# Ch. 6 Development of Reinforcement

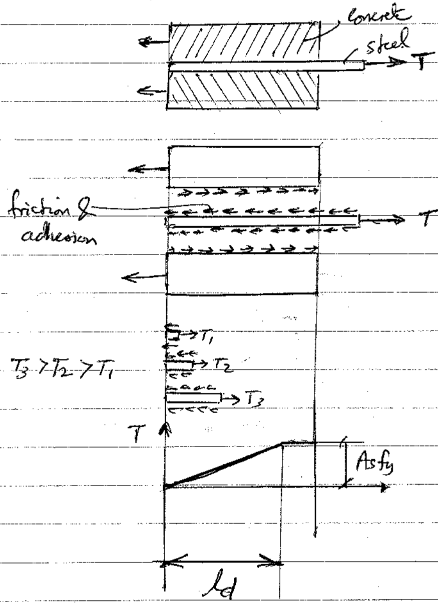
Behavior of the Reinforced concrete (RC) members:

- Bending (concrete + steel)
- Shear (concrete + stirrups)
- Bond (development of reinforcement)

Two types of failure under bond

① **Pullout failure**

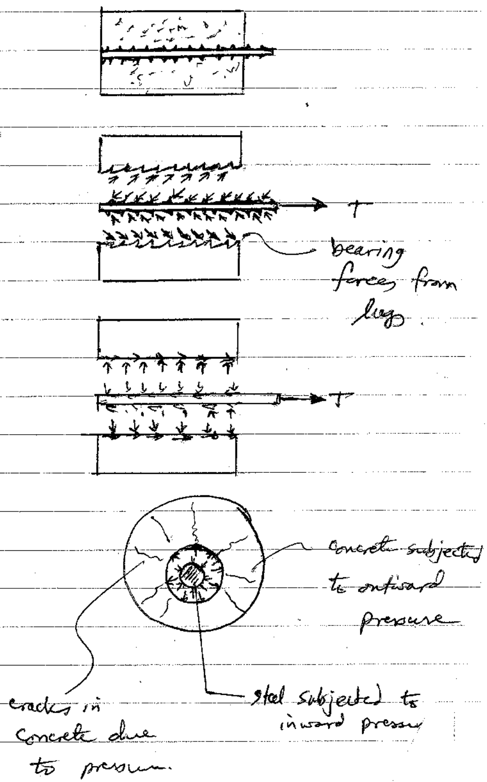
plain bars



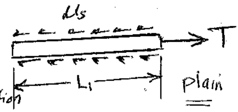

$\boxed{\text{Force in Steel developed gradually}}$

② **Splitting failure**

deformed bars with lugs



$L_1 = AB$. If $u_s$ is the failure stress against slippage acting over the nominal surface area $\pi d_b L_1$, then

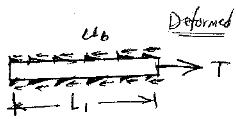



$$\underbrace{u_s \pi d_b L_1}_{\text{surface area}} = f_y \underbrace{\pi \frac{d_b^2}{4}}_{\text{cross section area}}$$

or

$$\boxed{L_1 = \frac{f_y}{4u_s} d_b} \qquad \textbf{(6.2.1)}$$

On the other hand, if $u_b$ is the failure stress against splitting and $A_{br}$ is the average bearing area per unit length, then




$$u_b A_{br} L_1 = f_y \pi \frac{d_b^2}{4}$$

or

$$\boxed{L_1 = \frac{f_y}{A_{br} u_b} \pi \frac{d_b^2}{4}} \qquad \textbf{(6.2.2)}$$

The same situation exists in free body *BC*, as shown in Fig. 6.2.1(c). Thus the maximum tensile force at *B* has to develop by embedment in both directions from *B*; that is, both the *AB* and *BC* distances. Where space limitations prevent providing the proper amount of straight embedment, such bars may be terminated by standard hooks (as defined in ACI-7.1). A standard hook is permitted to be considered as contributing to an equivalent development length by, mechanical action (ACI-12.5), thus reducing the total embedment dimension required. Section 6.11 provides treatment of development length with standard hooks.

