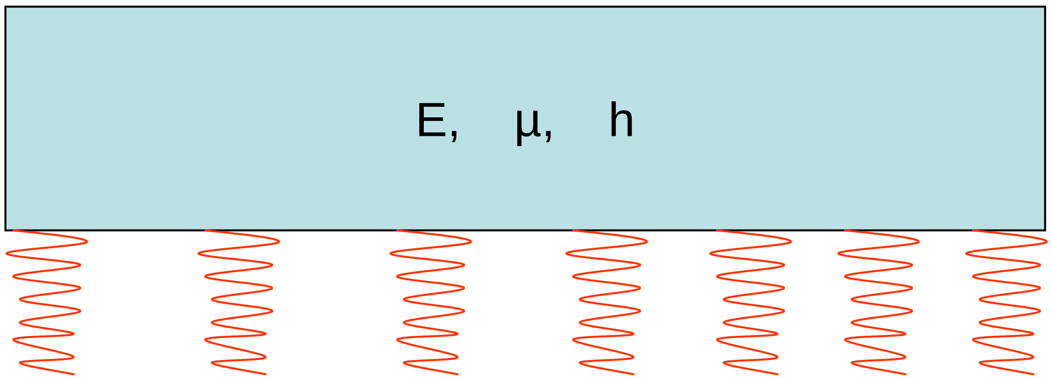
Chapter **3**

Stresses in Rigid Pavements

WESTERGARD'S THEORY



SUBRADE REACTION MODULUS - k

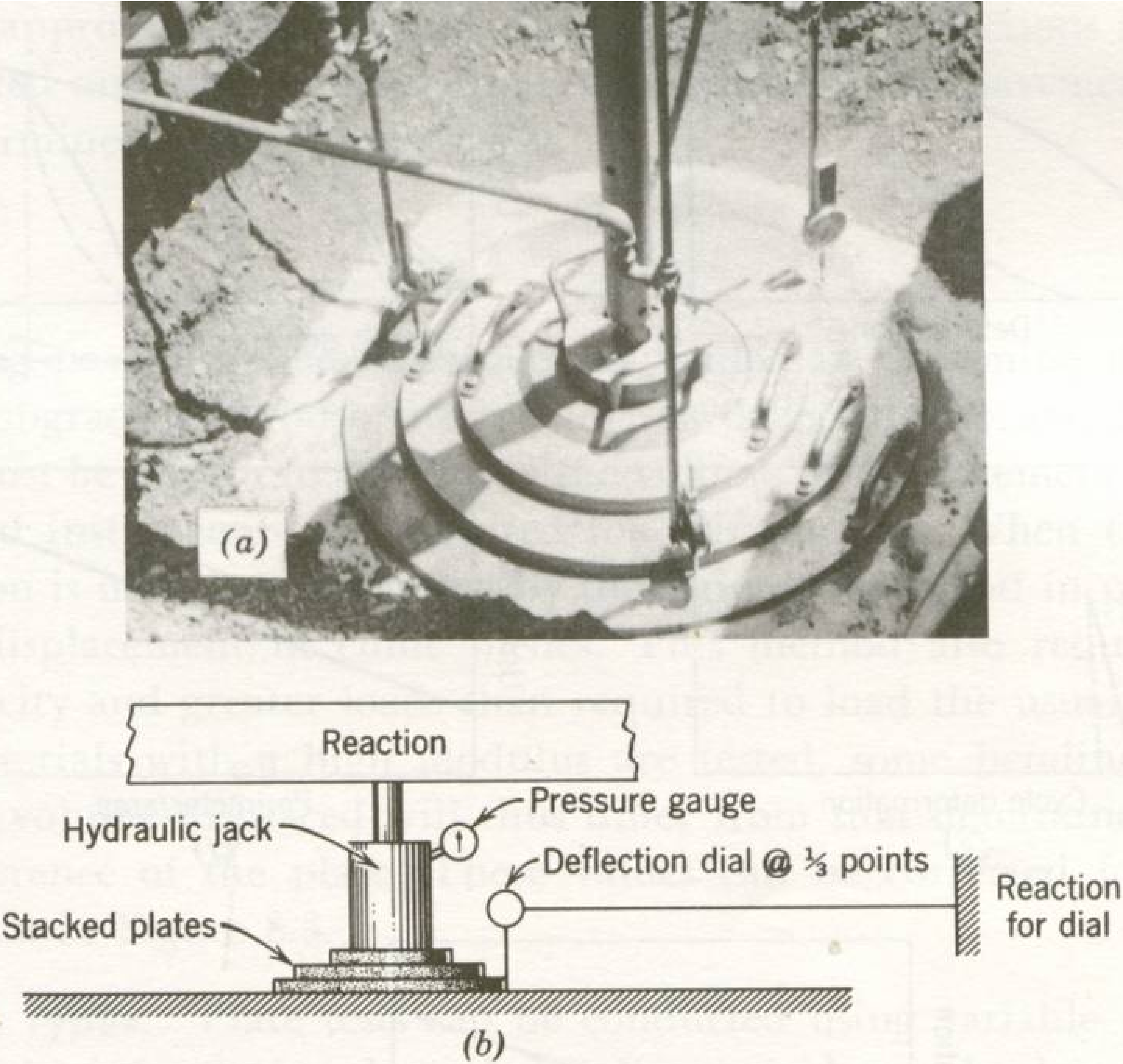


Figure 8.1. Plate-Bearing test. (*a*) View of plates and dials; (*b*) schematic diagram.

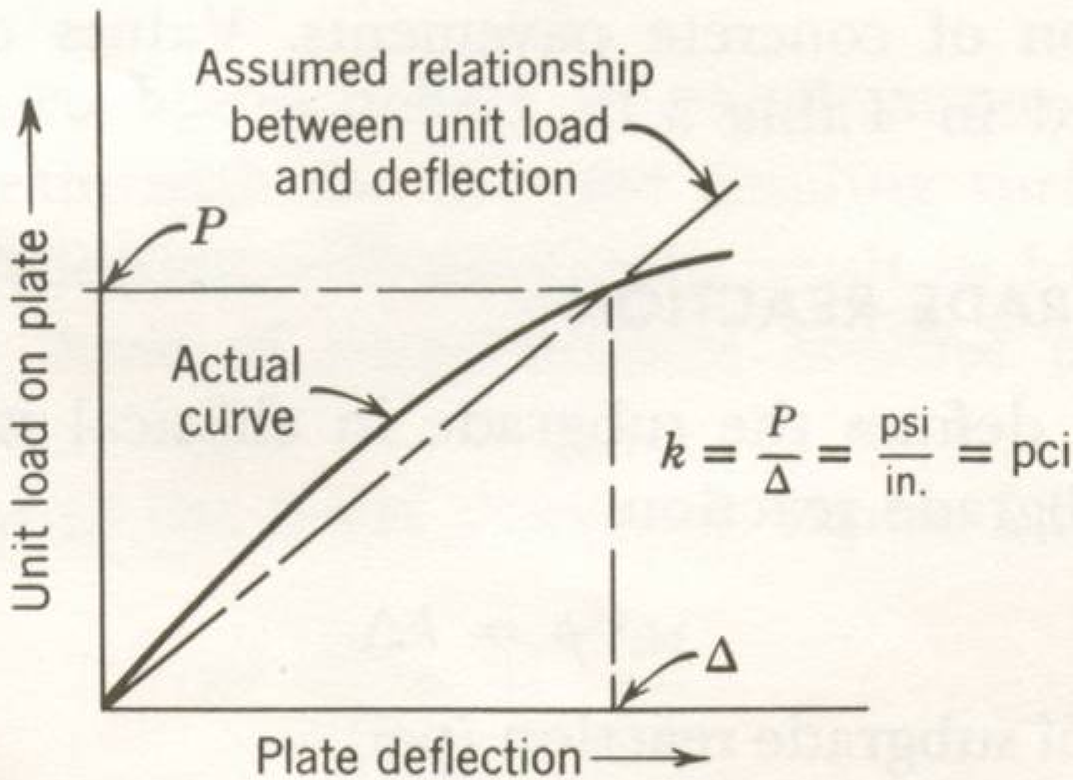


Figure 3.2. Basic assumptions in subgrade behavior.

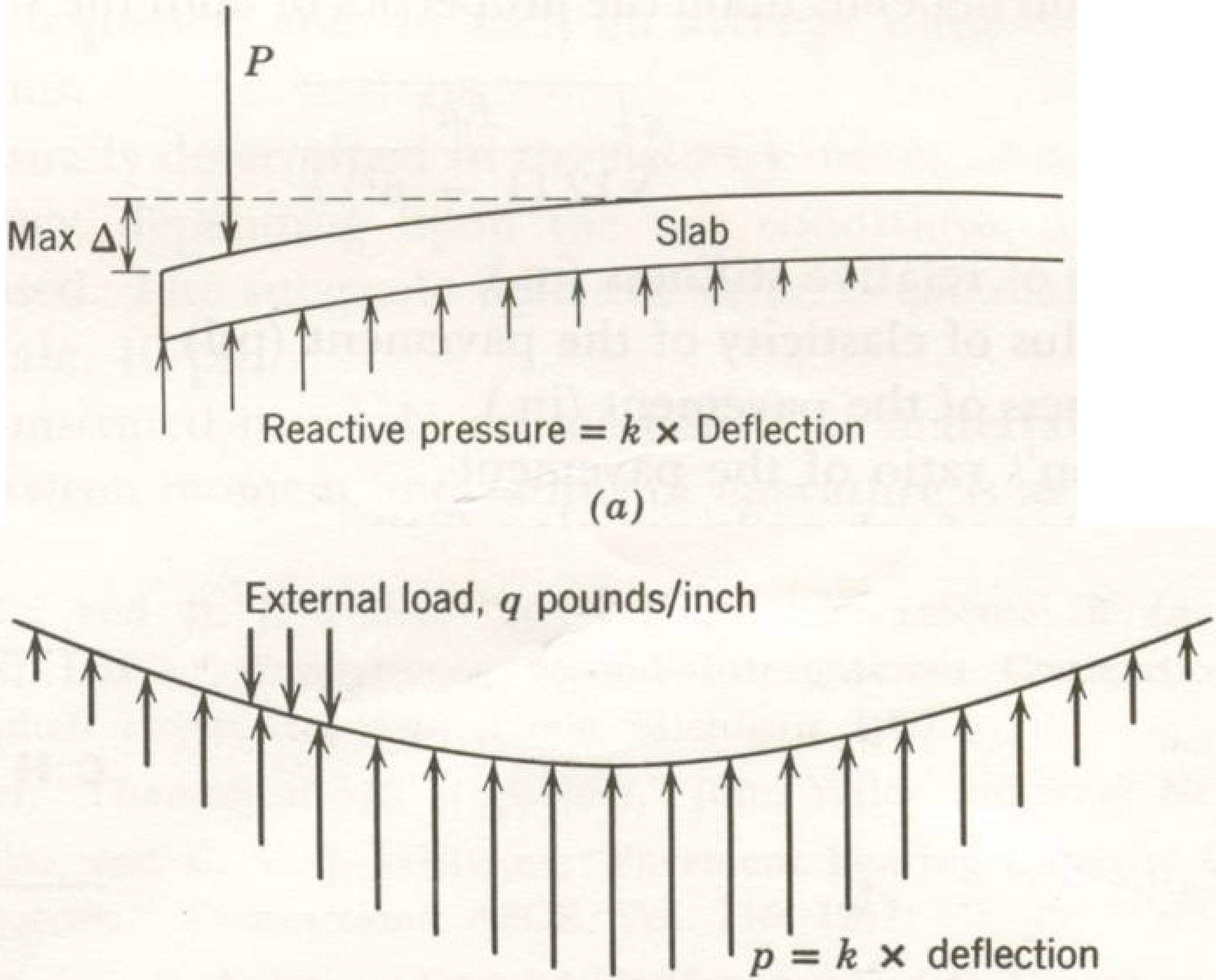


Figure 3.1. Deflected beam on elastic foundation.

$$l = \sqrt[4]{\frac{Eh^3}{12\,(1 - \mu^2)\,k}}$$

where l = radius of relative stiffness (in.)
E = modulus of elasticity of the pavement (psi)
h = thickness of the pavement (in.)
μ = Poisson's ratio of the pavement
k = modulus of subgrade reaction (pci)

TABLE 3.1. Radius of Relative Stiffness (ℓ)

($\mu = 0.15 \quad E = 4{,}000{,}000$ psi)

h (in.)	$k = 50$	$k = 100$	$k = 200$	$k = 300$	$k = 400$	$k = 500$
9.0	47.22	39.71	33.39	30.17	28.08	26.55
9.5	49.17	41.35	34.77	31.42	29.24	27.65
10.0	51.10	42.97	36.14	32.65	30.39	28.74
10.5	53.01	44.57	37.48	33.87	31.52	29.81
11.0	54.89	46.16	38.81	35.07	32.64	30.87
11.5	56.75	47.72	40.13	36.26	33.74	31.91
12.0	58.59	49.27	41.43	37.44	34.84	32.95
12.5	60.41	50.80	42.72	38.60	35.92	33.97
13.0	62.22	52.32	43.99	39.75	36.99	34.99
14.0	65.77	55.31	46.51	42.02	39.11	36.99
15.0	69.27	58.25	48.98	44.26	41.19	38.95
16.0	72.70	61.13	51.41	46.45	43.23	40.88
17.0	76.08	63.98	53.80	48.61	45.24	42.78
18.0	79.41	66.78	56.16	50.74	47.22	44.66
19.0	82.70	69.54	58.48	52.84	49.17	46.51
20.0	85.95	72.27	60.77	54.92	51.10	48.33
21.0	89.15	74.97	63.04	56.96	53.01	50.13
22.0	92.31	77.63	65.28	58.98	54.89	51.91
23.0	95.44	80.26	67.49	60.98	56.75	53.67
24.0	98.54	82.86	69.68	62.96	58.59	55.41

STRESSES IN RIGID PAVEMENTS

1- TRAFFIC (Single Tire, Multiple Tires)

