Examples

#10 Moment about an Axis

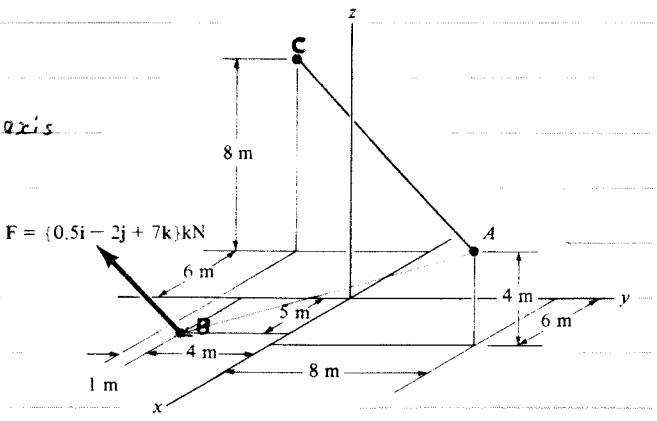
Example 1 :

Given :

The figure shown

Reqd. :

The moment of $\vec{F}$ about the AC axis



Soln. :

$M_{AC} = \vec{u}_{AC} \cdot (\vec{r} \times \vec{F})$

A (6, 8, 4)

C (-6, -5, 8)

$\vec{AC} = -12\,\vec{i} - 13\,\vec{j} + 4\,\vec{k}$ (m)

$\Rightarrow$ AC = 18.14 m

$\vec{u}_{AC} = \frac{\vec{AC}}{AC} = -0.6616\,\vec{i} - 0.7167\,\vec{j} + 0.2205\,\vec{k}$

B (5, -4, 0)

$\vec{r}_{AB} = -\vec{i} - 12\,\vec{j} - 4\,\vec{k}$

$$\Rightarrow M_{AC} = \begin{vmatrix} u_{AC}^x & u_{AC}^y & u_{AC}^z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} -0.6616 & -0.7167 & 0.2205 \\ -1 & -12 & -4 \\ 0.5 & -2 & 7 \end{vmatrix}$$

$\Rightarrow$ **$M_{AC} = 59.05$ kN.m** (Scalar)

$\vec{M}_{AC} = M_{AC}\,\vec{u}_{AC}$

$= 59.05\,(-0.6616\,\vec{i} - 0.7167\,\vec{j} + 0.2205\,\vec{k})$

$\Rightarrow$ **$\vec{M}_{AC} = -39.1\,\vec{i} - 42.3\,\vec{j} + 13.0\,\vec{k}$** (Vector)

* Solve the problem using the position vector $\vec{r}_{CB}$ instead of $\vec{r}_{AB}$. What conclusion can you make ?

#10

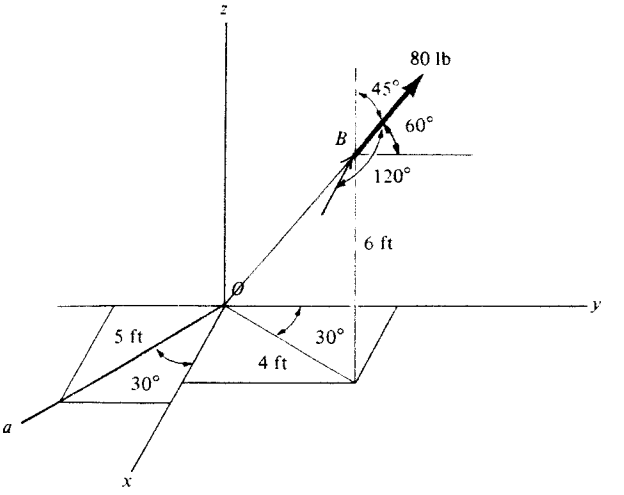
Example 2 :

Given :

The figure shown

Reqd :

The moment about axis oa



Soln. :

$$M_{oa} = \vec{u}_{oa} \cdot (\vec{r} \times \vec{F})$$

$$\vec{oa} = 5 \cos 30^\circ \vec{i} - 5 \sin 30^\circ \vec{j}$$

$$= 4.330 \vec{i} - 2.5 \vec{j} \quad (ft)$$

$oa = 5$ ft

$\vec{u}_{oa} = 0.866 \vec{i} - 0.5 \vec{j}$ }} ⇐ It can be obtained directly as $\vec{u}_{oa} = \cos 30^\circ \vec{i} - \sin 30^\circ \vec{j}$

Now, we need $\vec{r}_{oB}$

$B(4 \sin 30^\circ, 4 \cos 30^\circ, 6) = B(2, 3.464, 6)$

$$\vec{r}_{oB} = 2 \vec{i} + 3.464 \vec{j} + 6 \vec{k}$$

$$\vec{F} = 80 (\cos 120^\circ \vec{i} + \cos 60^\circ \vec{j} + \cos 45^\circ \vec{k})$$

$$= -40 \vec{i} + 40 \vec{j} + 56.57 \vec{k}$$

$$M_{oa} = \begin{vmatrix} 0.866 & -0.5 & 0 \\ 2 & 3.464 & 6 \\ -40 & 40 & 56.57 \end{vmatrix}$$

$$= 0.866[3.464(56.57) - 6(40)] - (-0.5)[2(56.57) - 6(-40)] + 0[\ldots]$$

$$\Rightarrow \boxed{M_{oa} = 138.4 \text{ ft-lb}}$$

$$\vec{M}_{oa} = M_{oa} \vec{u}_{oa} = 138.4 (0.866 \vec{i} - 0.5 \vec{j})$$

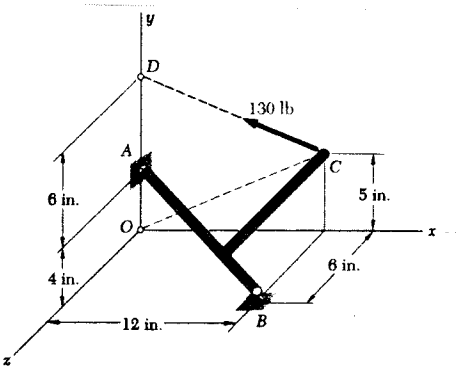
$$\Rightarrow \boxed{\vec{M}_{oa} = 120 \vec{i} - 69.2 \vec{j} \quad (ft\text{-}lb)}$$