Adequate development length must be provided for a reinforcing bar in compression as well as in tension.

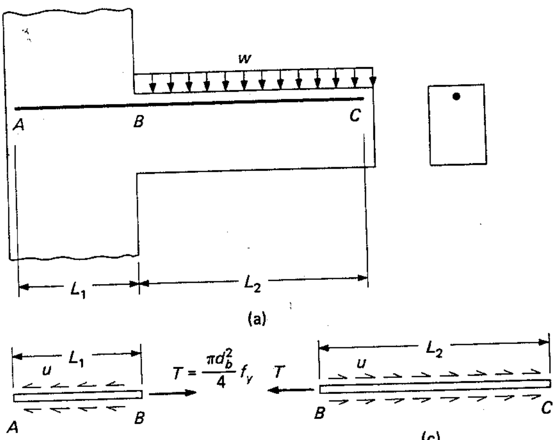



**Figure 6.2.1** Development of reinforcement.

## 6.4 Failure Modes

The term "bond failure" has been given to the mechanism by which failure occurs when inadequate development length is provided. Years ago, when plain bars (relatively smooth bars without lug deformations) were used, slip resistance ("bond") was thought of as adhesion between concrete paste and the surface of the bar. Yet even with low tensile stress in the reinforcement, there was sufficient slip immediately adjacent to a flexural crack in the concrete to break the adhesion, leaving only friction to resist bar movement relative to the surrounding concrete over the slip length.

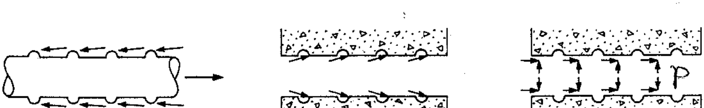
Shrinkage can also cause frictional drag against the bars. Typically, a hot-rolled *plain* bar may pull loose by longitudinal splitting when the adhesion and friction resistances are high, or just pull out leaving a cylindrical hole when adhesion and friction resistances are low.

Deformed bars were created to change the behavior pattern so that there would be less reliance on friction and adhesion (though they still exist) and more reliance on the bearing of the lugs against the concrete. The bearing forces act at an angle to the axis of the bar, causing radial outward components against the concrete, as shown in Fig. 6.4.1. When inadequate development length is provided, deformed bars in normal-weight concrete give rise to a splitting mode of failure (i.e., "bond failure") [6.1, 6.5, 6.7]. A splitting failure occurs when the wedging action of the steel lugs on a deformed bar causes cracks in the surrounding concrete parallel to the bar. These cracks occur between the bar and the nearest concrete face, as shown in Fig. 6.4.2(a, b), or over the short distance between bars when bars are closely spaced, as in Fig. 6.4.2(c).

When small size bars are used with large cover, the lugs may crush the concrete by bearing and result in a pullout failure without splitting the concrete. This nonsplitting failure has also been reported for larger bars in structural lightweight concrete [6.1].

Although splitting is the usual failure mode, an initial splitting crack on one face of a beam is *not* considered failure. The distress sign indicating failure is *progressive splitting*. Confinement of tension steel by stirrups, ties, or spirals usually will delay collapse (commonly defined as an increase in loading that results in no increase in resistance) until several splitting cracks have formed.

Originally, development length requirements were based on pullout tests [6.8] of plain bars, followed by pullout tests [6.9–6.15] of deformed bars, including the related load-slip data. Since confinement exists in pullout tests, the early work did not give sufficient emphasis to the splitting mode of failure. Splitting has been emphasized in the more recent studies by Orangun, Jirsa, and Breen [6.3, 6.4],