2- TEMPERATURE

3- FRICTION

TRAFFIC STRESS

Single Tire

$$\underset{\text{(circle)}}{\sigma_e} = \frac{3(1+\nu)P}{\pi(3+\nu)h^2}\left[\ln\left(\frac{Eh^3}{100ka^4}\right) + 1.84 - \frac{4\nu}{3} + \frac{1-\nu}{2} + \frac{1.18(1+2\nu)a}{\ell}\right]$$

$$\underset{\text{(circle)}}{\sigma_e} = \frac{0.803P}{h^2}\left[4\log\left(\frac{\ell}{a}\right) + 0.666\left(\frac{a}{\ell}\right) - 0.034\right]$$

$$\underset{\text{(circle)}}{\Delta_e} = \frac{\sqrt{2} + 1.2\nu P}{\sqrt{Eh^3k}}\left[1 - \frac{(0.76 + 0.4\nu)a}{\ell}\right]$$

$$\sigma_i = \frac{3(1 + \nu)P}{2\pi h^2}\left(\ln \frac{\ell}{b} + 0.6159\right)$$

$$\sigma_i = \frac{0.316P}{h^2}\left[4 \log_{10}\left(\frac{l}{b}\right) + 1.069\right]$$

$$b = \sqrt{1.6a^2 + h^2} - 0.675h$$

$$\Delta_i = \frac{P}{8k\ell^2}\left\{1 + \frac{1}{2\pi}\left[\ln\left(\frac{a}{2\ell}\right) - 0.673\right]\left(\frac{a}{\ell}\right)^2\right\}$$

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{\ell}\right)^{0.6}\right]$$

$$\Delta_c = \frac{P}{k\ell^2}\left[1.1 - 0.88\left(\frac{a\sqrt{2}}{\ell}\right)\right]$$

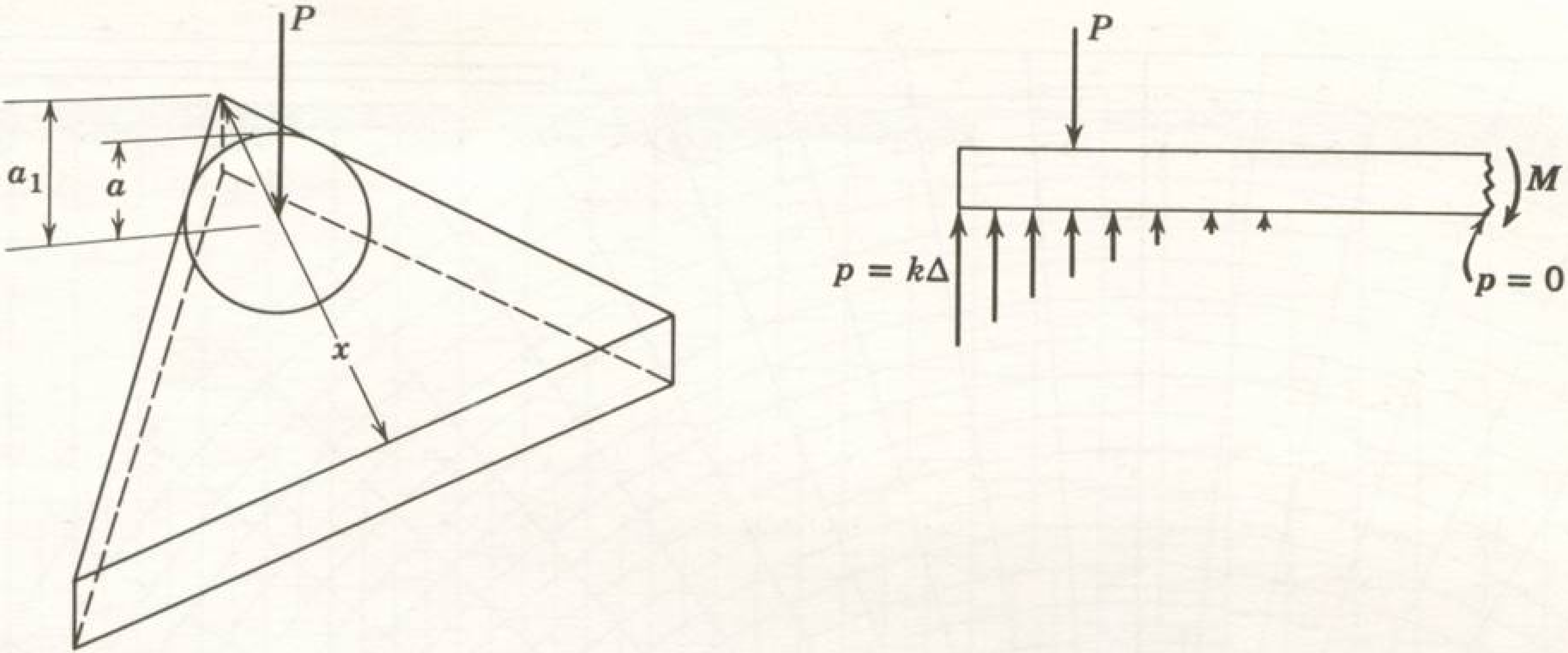


Figure 3.18. Stresses acting under corner load.

MULTIPLE TIRES

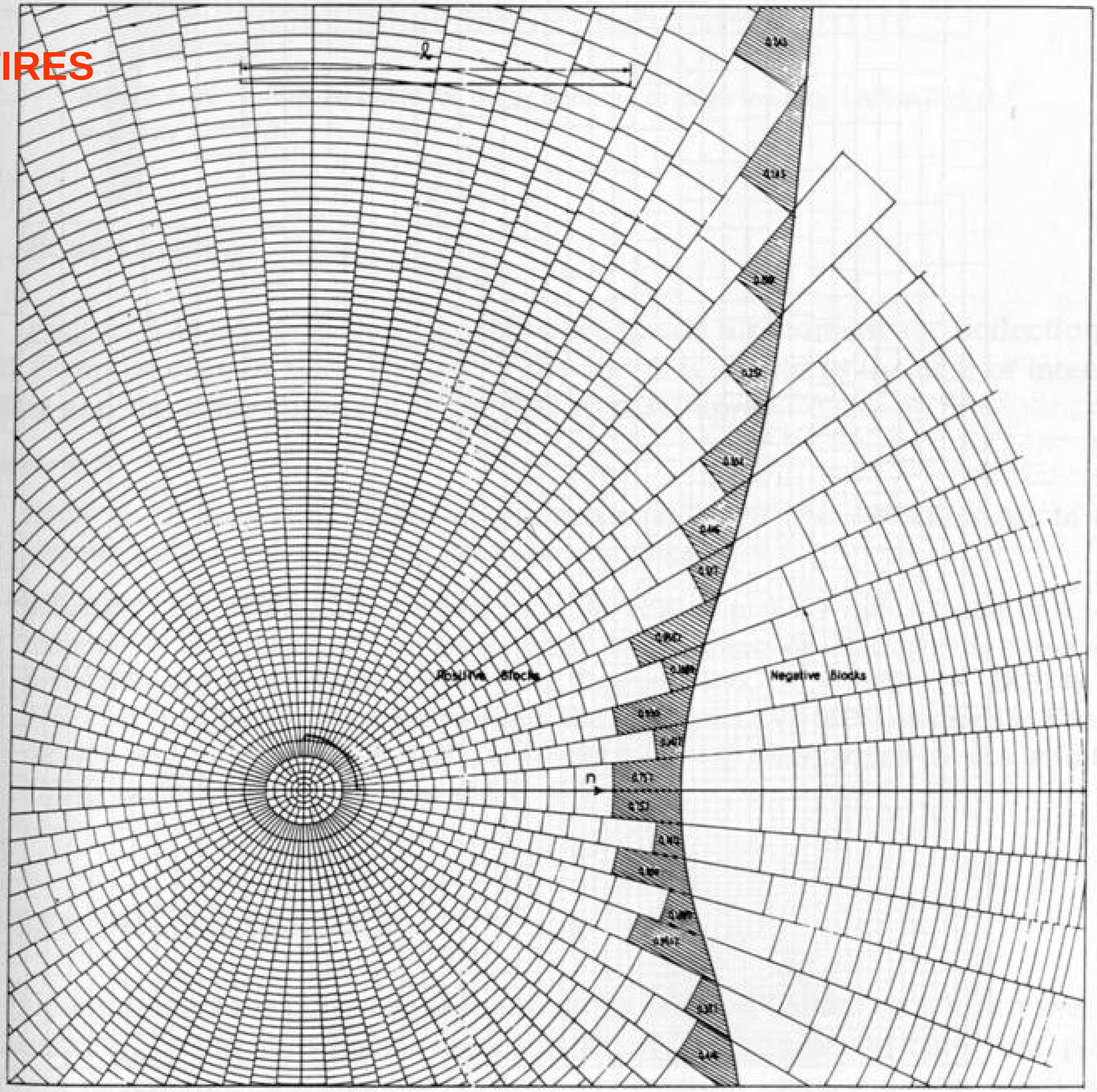


Figure 4.14 Influence chart for moment due to interior loading. (After Pickett and Ray (1951).)

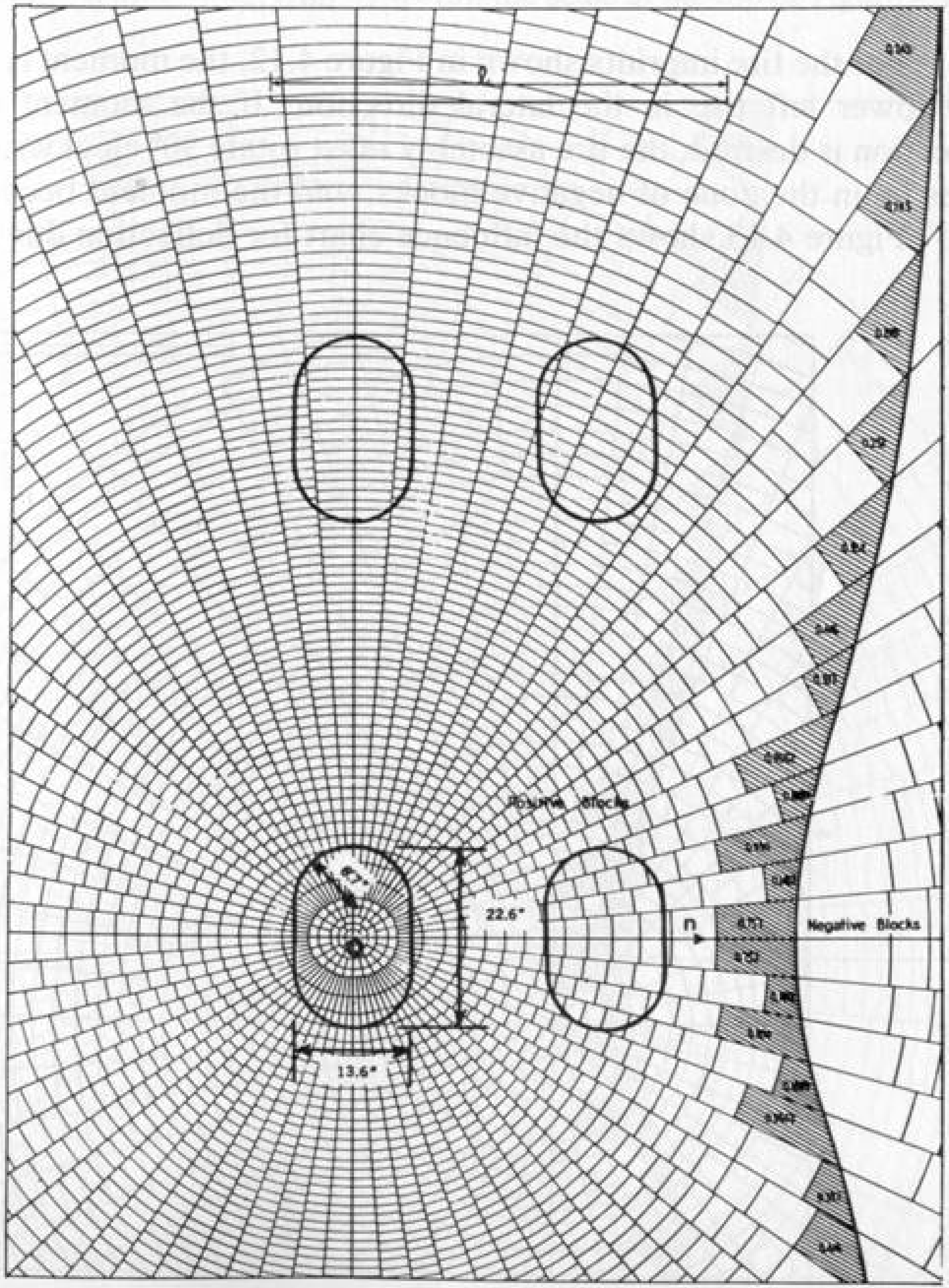


Figure 4.12 Application of influence chart for determining moment (1 in. = 25.4 mm). (After Pickett and Ray (1951).)

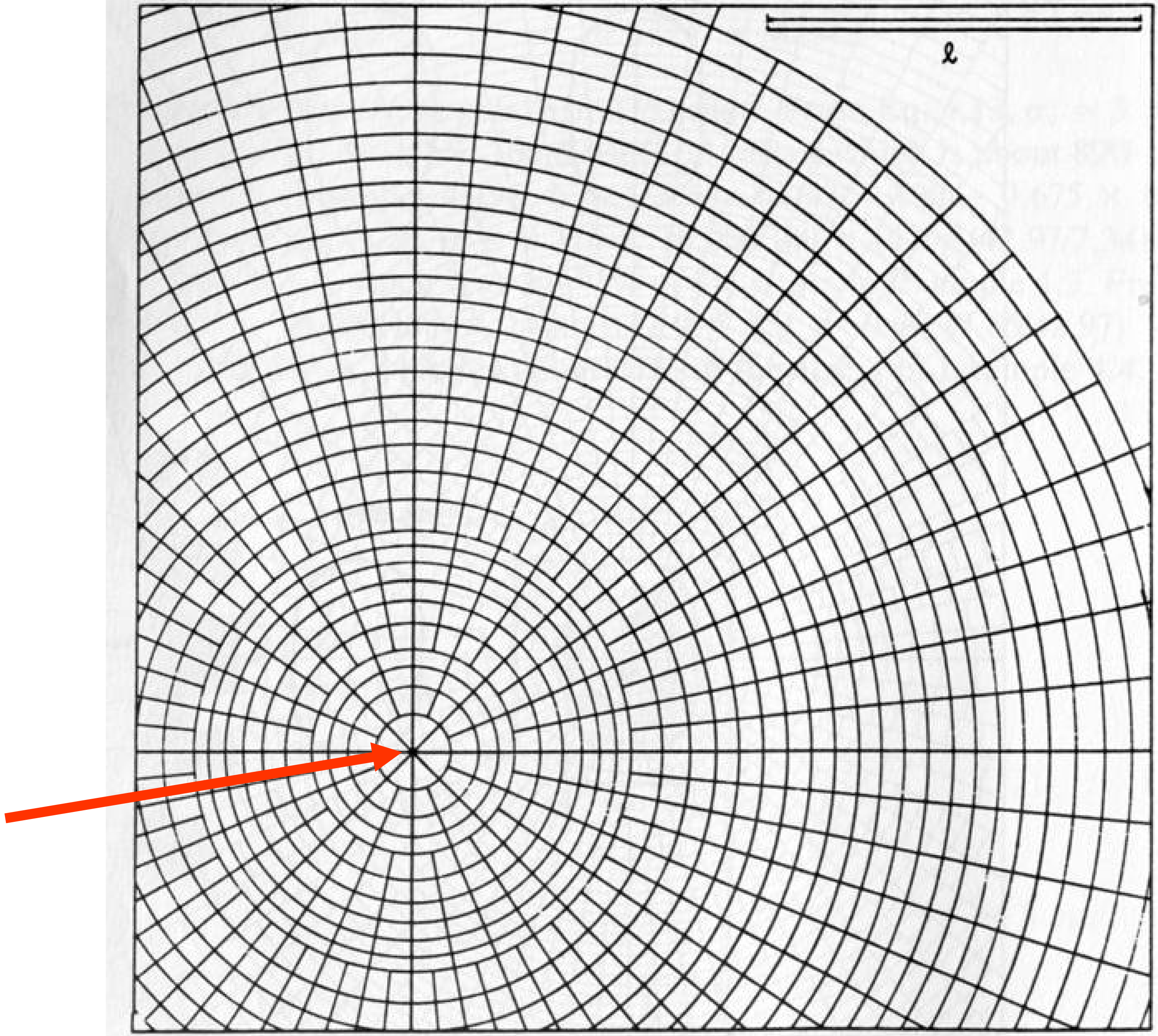


Figure 4.13 Influence chart for deflection due to interior loading. (After Pickett and Ray (1951).)

