

Examples

Moment of a Couple

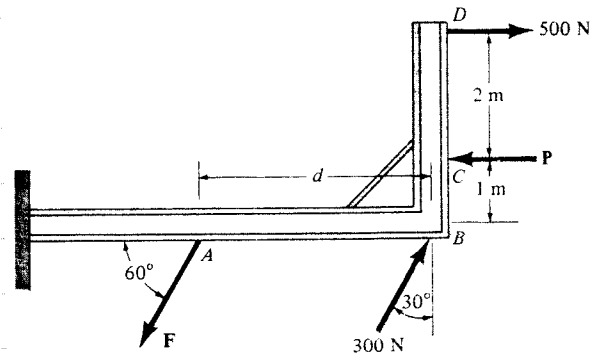
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Example 1:

Given:

The two couples which act on the beam shown.

The resultant couple is zero.



Req.d.:

- The forces P and F.
- The distance d.

Soln.:

- Since the two are couples, then, the forces have to be equal and opposite. \Rightarrow

$$P = 500 \text{ N}$$

in the direction shown.

$$F = 300 \text{ N}$$

" " " "

- To have the resultant couple equal to zero, then

$$M_P + M_F = 0$$

$$M_P = -500(2) = -1000 \text{ N}\cdot\text{m} = 1000 \text{ N}\cdot\text{m} \curvearrowright$$

$$\begin{aligned} M_F &= F_x y + F_y x \\ &= 300 \sin 30^\circ (0) + 300 \cos 30^\circ (d) \\ &= 259.8 d \curvearrowleft \end{aligned}$$

$$\sum M = 0 \Rightarrow -1000 + 259.8 d = 0$$

$$\Rightarrow d = 3.85 \text{ m}$$

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Example 2:

Given:

The two couples that act on the assembly shown

Member OB lies in the x-z plane

Req'd:

The resultant couple

Sol'n:

Vector analysis:

$$F = 400 \text{ N}$$

$$\vec{F}_B = 400 \vec{i} \quad (\text{N})$$

$$\begin{aligned} \vec{r}_{AB} &= (B - A) = (600 \cos 45^\circ - 0) \vec{i} + (0 - 500) \vec{j} + (-600 \sin 45^\circ - 0) \vec{k} \\ &= 423.3 \vec{i} - 500 \vec{j} - 423.3 \vec{k} \end{aligned}$$

$$\vec{M}_A = \vec{r}_{AB} \times \vec{F}_B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 424.3 & -500 & -424.3 \\ 400 & 0 & 0 \end{vmatrix} = 0 \vec{i} - 169.7 \vec{j} + 200 \vec{k}$$

Note: You may choose $\vec{F}_A = -400 \vec{i}$ and $\vec{r}_{BA} = -\vec{r}_{AB} \Rightarrow$ get the same answer.

$$P = 150 \text{ N}$$

$$\vec{P}_C = 150 \vec{j} \quad (\text{N})$$

$$\vec{r}_{OB} = 424.3 \vec{i} + 0 \vec{j} - 424.3 \vec{k}$$

$$\vec{M}_O = \vec{r}_{OB} \times \vec{P}_C = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 424.3 & 0 & -424.3 \\ 0 & 150 & 0 \end{vmatrix} = 63.645 \vec{i} - 0 \vec{j} + 63.645 \vec{k}$$

Note: Try \vec{M}_O with \vec{r}_{OC} and \vec{P}_C , or \vec{r}_{BO} , \vec{r}_{CO} with $\vec{P}_O = -150 \vec{j} \Rightarrow$ the same result

$$\vec{M}_R = \vec{M}_A + \vec{M}_O$$

$$= (0 + 63.645) \vec{i} + (-169.7 + 0) \vec{j} + (200 + 63.645) \vec{k} \Rightarrow \vec{M}_R = 63.6 \vec{i} - 170 \vec{j} + 264 \vec{k} \quad (\text{N.m})$$

Scalar analysis:

$$M_p = P d = 150(600) = 90000 \text{ N.mm} = 90 \text{ N.m} = M_o$$

$$M_{o_x} = 90 \cos 45^\circ = 63.64 \text{ N.m} \curvearrowright$$

$$M_{o_z} = 90 \sin 45^\circ = 63.64 \text{ N.m} \curvearrowright$$

$$M_{o_y} = 0$$

Note that M_F is not as easy to get it as M_p by scalar analysis \rightarrow try! \rightarrow which method is easier?!