(a) On bar (b) On concrete (c) Components

concrete cylinder under pressure P

**Figure 6.4.1** Forces between bar and surrounding concrete.

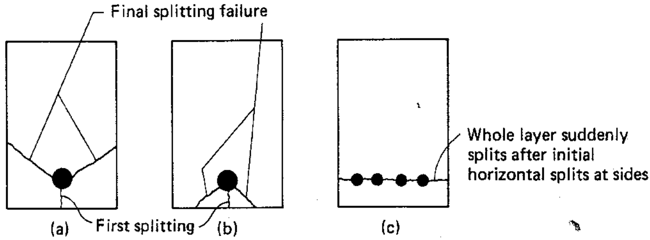



**Figure 6.4.2** Splitting cracks and ultimate splitting failure modes (from ACI Committee 408 [6.1]).

Untrauer and Warren [6.16], Kemp and Wilhelm [6.17], Morita and Kaku [6.18], Jimenez, White, and Gergely [6.19], Kemp [6.7], Mirza [6.20], Moehle, Wallace, and Hwang [6.21], Darwin, McCabe, Idun, Schoenekase [6.22], Lutz, Mirza, and Gosain [6.23], and Hwang, Leu, and Hwang [6.24].

The studies of Orangun, Jirsa, and Breen [6.3] and Untrauer and Warren [6.16] have hypothesized that the action of splitting arises from a stress condition analogous to a concrete cylinder surrounding a reinforcing bar and acted upon by the outward radial components [Fig. 6.4.1(c)] of the bearing forces from the bar. The cylinder would have an inner diameter equal to the bar diameter $d_b$ and a thickness $C$ equal to the smaller of $C_b$, the clear bottom cover, or $C_s$, half of the clear spacing to the next adjacent bar (see Fig. 6.4.3). The tensile strength of this concrete cylinder determines the resistance against splitting. If $C_s < C_b$, a side-split type of failure occurs [Fig. 6.4.2(c)]. When $C_s > C_b$, longitudinal cracks through the bottom cover form first [first splitting cracks in Fig. 6.4.2(a), (b)]. If $C_s$ is only nominally greater than $C_b$, the secondary splitting will be side splitting along the plane of the bars. If $C_s$ is significantly greater than $C_b$, the secondary splitting will also be through the bottom cover to create a V-notch failure [Fig. 6.4.2(b)].

The proposal of ACI Committee 408 [6.5, 6.25] recognizes the cylinder hypothesis for splitting failure. The portion of the proposal relating to hooks (see Section 6.11) was adopted for the 1983 ACI Code, and the portion relating to straight bar development length formed the basis for the relatively complex

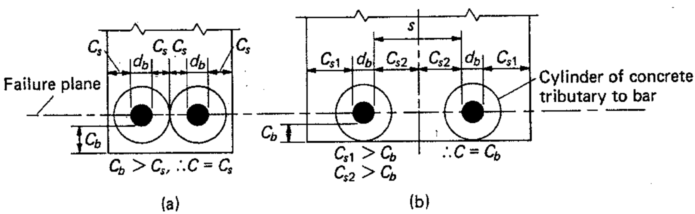



**Figure 6.4.3** Concrete cylinder hypothesis for splitting failure (from Orangun, Jirsa, and Breen [6.3]).

Thus, the localized situation, relating to rate of change in moment, does not directly correlate with the development-length-related strength of the member. When the bars are properly anchored, that is, they have adequate development length provided and continue to carry their required tensile force, the localized stress condition is not of concern.

## 6.6 Moment Capacity Diagram—Bar Bends and Cutoffs

As stated in Section 6.2, the moment capacity of a beam at any section along its length is a function of its cross-section and the actual embedment length of its reinforcement. The concept of a diagram showing this three-dimensional relationship can be a valuable aid in determining cutoff or bend points of longitudinal reinforcement. It may be recalled from Chapter 3 that in terms of the cross section, the moment capacity (i.e., strength) for a singly reinforced rectangular may be expressed

$$M_n = A_s f_y (d - a/2) \qquad \textbf{[3.8.1]}$$

Equation (3.8.1) assumes that the steel reinforcement comprising $A_s$ is adequately embedded *in each direction* by the required development length $L_d$ from the section where $M_n$ is computed such that the stress $f_y$ is reached.