Deflection

$$\Delta = \frac{0.0005pl^4N}{D}$$

Moments

$$M = \frac{pl^2N}{10{,}000}$$

$$l = \sqrt[4]{\frac{Eh^3}{12(1 - \mu^2)k}}$$

$$D = \frac{Eh^3}{12(1 - \mu^2)}$$

$$\text{Stress} = \frac{6M}{h^2}$$

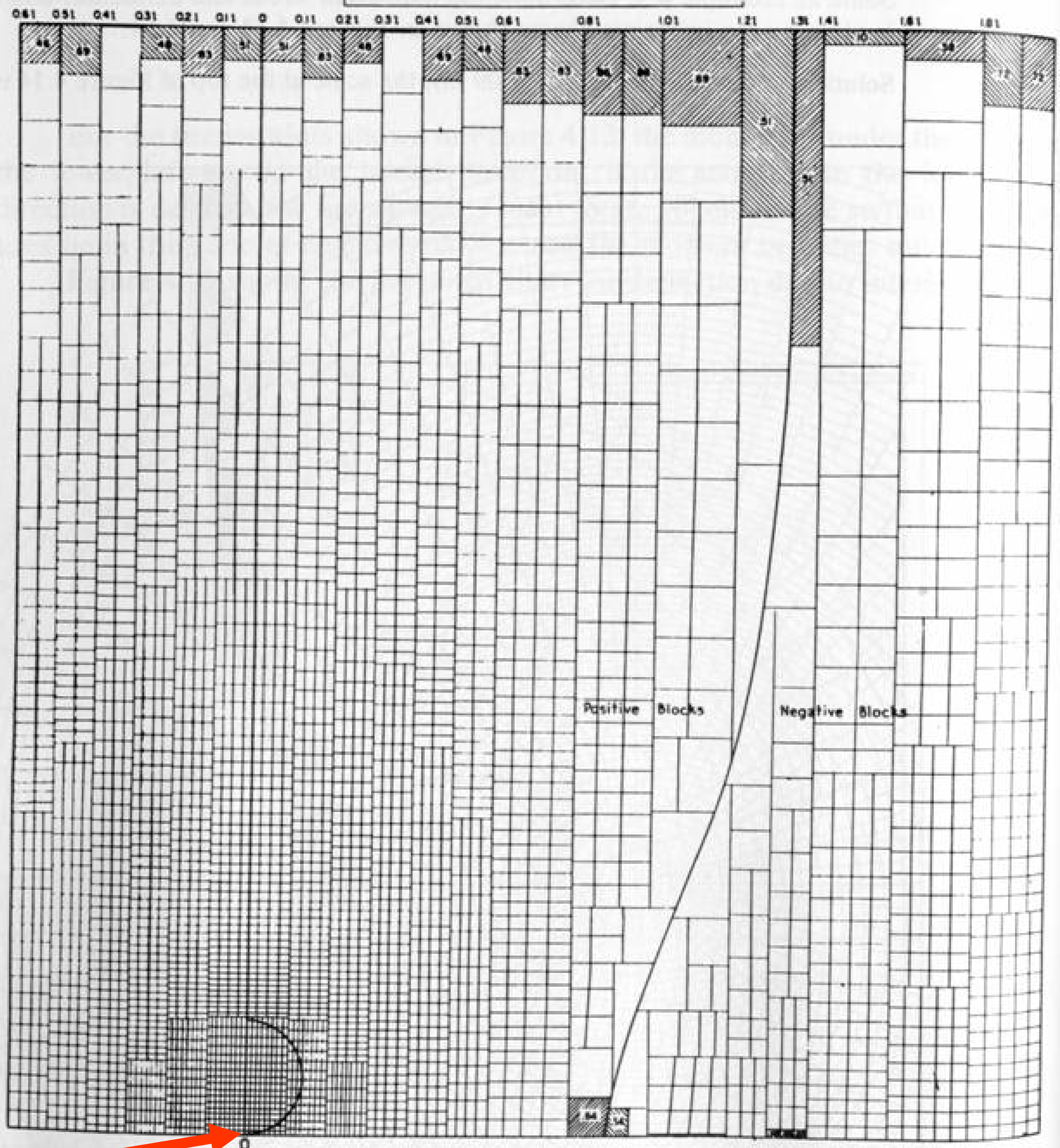


Figure 4.15 Influence chart for moment due to edge loading. (After Pickett and Ray (1951).)

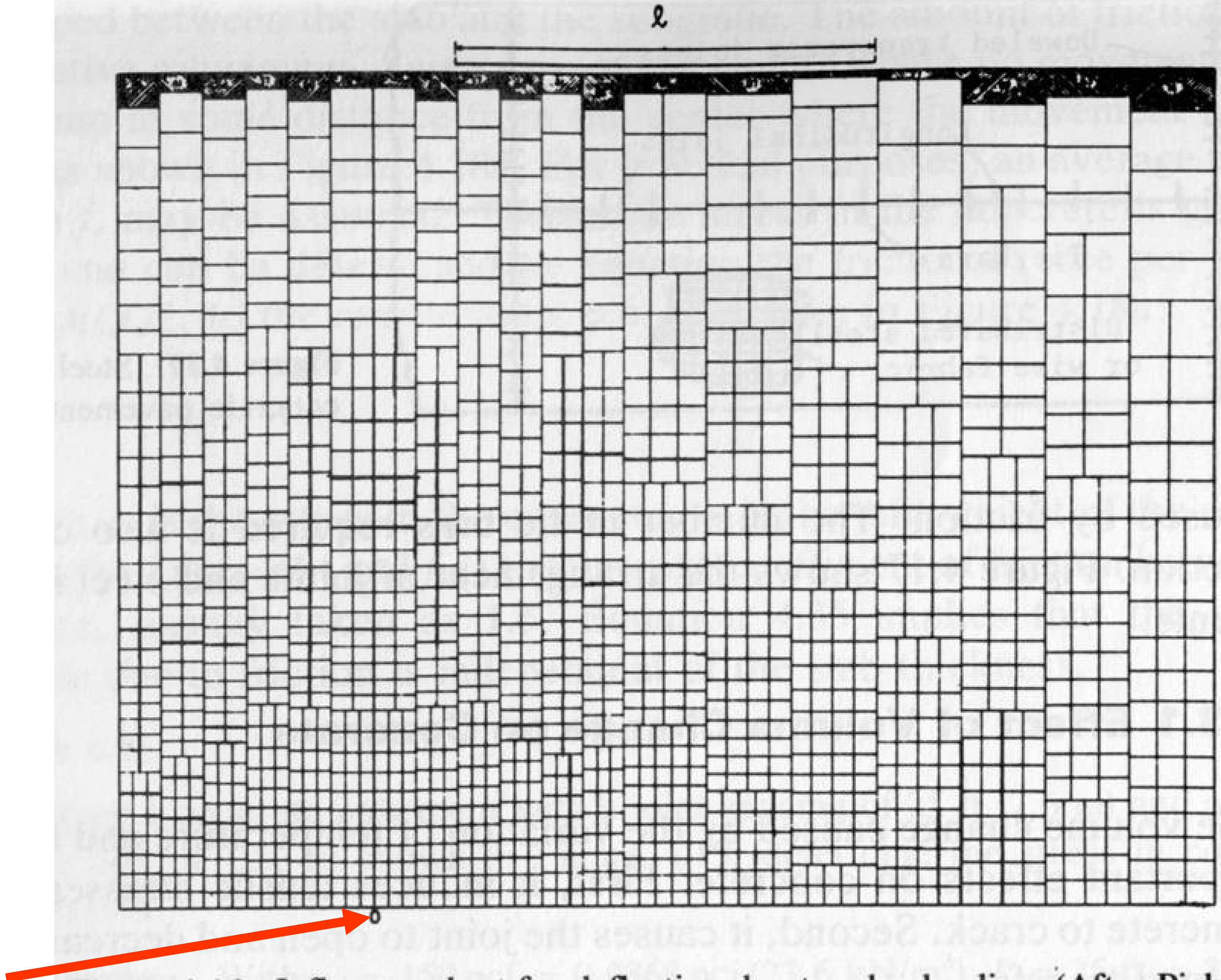
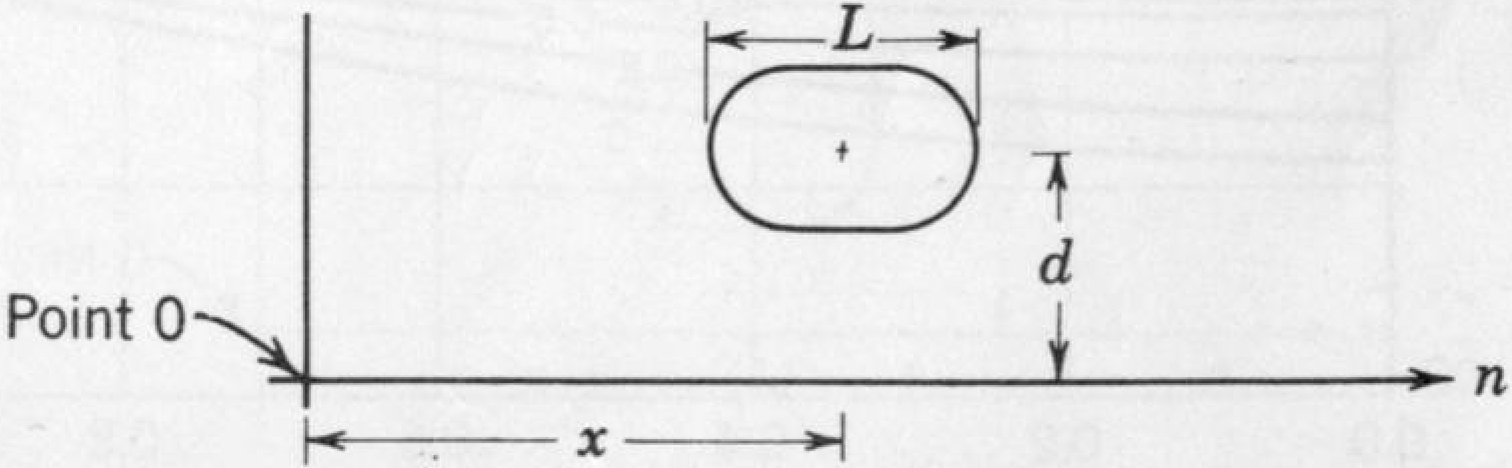
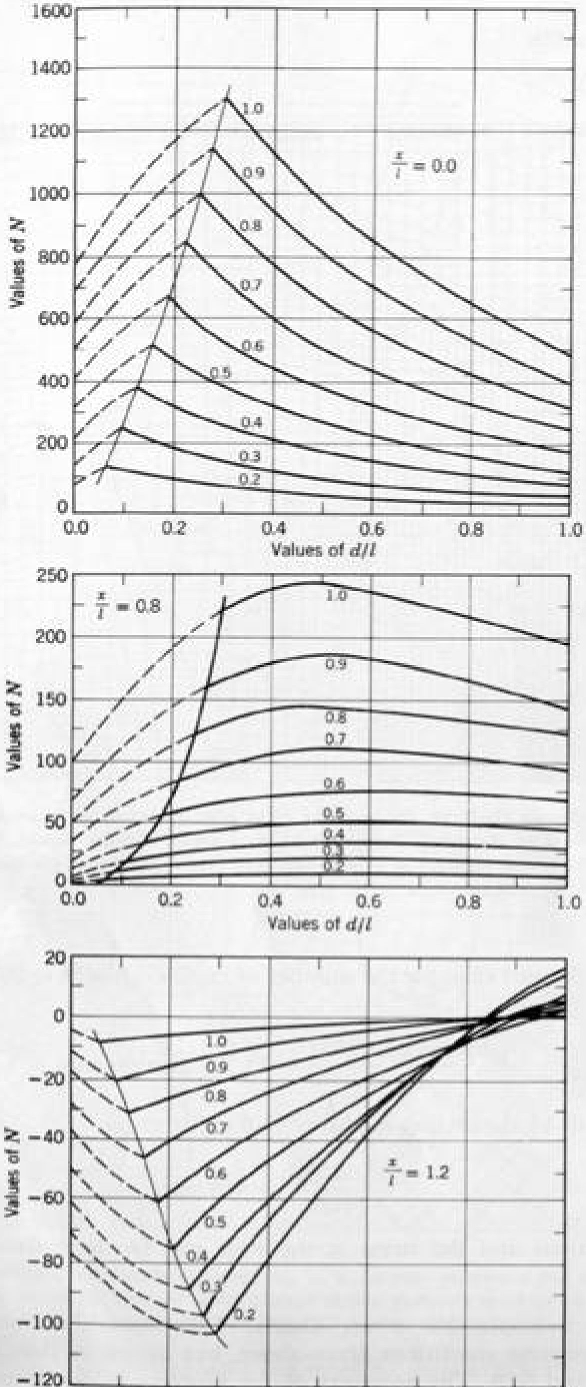


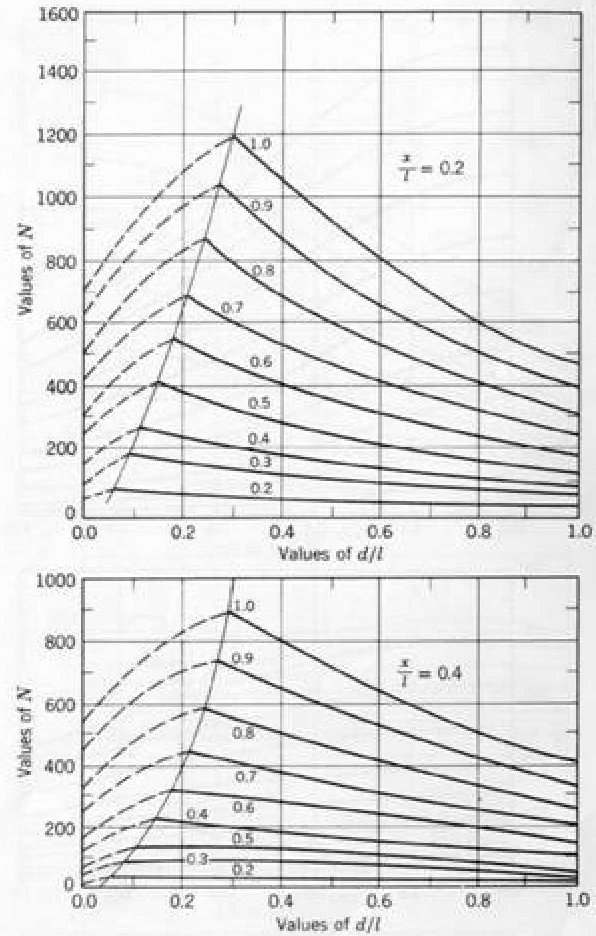
Figure 4.16 Influence chart for deflection due to edge loading. (After Pickett and Ray (1951).)

$$M = \frac{p\,l^2 N}{10{,}000}$$



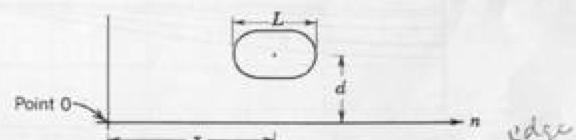
N values for moment at the pavement edge about point 0 in the n direction. The distances d and x are the distances from the tire center to the point 0 as shown.





