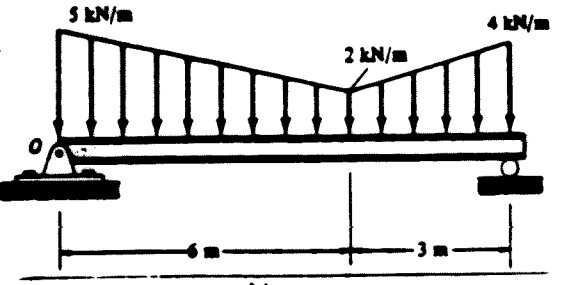
#13

Examples

Reduction of Distributed Loading

Example 1:



Given:

The loading on the beam shown

Reqd.:

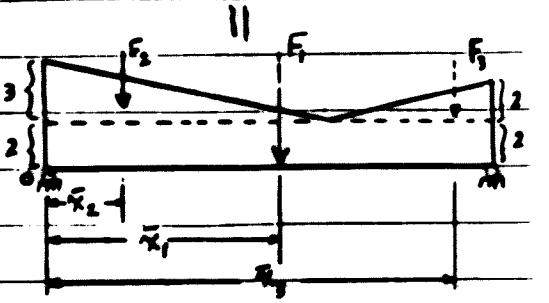
The magnitude & location of a single resultant force

Soln.:

$F_1 = 2(9) = 18$ kN

$F_2 = \frac{1}{2}(3)(6) = 9$ kN

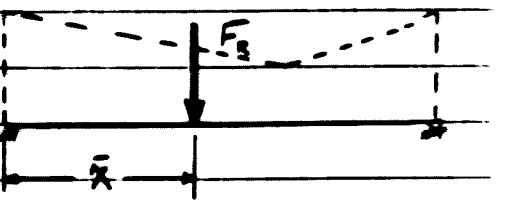
$F_3 = \frac{1}{2}(2)(3) = 3$ kN



$F_R = \Sigma F = F_1 + F_2 + F_3$

$= 18 + 9 + 3$

$\Rightarrow$ $\boxed{F_R = 30 \text{ kN} \downarrow}$



$\Sigma M = \bar{x} F_R$

$\Rightarrow (\bar{x}_1 F_1 + \bar{x}_2 F_2 + \bar{x}_3 F_3) = \bar{x} F_R$

$(+\circlearrowleft) \quad -18(4.5) - 9(2) - 3(6+1) = -\bar{x}(30)$

$\Rightarrow \quad 123 = 30\bar{x} \quad \Rightarrow \quad \boxed{\bar{x} = 4.10 \text{ m}}$ as shown

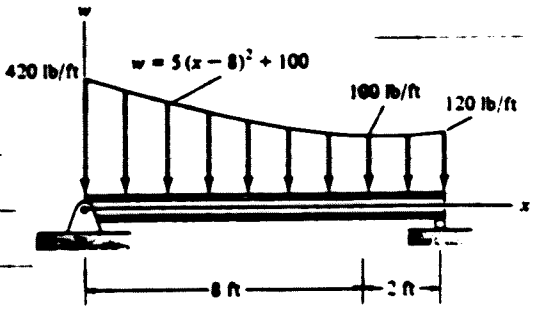
Note that $F_R = \sum_{i=1}^{n} A_i$

$$\bar{x} = \frac{\sum_{i=1}^{n} \bar{x}_i A_i}{\sum_{i=1}^{n} A_i}$$

Example 2:

Given:

The beam loaded as shown

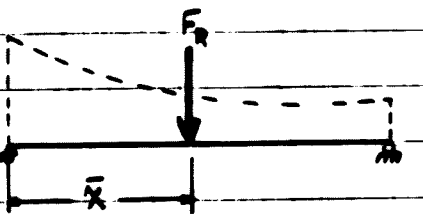


Reqd:

The magnitude and location of the resultant force

Soln:

First, guess the answers!



$$F_R = \int_A dA$$

$$= \int_0^{10} [5(x-8)^2 + 100]\, dx$$

$$= \left[\frac{5}{3}(x-8)^3 + 100x\right]\Big|_0^{10}$$

$$= \left[\frac{5}{3}(10-8)^3 + 100(10)\right] - \left[\frac{5}{3}(0-8)^3 + 100(0)\right]$$

$$= \frac{40}{3} + 1000 + \frac{2560}{3} \Rightarrow$$

$$\boxed{F_R = 1867 \text{ lb} \downarrow}$$

$$\bar{x} = \frac{\int_A x\, dA}{\int_A dA} = \frac{\int_0^{10} [5x(x-8)^2 + 100x]\, dx}{1867}$$

$$= \frac{\left(\frac{5}{4}x^4 - \frac{80}{3}x^3 + \frac{320}{2}x^2 + \frac{100}{2}x^2\right)\Big|_0^{10}}{1867}$$

$$= \frac{6833.3}{1867} \Rightarrow$$

$$\boxed{\bar{x} = 3.66 \text{ ft}}$$

Are the answers "reasonable"? Explain!

How do the answers compare with your "guessing"? Comment!