■ **EXAMPLE 6.6.1** Compute and draw the moment capacity diagram qualitatively for the beam of Fig. 6.6.1.

***Solution:*** The procedure is basically the same whether strength ($M_n$ or $\phi M_n$) or working stress moment capacity is desired.

The maximum capacity in each region is represented by the horizontal portions of the diagram in Fig. 6.6.1. In this example, there are five bars of one size in section *C–C*; thus the maximum moment capacity represented by each bar is in this case approximately one-fifth of the total capacity. Actually, the sections with four and two bars will have a little more than four-fifths and two-fifths, respectively, of the total capacity of the section containing five bars, due to the slight increase in moment arm when the number of bars in the section decreases.

At point *a*, the location where the fifth bar terminates, this bar has zero embedment length to the left and thus has zero capacity. Proceeding to the right from point *a*, the bar may be counted on to carry a tensile force proportional to its embedment from point *a* up to the development length $L_d$. Thus, in Fig. 6.6.1, point *b* represents the point where the fifth bar is fully developed through the distance $L_d$ and can therefore carry its full tensile capacity. The other cutoff points are treated in the same way. ■

■ **EXAMPLE 6.6.2** Demonstrate qualitatively the practical use of the moment capacity $\phi M_n$ diagram for verification of the locations of cutoff or bend points in a design. Assume that the main cross-section with five equal-sized bars provides exactly the required strength at midspan for this simply supported beam with uniform load, as shown in Fig. 6.6.2.

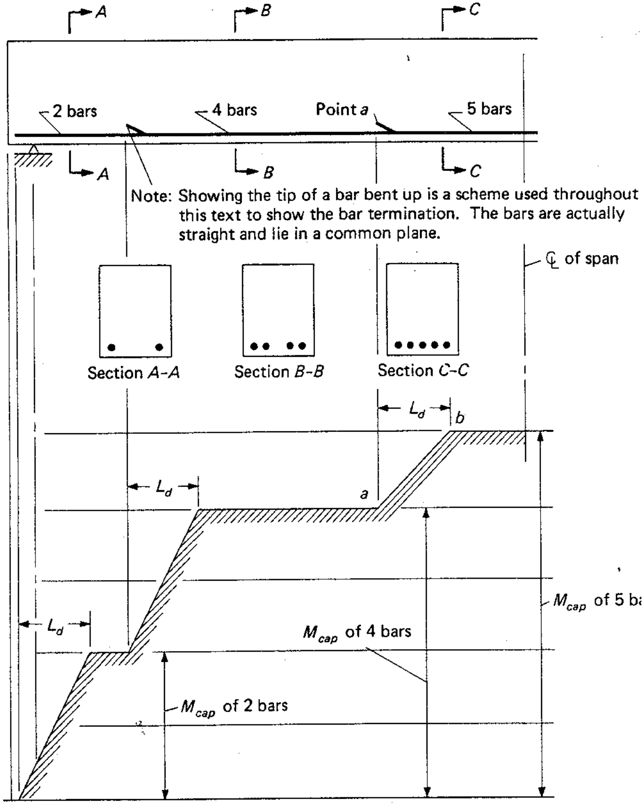
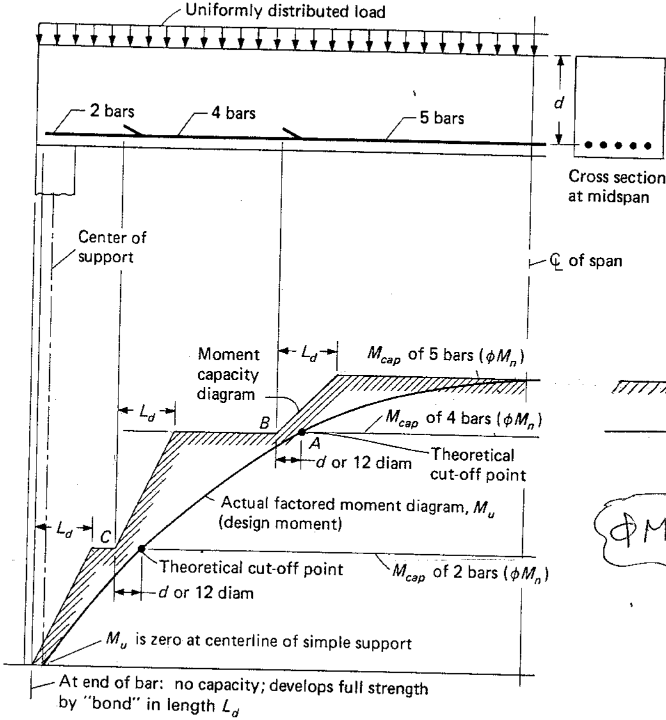