#10

Example 3 :

Given :

The figure shown



Req.d :

The moment about axis AB

Soln. :

$$M_{AB} = \vec{u}_{AB} \cdot (\vec{r} \times \vec{F})$$

$A(0,4,0)$, $B(12,0,6)$, $C(12,5,0)$, $D(0,10,0)$

$\vec{AB} = 12\vec{i} - 4\vec{j} + 6\vec{k} \Rightarrow AB = 14$ in

$\vec{u}_{AB} = \frac{1}{14}(12\vec{i} - 4\vec{j} + 6\vec{k})$

$\vec{CD} = -12\vec{i} + 5\vec{j} \Rightarrow CD = 13$ in

$\vec{u}_{CD} = \frac{1}{13}(-12\vec{i} + 5\vec{j})$

$\vec{F}_{CD} = F_{CD}\,\vec{u}_{CD} = -120\vec{i} + 50\vec{j}$

Take $\vec{r}$ as $\vec{AD}$

$\Rightarrow \vec{r}_{AD} = 6\vec{j}$

$$\Rightarrow M_{AB} = \begin{vmatrix} \frac{12}{14} & -\frac{4}{14} & \frac{6}{14} \\ 0 & 6 & 0 \\ -120 & 50 & 0 \end{vmatrix} = \frac{12}{14}(0-0) + \frac{4}{14}(0-0) + \frac{6}{14}(0+6\times120)$$

$$= 0 + 0 + 309$$

$$= 309 \text{ in-lb}$$

Take $\vec{r}$ as $\vec{AC}$ $\Rightarrow$ $\vec{AC} = 12\vec{i} + 1\vec{j}$

$$M_{AB} = \begin{vmatrix} \frac{12}{14} & -\frac{4}{14} & \frac{6}{14} \\ 12 & 1 & 0 \\ -120 & 50 & 0 \end{vmatrix} = \frac{12}{14}(0-0) - \left(-\frac{4}{14}\right)(0-0) + \frac{6}{14}[12(50) - 1(-120)]$$

$\Rightarrow$ **$M_{AB} = 309$ in-lb**

Which is easier ?

$\vec{M}_{AB} = M_{AB}\,\vec{u}_{AB}$

$= \frac{309}{14}(12\vec{i} - 4\vec{j} + 6\vec{k}) \Rightarrow$ **$\vec{M}_{AB} = 264\vec{i} - 88.2\vec{j} + 132\vec{k}$ (in-lb)**

#10

Example 4 :

Given :

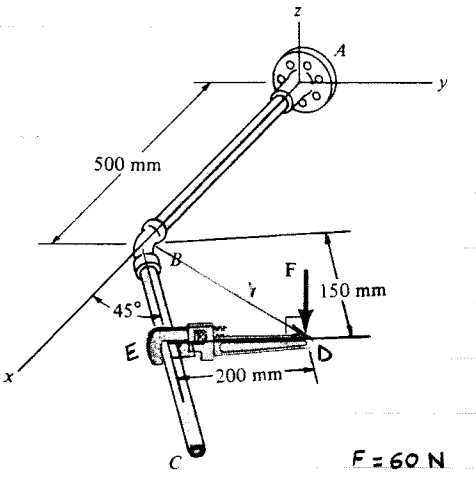
The force on the wrench and pipe shown

Req'd :

The moment about AB (x axis)

Soln. :

Note that Both the wrench and pipe assembly ABC lie in the x-y plane.



Solution (a) : Vector analysis

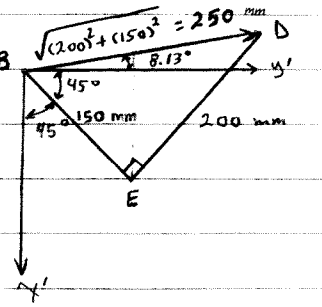
$$M_{AB} = \vec{u}_{AB} \cdot (\vec{r} \times \vec{F})$$

$$\vec{u}_{AB} = 1\,\vec{i}$$

$$\vec{F} = -60\,\vec{k}$$

$$\vec{r} = 250(-\sin 8.13^\circ\,\vec{i} + \cos 8.13^\circ\,\vec{j})$$

$$= -35.35\,\vec{i} + 247.5\,\vec{j}$$



$$M_{AB} = \begin{vmatrix} 1 & 0 & 0 \\ -35.35 & 247.5 & 0 \\ 0 & 0 & -60 \end{vmatrix} = 1(247.5)(-60)$$

$$\Rightarrow \boxed{M_{AB} = M_x = -14.8 \text{ N.m} = 14.8 \text{ N.m} \circlearrowleft}$$

Solution (b) : Scalar analysis

$$M_{AB} = M_x = Fd$$

$$= -60(150 \sin 45^\circ + 200 \sin 45^\circ)$$

$$= -60(247.5) = -14.8 \text{ N.m} = 14.8 \text{ N.m} \circlearrowleft$$ as above

In this particular example, which method is easier ?!