Notes:
Numbers on curves are L/l values
Assumptions: Subgrade is dense liquid,
Poisson's ratio = 0.15

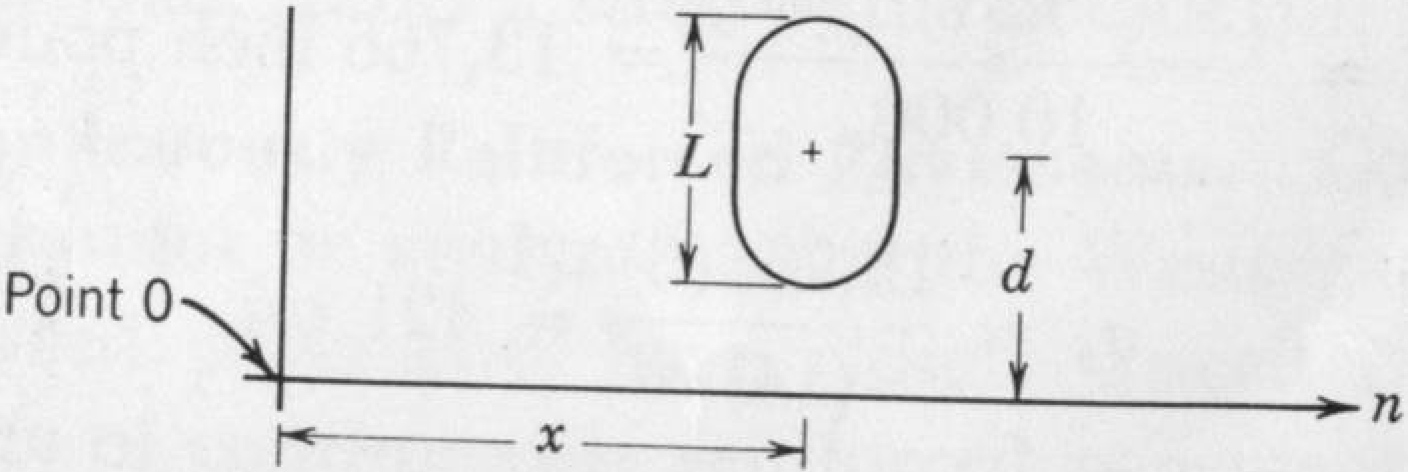
$$M = \frac{p l^2 N}{10{,}000}$$



N values for moment at the pavement edge about point 0 in the n direction. The distances d and x are the distances from the tire center to the point 0 as shown.

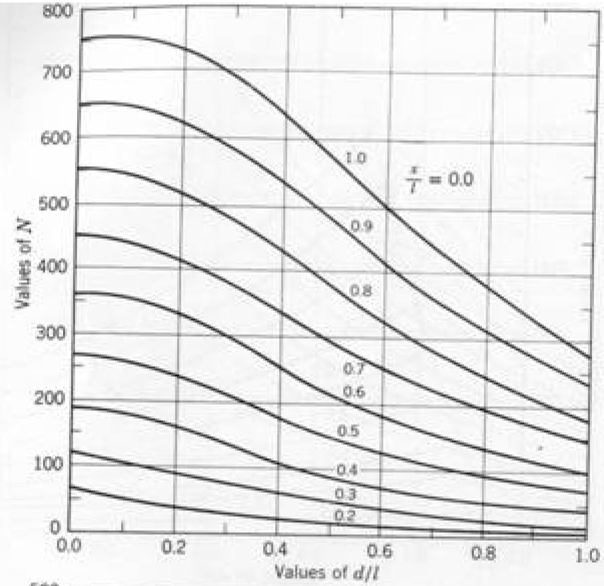
Figure 3.23. N values for moment at the pavement edge about point 0 in the direction o

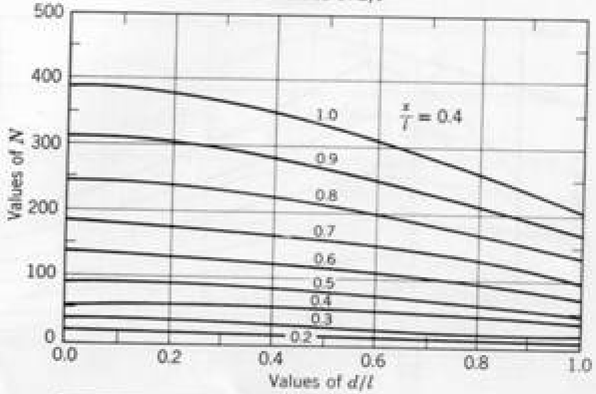
$$M = \frac{p l^2 N}{10{,}000}$$

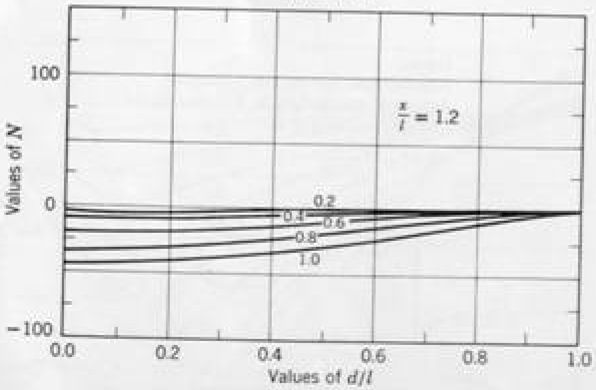


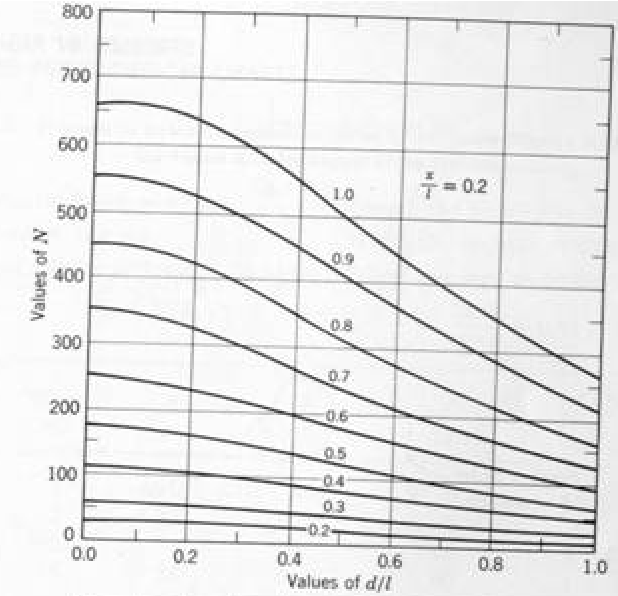
N values for moment at the pavement interior about point 0 in the *n* direction. The distances *d* and *x* are the distances from the tire center to the point 0 as shown.

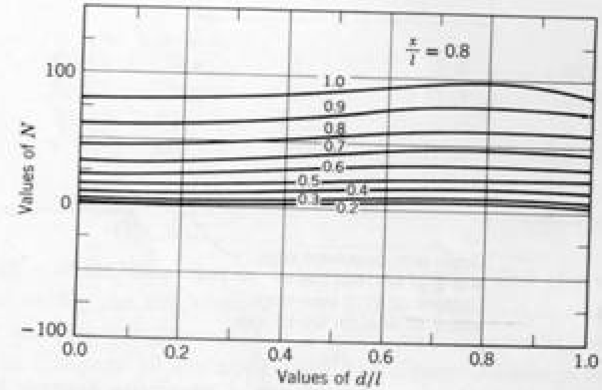
joint
interior









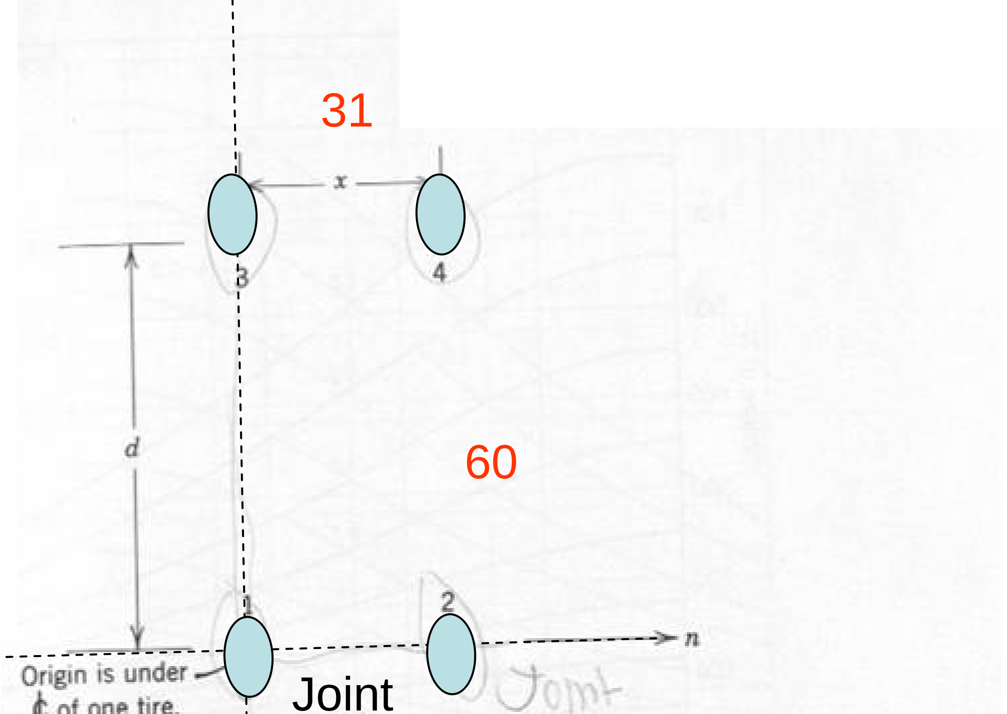


Notes:
Numbers on curves are L/l values
Assumptions: Subgrade is dense liquid,
Poisson's ratio = 0.15

$$M = \frac{pl^2N}{10{,}000}$$

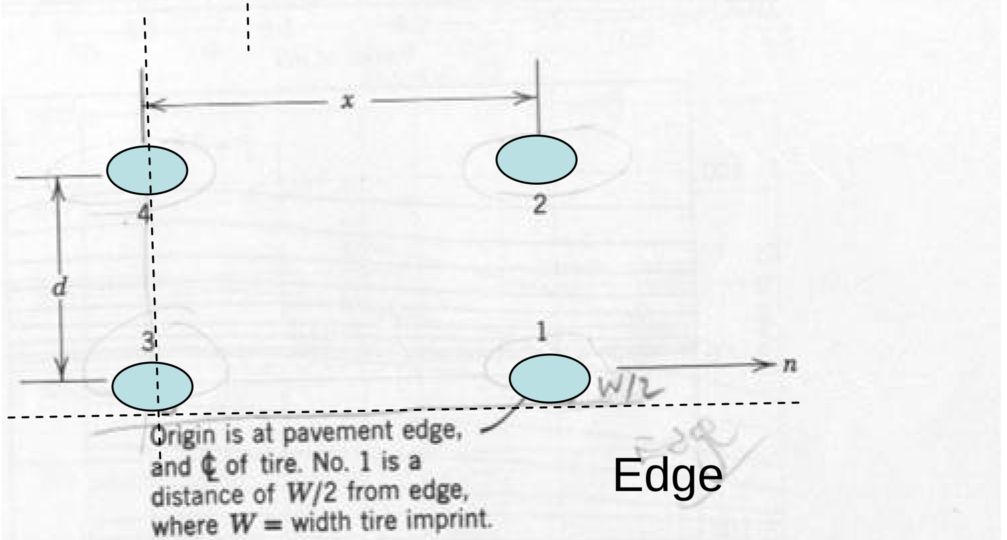
N values for moment at the pavement interior about point 0 in the n direction. The distances d and x are the distances from the tire center to the point 0 as shown

P=160,000 lb
p=150 psi
60 in
31 in



31
x
3
4
d
60
1
2
n
Origin is under ℄ of one tire.
Joint

(a)



x
4
2
d
3
1
n
W/2
Origin is at pavement edge, and ℄ of tire. No. 1 is a distance of W/2 from edge, where W = width tire imprint.
Edge

(b)

P =160,000 / 4 = 40000 lb , A = 40,000 / 150 psi = 267 in^2

TABLE 3.2. Example of Stress Computations Using Stress Charts (Figures 3.23 and 3.24). See Figure 3.25 for Layout of the Problem.

Pavement thickness = 14 in.
Modulus k = 100 pci
Radius of relative stiffness = 55.31 in. ℓ (T3.1)

Length tire imprint = 22.6 in.
Width tire imprint = 13.6
Wheel spacings = 60 in. × 31 in.