**Figure 6.6.1** Moment capacity diagram.

***Solution:*** **(a)** Compute the actual $\phi M_n$ for each potential bar grouping that may be used; in the present case, for five bars, four bars, and two bars.

**(b)** Decide which bars must extend entirely across the span and into the support. ACI-12.11.1 states that "At least one-third the positive moment reinforcement in simple members . . . shall extend along the same face of member into the support." In beams, the reinforcement must extend into the support at least 6 in. In this case, two bars should extend into the support.

**(c)** Decide on the order of cutting or bending the remaining bars. The least amount of longitudinal reinforcement will be obtained when the resulting moment capacity $\phi M_n$ diagram is closest to the factored moment $M_u$ diagram. With that thought in mind, and proceeding from maximum moment region to the support, cut off one bar as soon as permissible.




$\phi M_n$

$M_u$

$\phi M_n \geq M_u$

**Figure 6.6.2** Verification of bar cutoffs with the moment capacity diagram.

**(d)** Cutoff restrictions. Point $A$ of Fig. 6.6.2 is the theoretical location to the left of which the capacity represented by the remaining four bars is adequate. To provide for a safety factor against shifting of the moment $M_u$ diagram (especially in continuous spans) and to provide partially for the complexity arising from a potential diagonal crack, the ACI Code provides that there must be an extension beyond the point where a bar theoretically may be terminated, or it may be bent into the compression face. In ACI-12.10.3 is the statement, "Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the effective depth of the member or 12 bar diameters, whichever is greater, except at supports of simple spans and at free end of cantilevers."

**(e)** Once cutoff or bend points are located, a check is made by drawing the moment capacity $\phi M_n$ diagram to ensure no encroachment on the factored moment $M_u$ diagram.

**(f)** Other restrictions. Since points $B$ and $C$ of Fig. 6.6.2 are bar terminations in a tension zone, the stress concentrations described in Section 6.5 are present, effectively reducing the shear strength of the beam [6.26, 6.27]. Thus, *one* of the three special conditions of ACI-12.10.5 must be satisfied for cutoffs to be acceptable. However, if these bars were bent up and anchored in the compression zone, no further investigation would be necessary. ■

## 6.7 Development Length for Tension Reinforcement —ACI Code

The term "development length" has been defined in Sec. 6.2 as the length of embedment needed to develop the yield stress in the reinforcement. As described in Section 6.4, the development length requirement is primarily a function of the splitting resistance of the concrete surrounding the bars rather than a frictional-adhesional pullout resistance. The splitting resistance is roughly proportional to the bar area, indicated by Eq. (6.2.2); whereas the pullout resistance is roughly proportional to the bar diameter, indicated by Eq. (6.2.1).

In the 1989 ACI Code, completely new bar development provisions were adopted (ACI-12.2), recognizing the effects of (a) lateral spacing of bars being developed, (b) clear cover over bars being developed, and (c) confinement, if any, by stirrups, ties, or spirals around the bars being developed. Those provisions are described in detail in the 5th edition of this text, and are not repeated here.

Because of the seeming complexity of the 1989 provisions for bar development, and in response to strong encouragement from the profession, ACI Committee 318 revised the requirements for the 1995 ACI Code.

The 1995 Code provisions are based on the same basic relationship developed by Orangun, Jirsa, and Breen [6.3, 6.4] that formed the basis for the 1989 Code provisions. The 1995 provisions are also influenced by a more recent study Sozen and Moehle [6.28].

