 0 & 100 \\ 0 & -800 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 500 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 800 & 200\cos 60° & -200\cos 30° \\ 0 & P\cos 30° & P\cos 60° \end{vmatrix} = \vec{0}$$

$$[80\,000 + 200P\underbrace{(\cos^2 60° + \cos^2 30°)}_{1}]\,\vec{i} + [-500B_z - 800(-400)\cos 60°]\,\vec{j} + [-200\,000 + 500B_y + 800(-400)\cos 30°]\,\vec{k} = \vec{0}$$

$\Sigma M_x = 0 \Rightarrow$

$80\,000 + 200P = 0 \Rightarrow \boxed{P = -400\text{ N}} \Rightarrow \boxed{\vec{P} = -400\cos 30°\,\vec{j} - 400\cos 60°\,\vec{k}}$ (N)

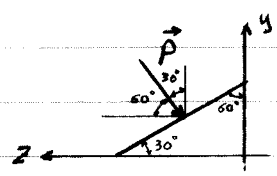
$\Sigma M_y = 0 \Rightarrow$

$-500B_z - 800(-400)\cos 60° = 0 \Rightarrow \boxed{B_z = 320\text{ N}}$

$\Sigma M_z = 0 \Rightarrow -200\,000 + 500B_y + 800(-400)\cos 30° = 0 \Rightarrow \boxed{B_y = 954\text{ N}}$

$\Rightarrow \boxed{\vec{B} = 954\,\vec{j} + 320\,\vec{k}}$ (N)

$\Sigma F_y = 0 \Rightarrow A_y + 954 - 800 - 400\cos 30° = 0 \Rightarrow \boxed{A_y = 192.4\text{ N}}$



$\Sigma F_z = 0 \Rightarrow A_z + 320 - 400\cos 60° = 0 \Rightarrow \boxed{A_z = -120\text{ N}}$

$\Rightarrow \boxed{\vec{A} = 192.4\,\vec{j} - 120\,\vec{k}\ \text{(N)}}$

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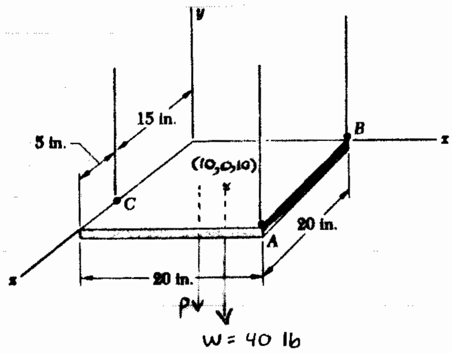
Example 4 :

Given :

The plate and cables shown

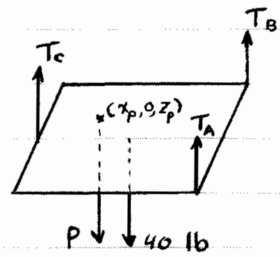


Reqd. :

Location of smallest P in order to have equal T in all cables

Soln. : " Guess the answers ! "

The FBD is shown below.



$\Sigma F_y = 0 \Rightarrow \quad 3T - P - 40 = 0 \quad \Rightarrow \quad T = \frac{P+40}{3}$

$\Sigma \vec{M}_A = 0$

$$\vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -20 \\ 0 & T_B & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -20 & 0 & -5 \\ 0 & T_C & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -10 & 0 & -10 \\ 0 & -40 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & 0 & z \\ 0 & -P & 0 \end{vmatrix} = 0$$

$(20 T_B \vec{i}\,) + (5 T_C \vec{i} - 20 T_C \vec{k}) + (-400 \vec{i} + 400 \vec{k}) + (Pz \vec{i} - Px \vec{k}) = 0$

$\Rightarrow (20T + 5T - 400 + Pz)\vec{i} + (-20T + 400 - Px)\vec{k} = 0$

Note that $T_A = T_B = T_C = T$

$\Sigma M_z = -20T + 400 - Px = 0 \Rightarrow \frac{-20(P+40)}{3} + 400 - Px = 0 \Rightarrow P = \frac{400}{20 + 3x}$

To minimize P, maximize $(20 + 3x)$.

$x = x_p - x_A = x_p - 20 \Rightarrow P = 400/[20 + 3(x_p - 20)]$

$\Rightarrow$ For P_{min}, take $x_{p_{max}} \Rightarrow x_p \equiv 20$ in (We still need to check $z \leq 20$ in)

Check z_p :

$\Sigma M_x = 25T - 400 + Pz = 0 \Rightarrow \frac{25(P+40)}{3} - 400 + Pz = 0 \Rightarrow P = \frac{200}{25 + 3z}$

As above, for P_{min}, take $(25 + 3z)$ to be max. $\Rightarrow$

$z = z_p - z_A = z_p - 20$

$\Rightarrow P = 200/[25 + 3(z_p - 20)]$

For P_{min}, take $z_{p_{max}} \Rightarrow z_p \leq 20$ in

$\Rightarrow z_{p_{max}} = 20$ in

$\Rightarrow P = 200/[25 + 3(0)] = 8$ lb

$\Rightarrow$ check x_p : $8 \equiv 400/[20 + 3x_p - 60]$

$\Rightarrow x_p = 30$ in > 20 in $\Rightarrow$ impossible $\Rightarrow x_{p_{max}} = 20$ in (above)

$\Rightarrow P_{min} = 400/[20 + 3(20-20)] = 20$ lb $\Rightarrow z_p = 15$ in < 20 in $\Rightarrow$ ok

Answers seem ok ?! $\Rightarrow$ $\boxed{P_{min} = 20 \text{ lb}}$ $\boxed{x_p = 20 \text{ in}}$ $\boxed{z_p = 15 \text{ in}}$

There are different ways for solving this problem. Try some!