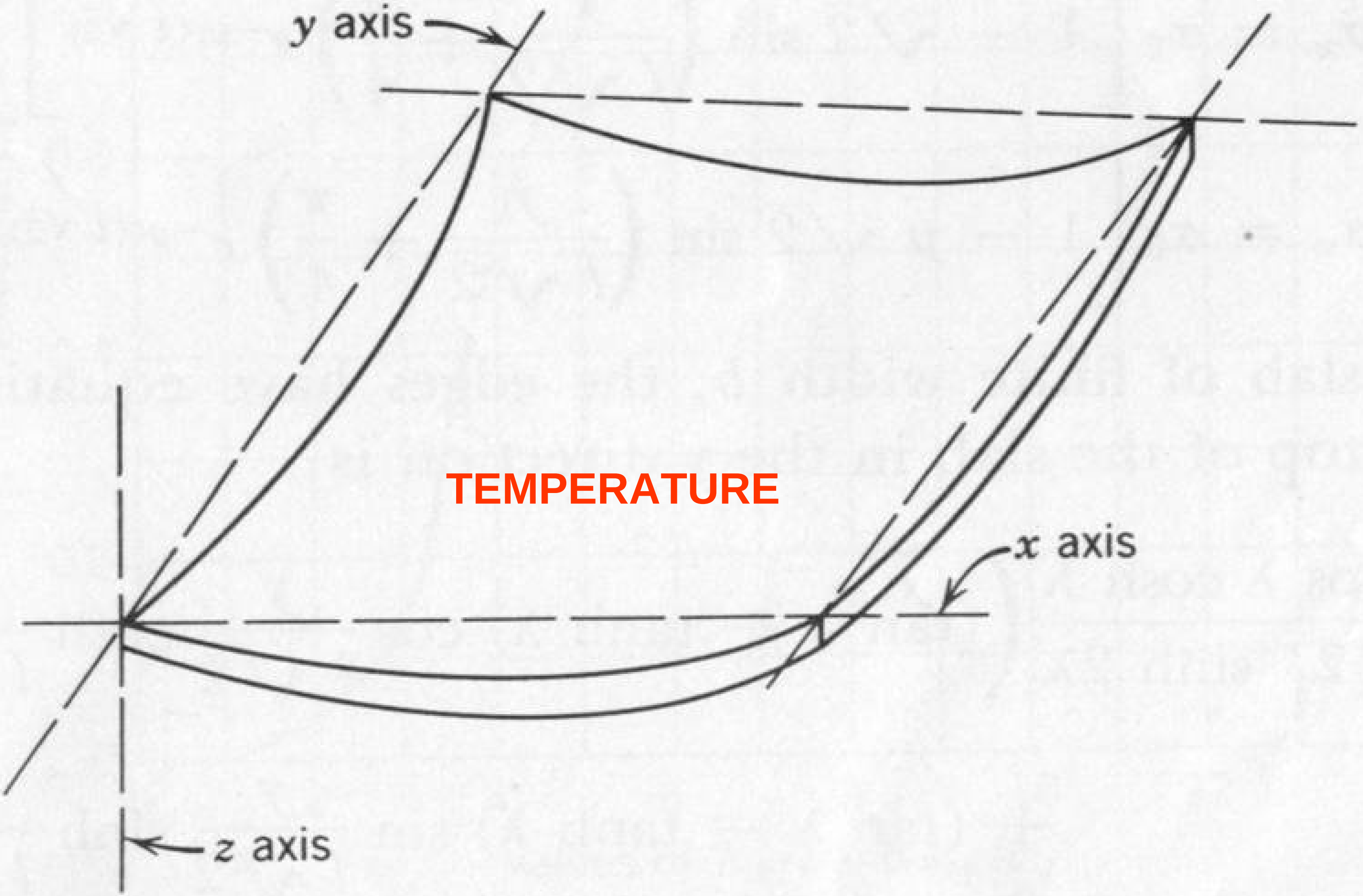
$$\frac{L}{l} = \frac{22.6}{55.31} = 0.41$$

$L_1 = \sqrt{\frac{A}{0.523}}$

$A = \frac{P}{p}$

width = 0.6 L_1

Case	Wheel No	x (in.)	$\frac{x}{l}$	d (in.)	$\frac{d}{l}$	N
Interior (Joint)	1	0	0	0	0	190
	2	31	0.56	0	0	50
	3	0	0	60	1.08	45
	4	31	0.56	60	1.08	15
					Total N	300
Edge	1	0	0	6.8	0.12	395
	2	0	0	37.8	0.68	190
	3	60	1.08	6.8	0.12	−29
	4	60	1.08	37.8	0.68	−10
					Total N	546



. Curvature of elastic surface due to temperature warping.

Edge stresses

$$\sigma = \frac{CE\epsilon_t \Delta t}{2}$$

Interior stresses

$$\sigma = \frac{E\epsilon_t \Delta t}{2}\left(\frac{C_1 + \mu C_2}{1 - \mu^2}\right)$$

ε_t = strain/ 1.0 F = 5* 10^{-6} in / F
Δ_t = Total temp. difference
μ = 0.15
E = 4*10^6 psi
C = C1 = Max of C1 or C2

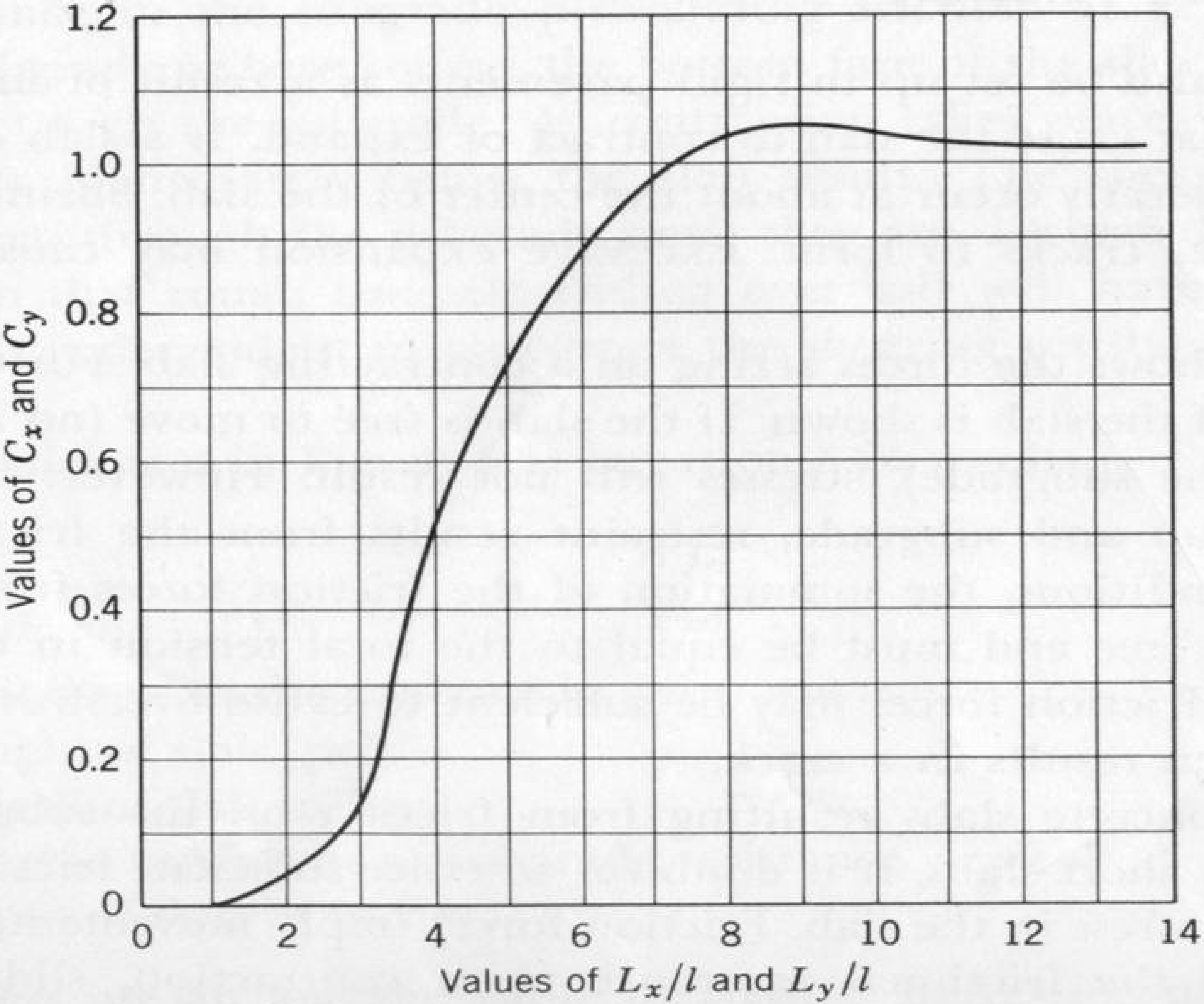
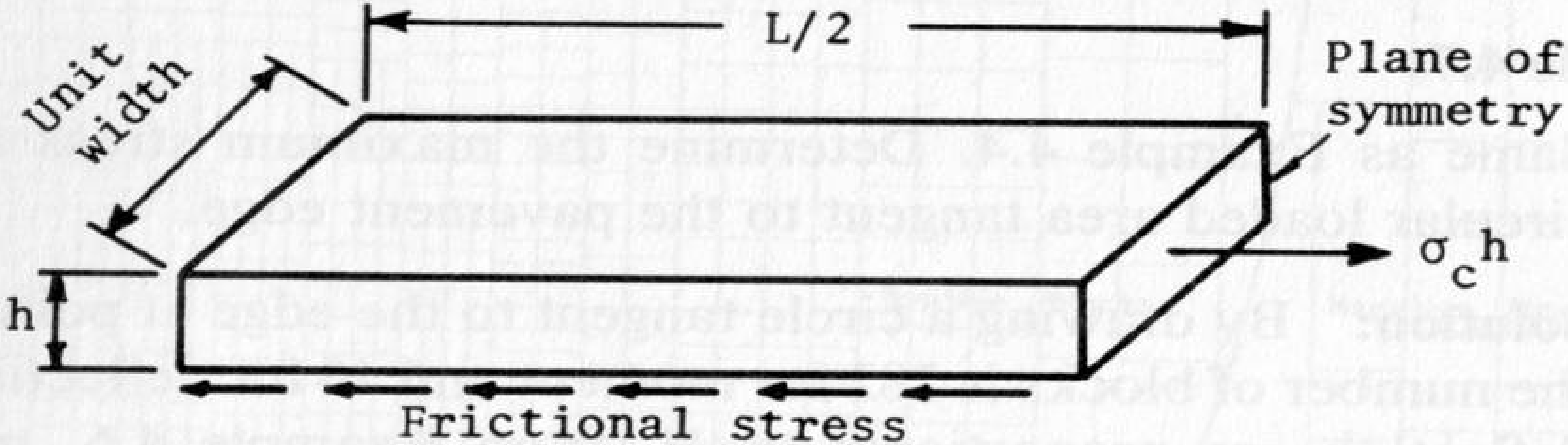
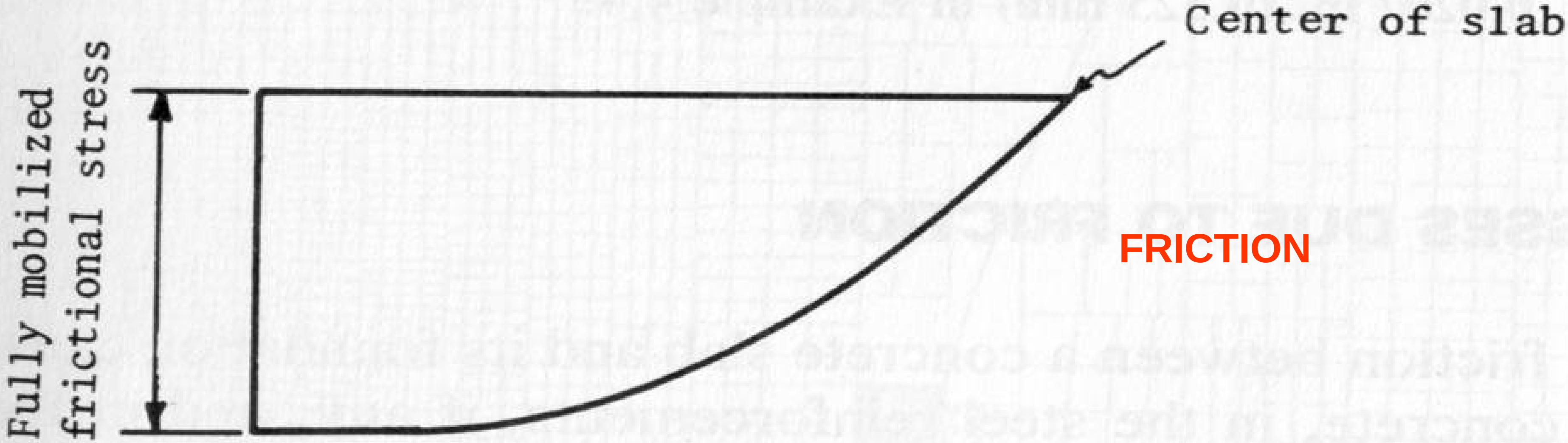


Figure 3.4. Warping stress coefficients. (From Bradbury.)