The general equation, after some tampering with the Orangun, Jirsa, and Breen [6.3, 6.4] format, is given in ACI-12.2.3 as ACI Formula (12-1),

$$\frac{L_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c + K_{tr}}{d_b}\right)} \qquad \textbf{(6.7.1)*}$$

where

$L_d$ = development length

$d_b$ = nominal diameter of bar or wire

$c$ = cover or spacing dimension

= the smaller of (1) distance from center of bar being developed to the nearest concrete surface, and (2) one-half the center-to-center spacing of bars being developed

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* For SI, with $f_y$ and $f'_c$ in MPa,

$$\frac{L_d}{d_b} = \frac{15}{16}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c + K_{tr}}{d_b}\right)} \qquad \textbf{(6.7.1)}$$

The transverse reinforcement term $K_{tr}$ is defined as follows:

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500sn} \tag{6.7.2}^*$$

where

$A_{tr}$ = total cross-sectional area of all transverse reinforcement which is within the spacing $s$ and which crosses the potential plane of splitting through the reinforcement being developed

$f_{yt}$ = specified yield strength of transverse reinforcement, psi

$s$ = maximum center-to-center spacing of transverse reinforcement within development length $L_d$

$n$ = number of bars being developed along the plane of splitting

In the use of Eq. (6.7.1), the cover and transverse reinforcement term cannot be taken greater than 2.5; thus,

$$\left(\frac{c + K_{tr}}{d_b}\right) \le 2.5 \tag{6.7.3}$$

The symbols $\alpha$, $\beta$, $\gamma$ and $\lambda$ in Eq. (6.7.1) represent the following modification factors:

$\alpha$ = modification factor for reinforcement location
= 1.3 for top bars†
= 1.0 for other bars

$\beta$ = modification factor for epoxy-coated reinforcement
= 1.5 when cover $< 3d_b$ or clear spacing $< 6d_b$
= 1.2 other epoxy-coated reinforcement
= 1.0 non-epoxy-coated reinforcement

$\alpha\beta$ = need not exceed 1.7

$\gamma$ = modification factor for bar size
= 0.8 for #6 and smaller bars and deformed wire
= 1.0 for #7 and larger bars

$\lambda$ = modification factor for lightweight aggregate concrete
= 1.3 for lightweight aggregate concrete
(or $6.7\sqrt{f'_c}/f_{ct} \ge 1.0$ when $f_{ct}$ is specified)
= 1.0 for normal-weight concrete

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*For SI, with $f_{yt}$ in MPa,

$$K_{tr} = \frac{A_{tr} f_{yt}}{260sn} \tag{6.7.2}$$

†Top bars are defined in ACI-12.2.4 as "Horizontal reinforcement so placed that more than 12 in. of fresh concrete is cast in the member below the development length or splice."

**Table 6.7.3** Development Length for Category A*, Eqs. (6.7.4) and (6.7.5) with $\alpha\beta\lambda = 1.0$

1996 ASTM METRIC BARS WITH $L_d$ IN CENTIMETERS

| | $f_y = 300$ MPa | | | $f_y = 420$ MPa | | |
|---|---|---|---|---|---|---|
| | $f'_c$ (MPa) | | | $f'_c$ (MPa) | | |
| BAR | 25 | 30 | 35 | 25 | 30 | 35 |
| #10M | 30.0 | 30.0 | 30.0 | 39.9 | 36.4 | 33.7 |
| #13M | 38.1 | 34.8 | 32.2 | 53.3 | 48.7 | 45.1 |
| #16M | 47.7 | 43.5 | 40.3 | 66.8 | 61.0 | 56.4 |
| #19M | 57.3 | 52.3 | 48.4 | 80.2 | 73.2 | 67.8 |
| #22M | 83.3 | 76.0 | 70.4 | 117 | 106 | 98.5 |
| #25M | 95.3 | 87.0 | 80.5 | 133 | 122 | 113 |
| #29M | 108 | 98.2 | 91.0 | 151 | 138 | 127 |
| #32M | 121 | 111 | 102 | 170 | 155 | 143 |
| #36M | 134 | 123 | 113 | 188 | 172 | 159 |
| #43M | 161 | 147 | 136 | 226 | 206 | 191 |
| #57M | 215 | 196 | 182 | 301 | 275 | 254 |