(a) Free Body Diagram



FRICTION

(b) Variation of Frictional Stress

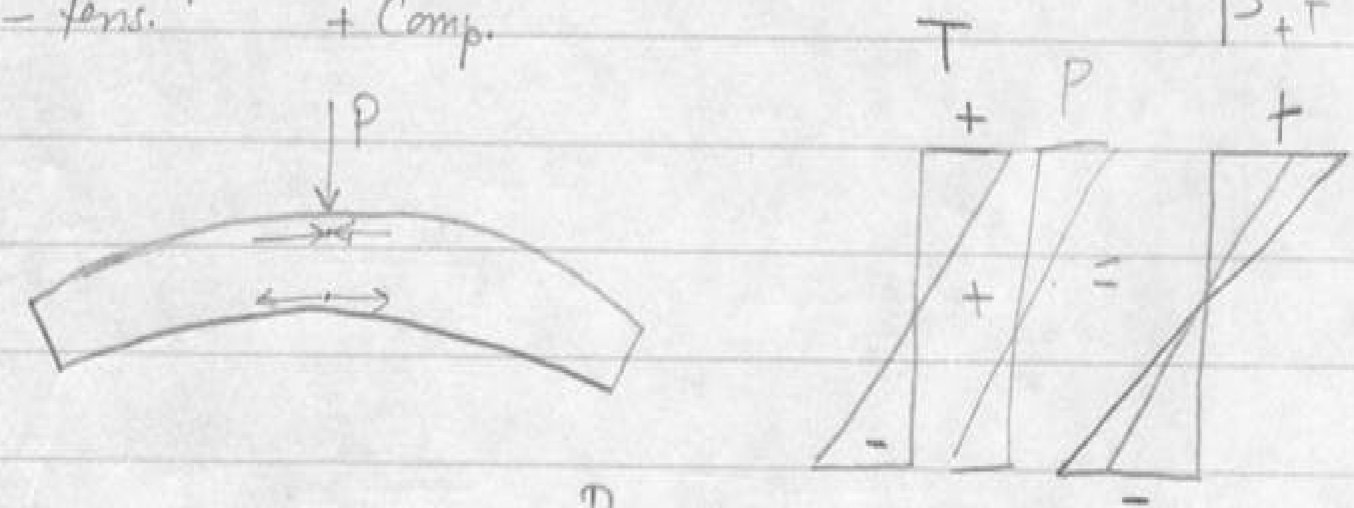
$$\sigma_c = \frac{WLf}{24h}$$

where σ_c = unit stress in the concrete in psi
W = weight of slab (psf)
L = length of slab in feet
f = average coefficient of subgrade resistance
h = depth of slab in inches

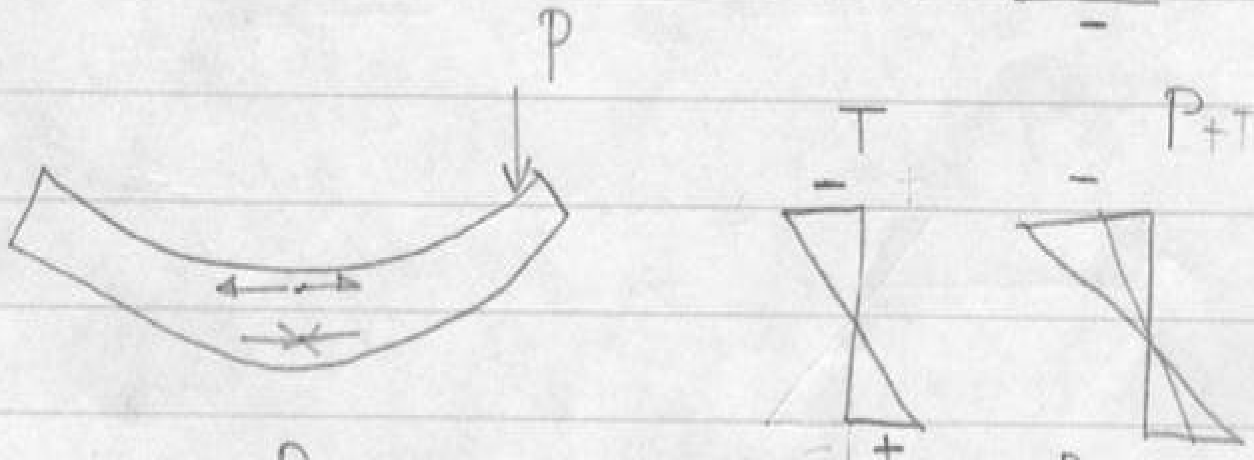
Interaction of temperature and load stresses:–

a) Warping – tens. + Comp.

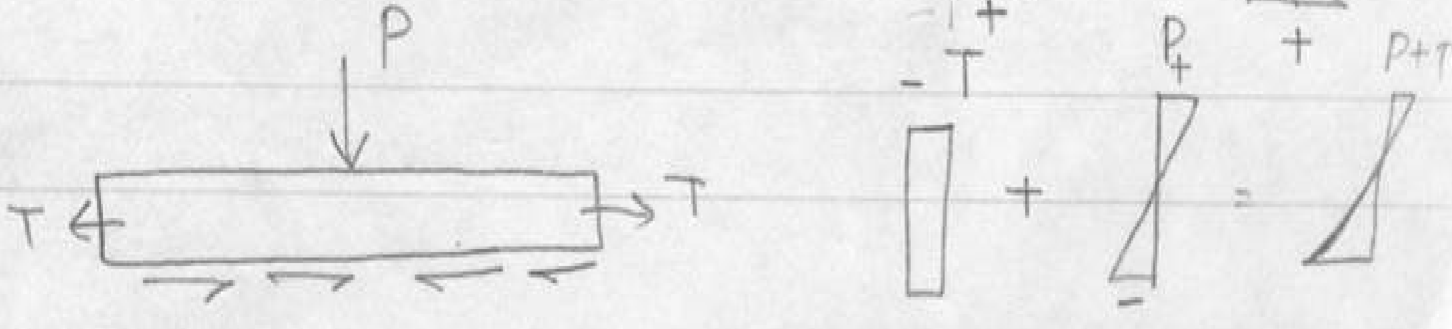
Day.

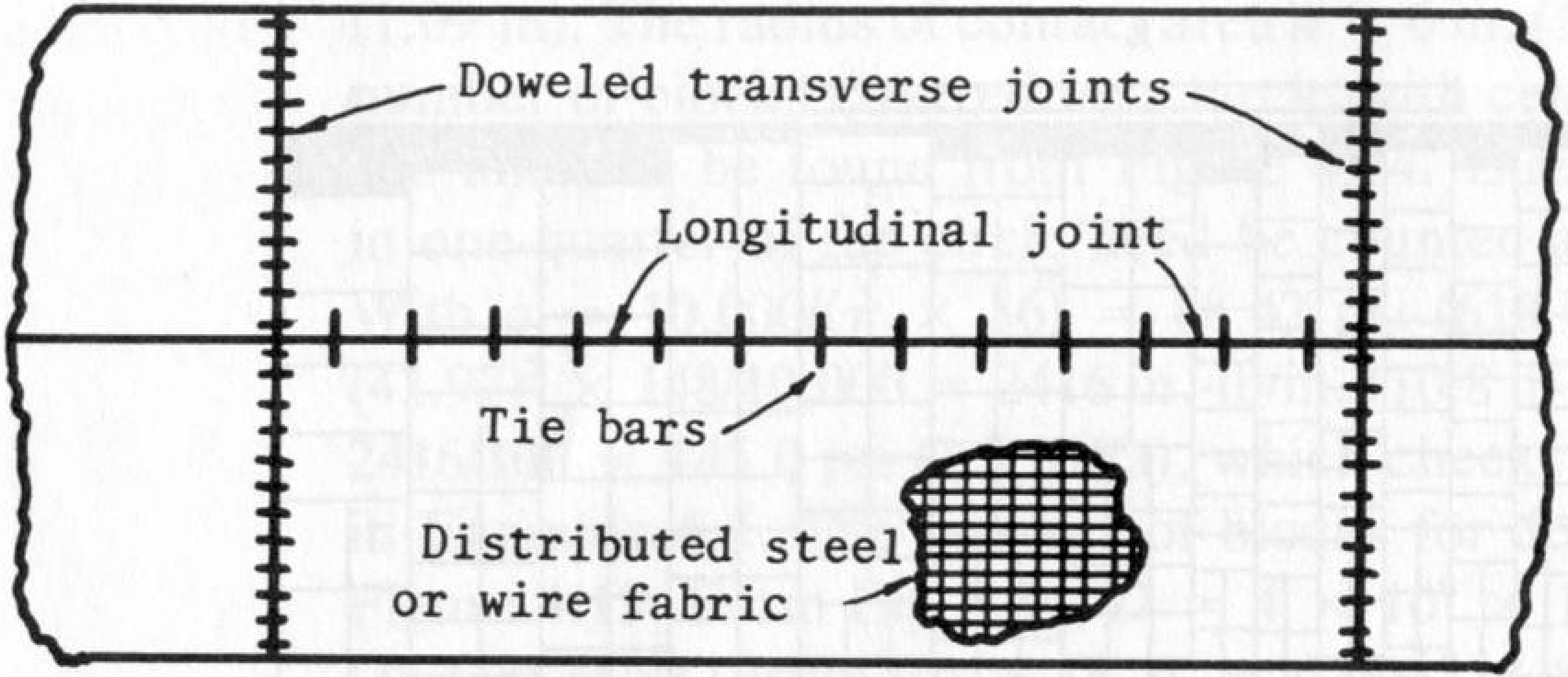


NIGHT



b) Friction




Doweled transverse joints
Longitudinal joint
Tie bars
Distributed steel
or wire fabric

1-Thermal Reinforcement

$$A_s = \frac{WfL}{2f_s}$$ Temp.

where A_s = required steel per foot of width

W = weight of slab (lb/ft^2) ($144\ lb/ft^3$)($h/12$)

f = coefficient of resistance (generally assumed to be 1.5)

f_s = allowable stress in steel

L = length of slab

2-Tie Bars

$$A_s = \frac{WfLd}{f_s}$$

where W = weight of the slab (psf)

f = coefficient of resistance

L = lane width (ft)

f_s = allowable stress in steel (psi)

d = tie bar spacing

3 - Dowel Bars

Step 1: Find Joint Opening (z)

$$z = L(12)[\epsilon \Delta t + \delta]$$

where L = slab length (ft)

z = joint opening (amount a joint will open)

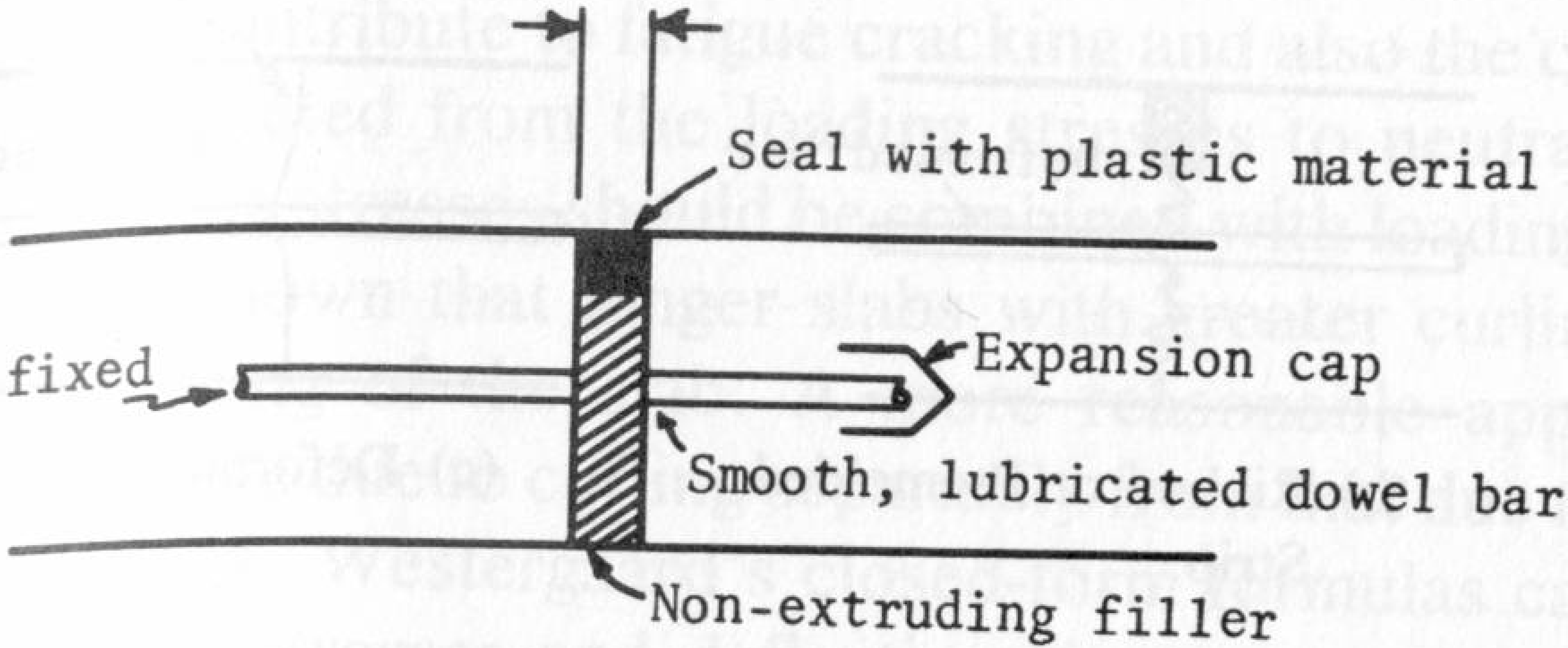
ϵ = coefficient of thermal volume change (0.000005 in./in./°F)

δ = coefficient of shrinkage (0.00005 in./in.)

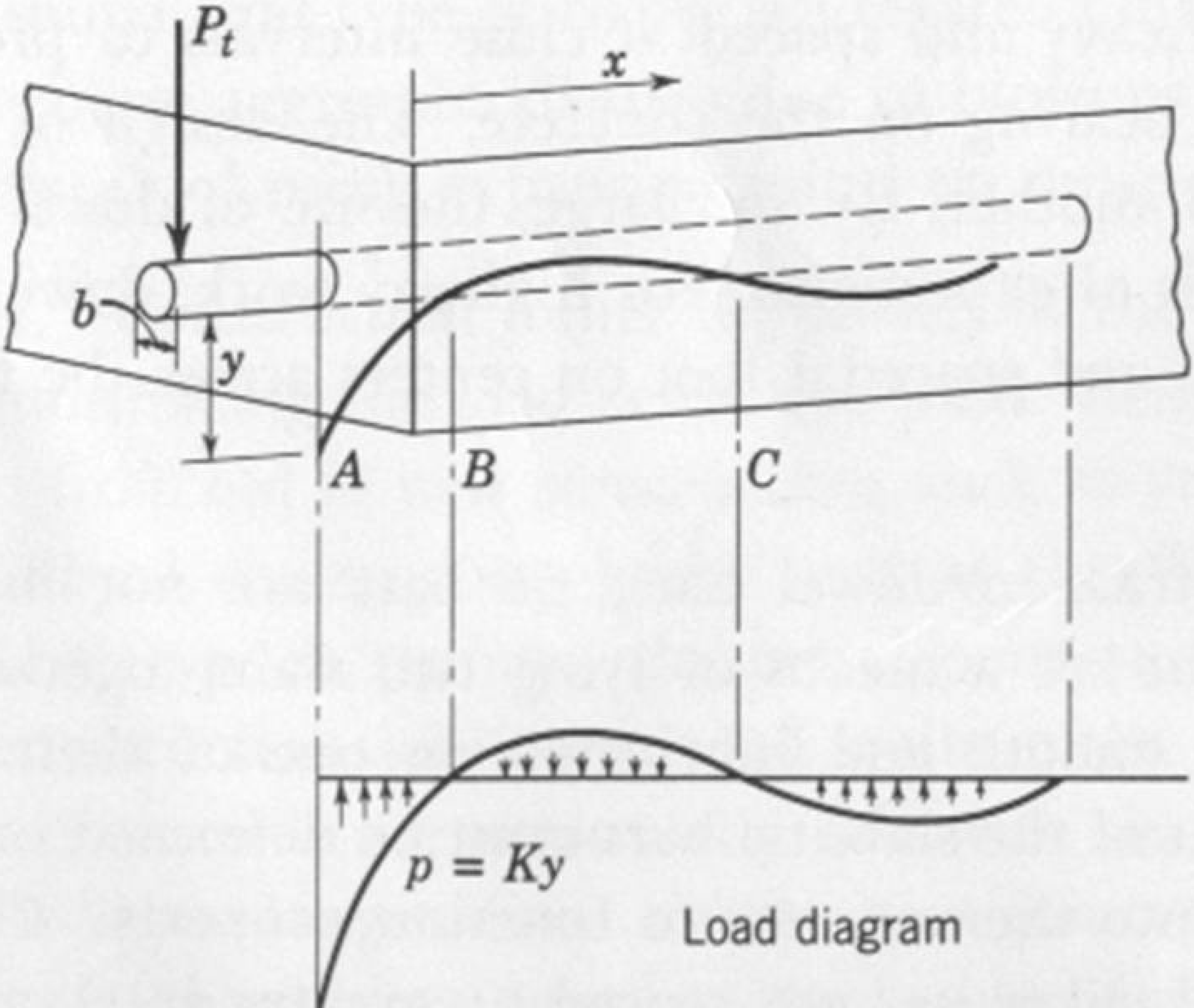
Δt = total temperature drop

Example: slab length= 50ft, Δt= 60^{o}F

Z= 50 (12)[0.000005 X 60+ 0.00005] = 0.21 in


Seal with plastic material
Expansion cap
fixed
Smooth, lubricated dowel bar
Non-extruding filler

Load transfer = 50 % of the applied load max.



LANE WIDTH = 12 ft , 6 ft - Axle

Step 2: Find critical Dowel load

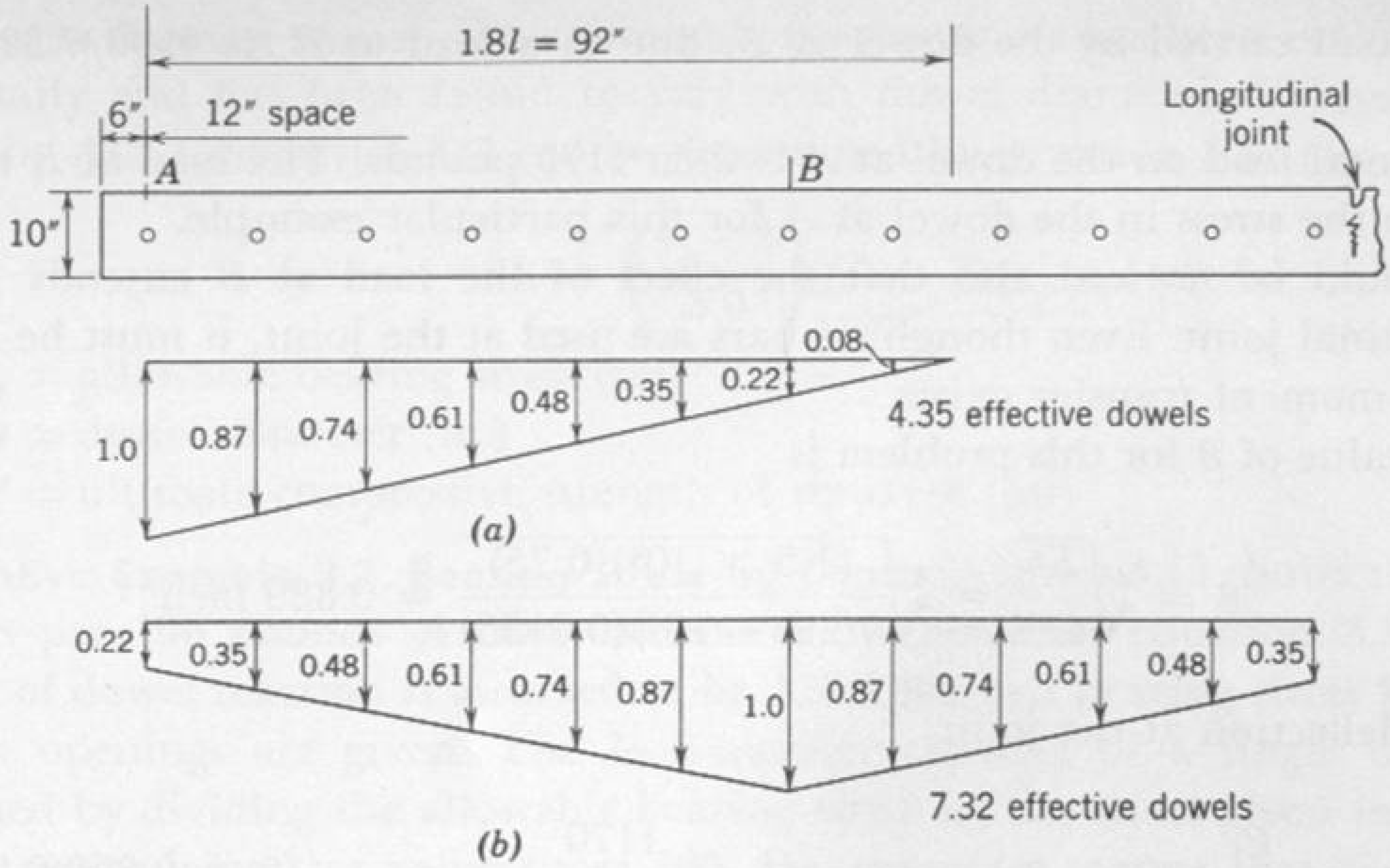
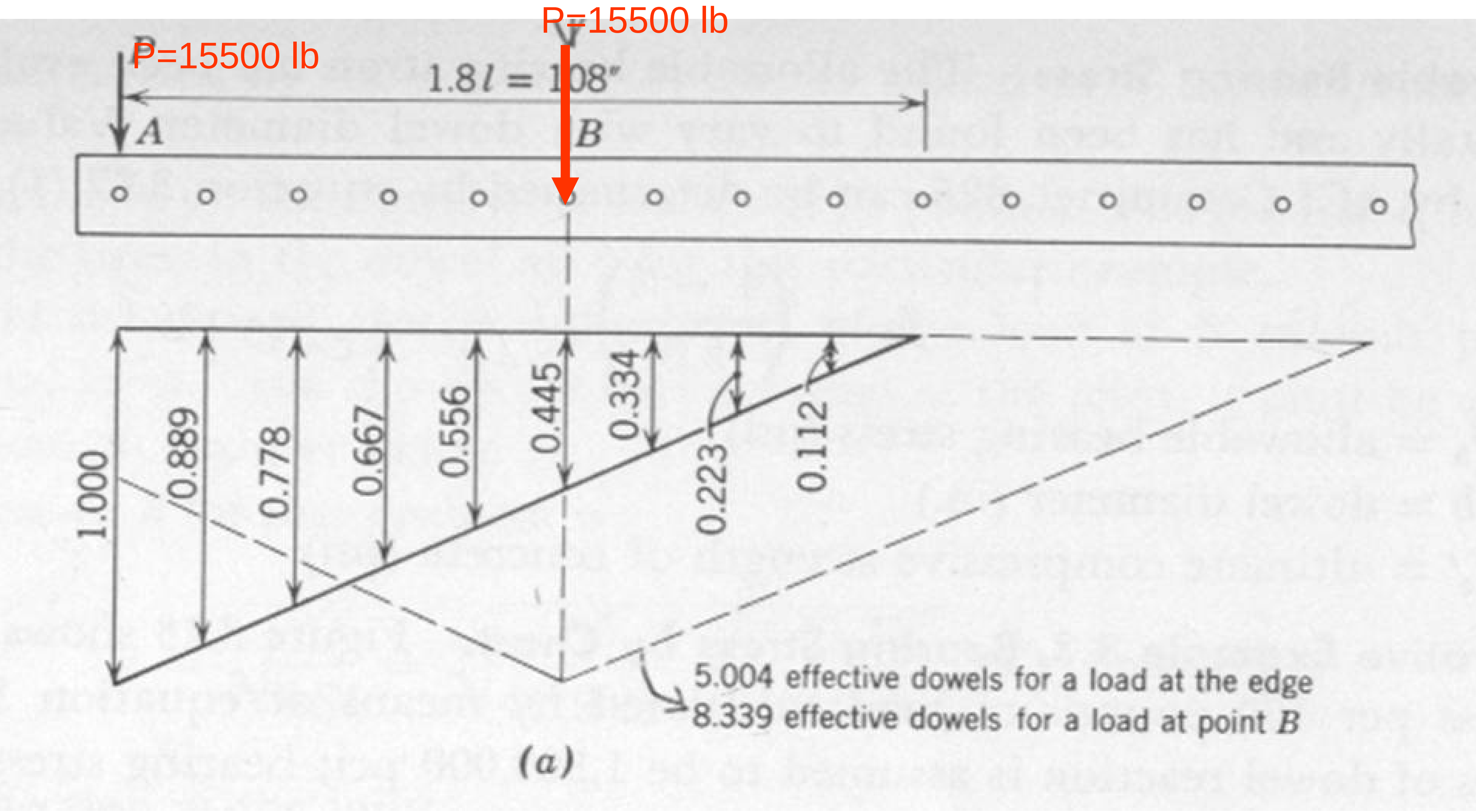


Figure 3.12. Loads on dowel group; pave = 10 inches, k = 50 psi, ¾-inch round dowels spaced 12 inches c–c. (*a*) Effective dowels due to load at *A*; (*b*) effective dowels due to load at *B*.



Given: Z= 0.25, Assume dowel diameter = 1 in,
Load transfer = 50 % of the applied load

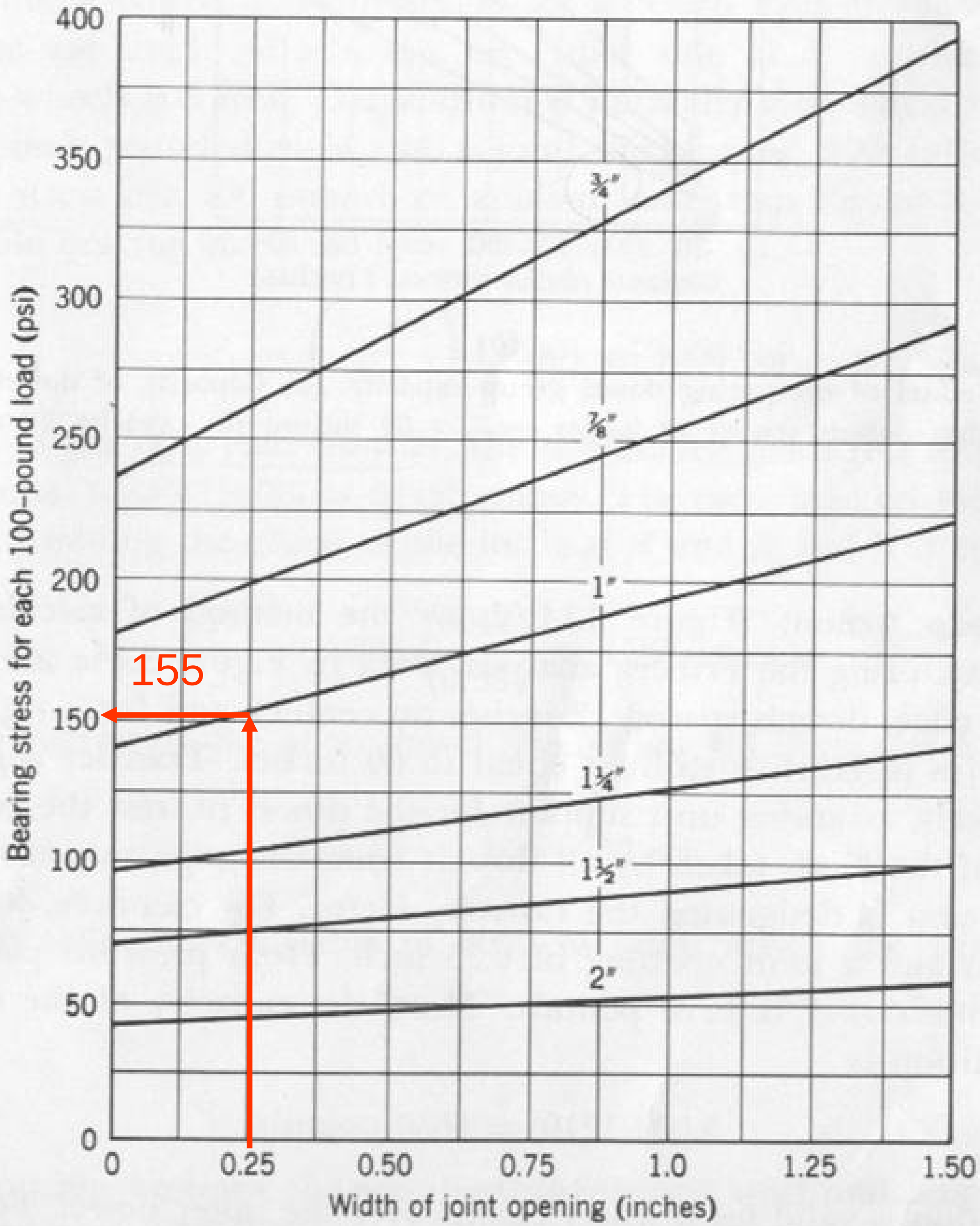
$$f_b = \left(\frac{4 - b}{3.0}\right) f_c'$$

where f_b = allowable bearing stress (psi)

b = dowel diameter (in.)

f_c' = ultimate compressive strength of concrete (psi)

= 3000 psi



Bearing stress on single dowels (modulus of dowel reaction K = 1,500,000 pci).

Allowable stress =
[(4-1)/3] X 3000= 3000 psi

Transferred load =
(50/100)(15500)[(1/5)+(0.44/8.3)]
= 1763 lb

Bearing stress =
(155/100) X 1763 =
2732 psi < 3000 OK.

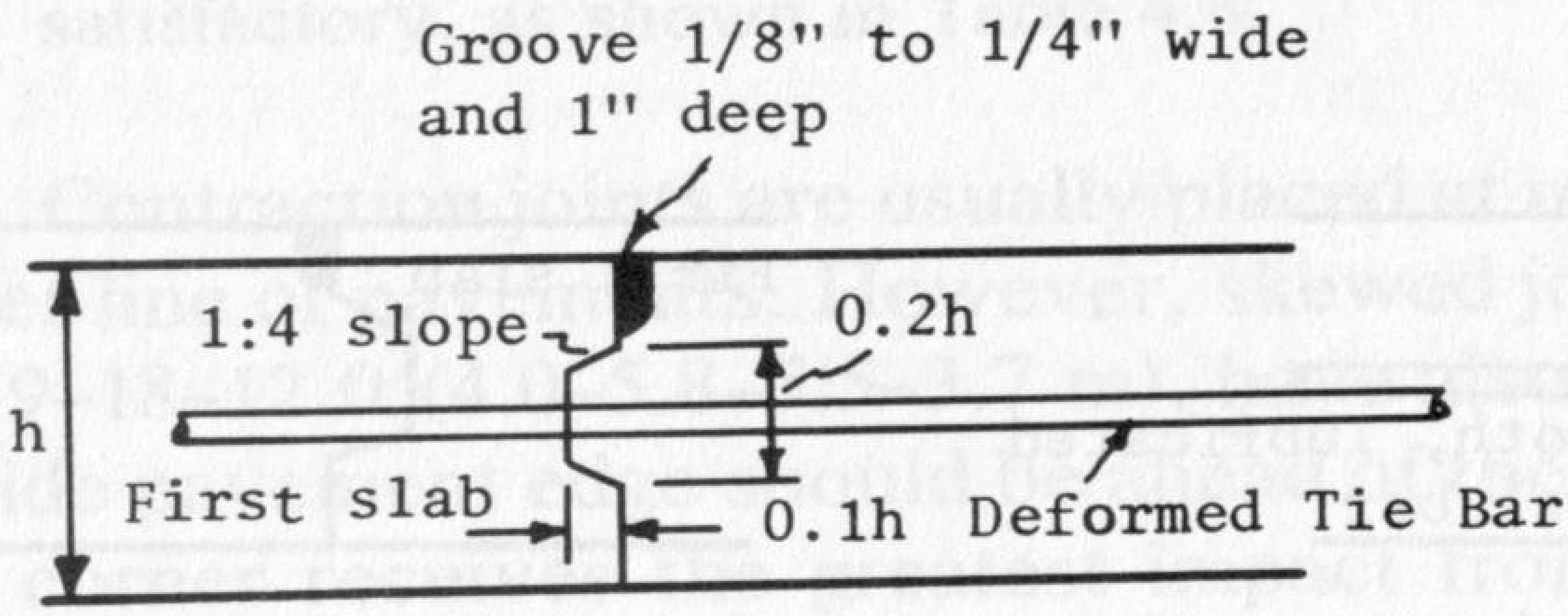


Figure 4.31 Longitudinal joints for lane-at-a-time construction (1 in. = 25.4 mm).

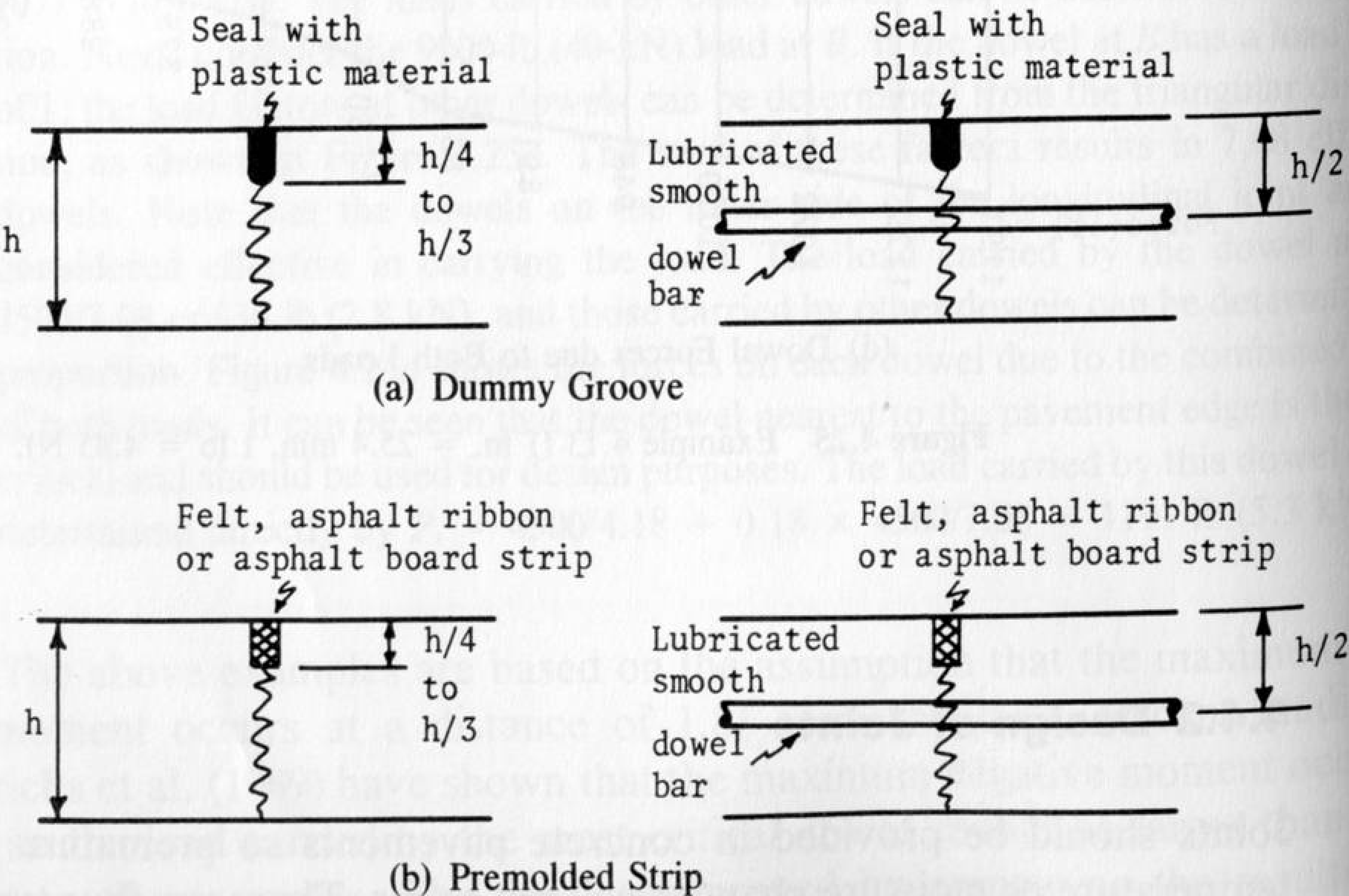
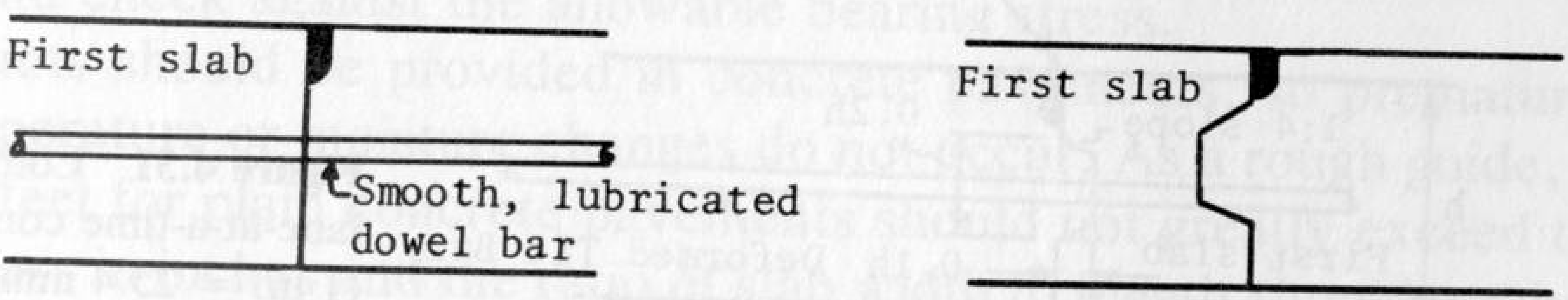


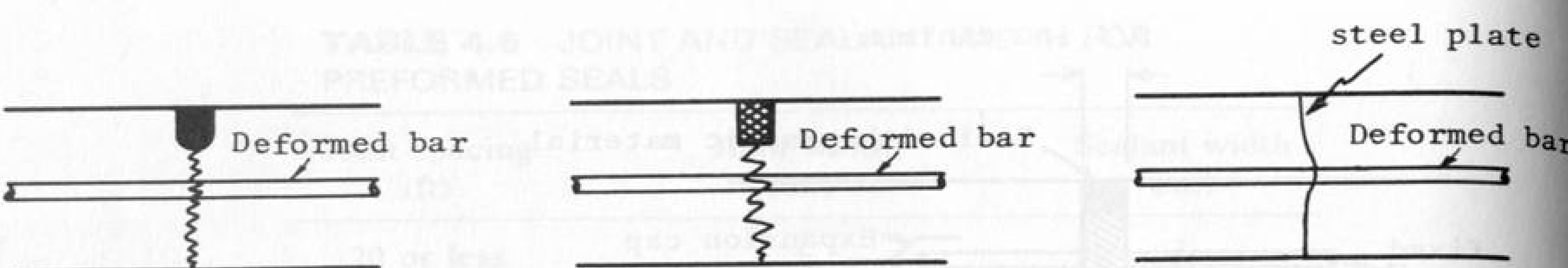
Figure 4.26 Typical contraction joints.



(a) Butt Joint at Contraction Joint.

(b) Key Joint for Emergency.

Figure 4.29 Construction joints.



(a) Dummy Groove.

(b) Ribbon or Premolded Strip.

(c) Deformed Plate.

Figure 4.30 Longitudinal joints for full-width construction.

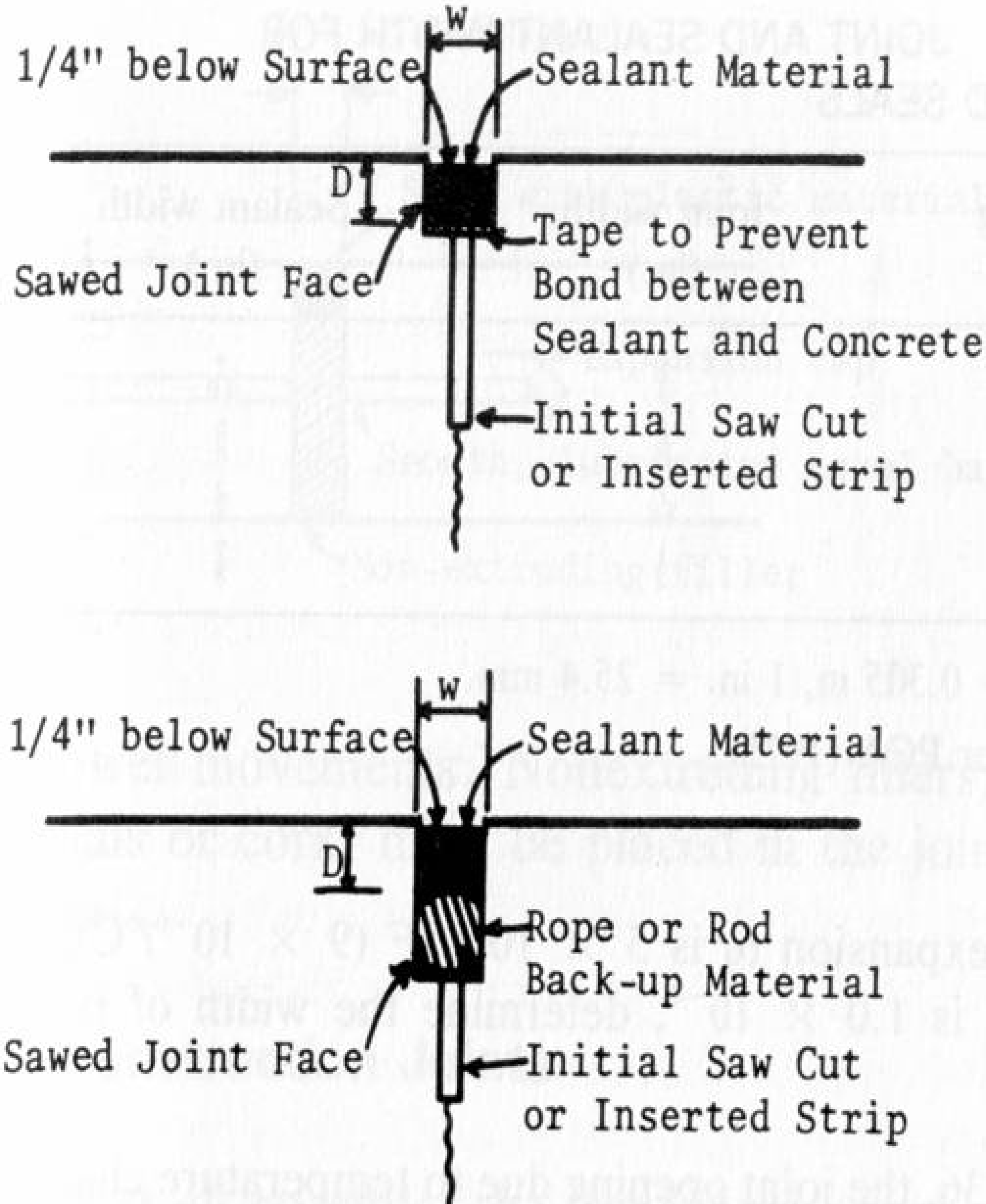


Figure 4.27 Design of joint sealant reservoir (1 in. = 25.4 mm). (After PCA (1975).)

TABLE 4.5 RESERVOIR DIMENSIONS FOR FIELD-MOLDED SEALANTS

Joint spacing (ft)	Reservoir width (in.)	Reservoir depth (in.)
15 or less	$\frac{1}{4}$	$\frac{1}{2}$ minimum
20	$\frac{3}{8}$	$\frac{1}{2}$ minimum
30	$\frac{1}{2}$	$\frac{1}{2}$ minimum
40	$\frac{5}{8}$	$\frac{5}{8}$

Note. 1 ft = 0.305 m, 1 in. = 25.4 mm.

Source. After PCA (1975).

TABLE 4.6 JOINT AND SEALANT WIDTH FOR PREFORMED SEALS

Joint spacing (ft)	Joint width (in.)	Sealant width (in.)
20 or less	$\frac{1}{4}$	$\frac{7}{16}$
30	$\frac{3}{8}$	$\frac{5}{8}$
40	$\frac{7}{16}$	$\frac{3}{4}$
50	$\frac{1}{2}$	$\frac{7}{8}$

Note. 1 ft = 0.305 m, 1 in. = 25.4 mm.

Source. After PCA (1975).