*(a) Clear spacing and clear cover $\geq d_b$ and minimum stirrups, ***or***
**(b)** clear spacing $\geq 2d_b$ and clear cover $\geq d_b$

**Table 6.7.4** Development Length for Category B*, Eqs. (6.7.6) and (6.7.7) with $\alpha\beta\lambda = 1.0$

1996 ASTM METRIC BARS WITH $L_d$ IN CENTIMETERS

| | $f_y = 300$ MPa | | | $f_y = 420$ MPa | | |
|---|---|---|---|---|---|---|
| | $f'_c$ (MPa) | | | $f'_c$ (MPa) | | |
| BAR | 25 | 30 | 35 | 25 | 30 | 35 |
| #10M | 42.8 | 39.0 | 36.1 | 59.9 | 54.6 | 50.6 |
| #13M | 57.2 | 52.2 | 48.3 | 80.0 | 73.0 | 67.6 |
| #16M | 71.6 | 65.3 | 60.5 | 100 | 91.4 | 84.7 |
| #19M | 86.0 | 78.5 | 72.6 | 120 | 110 | 102 |
| #22M | 125 | 114 | 106 | 175 | 160 | 148 |
| #25M | 143 | 130 | 121 | 200 | 183 | 169 |
| #29M | 161 | 147 | 136 | 226 | 206 | 191 |
| #32M | 182 | 166 | 154 | 254 | 232 | 215 |
| #36M | 201 | 184 | 170 | 282 | 257 | 238 |
| #43M | 242 | 221 | 204 | 339 | 309 | 286 |
| #57M | 322 | 294 | 272 | 451 | 412 | 381 |

*Everything not in Category A.

**Practical Application of ACI-12.2 Development Length Rules.** The practicality for applying the rules in ordinary reinforced concrete construction is that most beams will contain at least ACI Code-specified minimum stirrups (thereby satisfying Category A, item 1c), clear spacing must satisfy the larger of the bar diameter $d_b$ or 1 in. (ACI-7.6.1), and cover must satisfy the minimum specified in ACI-7.7.1 in any case. Using the minimum 1.5 in. of cover on beams will commonly provide the Category A minimum of $d_b$. For slab-like elements without shear reinforcement, clear spacing will usually satisfy the Category A, item 2a,

■ **EXAMPLE 6.8.1** Determine the development length $L_d$ required for the #9 epoxy-coated bars $A$ on the top of a 15-in. slab, as shown in Fig. 6.8.1. Use $f_y = 60{,}000$ psi, and $f'_c = 4000$ psi with lightweight aggregate concrete.

***Solution:*** **(a)** Determine the development length $L_d$ using the simplified equations. Since cover of 1.5 in. exceeds $d_b$ of 1.128 in., and the 8 in. bar spacing exceeds clear spacing of $2d_b$ (i.e., 2.3 in.), the situation is Category A, item 2, and for #9 bars Eq. (6.7.5) applies,

$$\frac{L_d}{d_b} = \frac{f_y}{20\sqrt{f'_c}}\alpha\beta\lambda \qquad [6.7.5]$$

$$= \frac{60{,}000}{20\sqrt{4000}}\alpha\beta\lambda = 47.4\alpha\beta\lambda$$

$$L_d = 47.4 d_b \alpha\beta\lambda = 47.4(1.128)\alpha\beta\lambda = 53.5\alpha\beta\lambda$$

Note that 53.5 in. agrees with the value in Table 6.7.1.

Referring to Fig. 6.8.1, when checking the bar spacing, bars $A$ are developed over distance 1-2, while bars $B$ are developed over the distance 2-3. The spacing

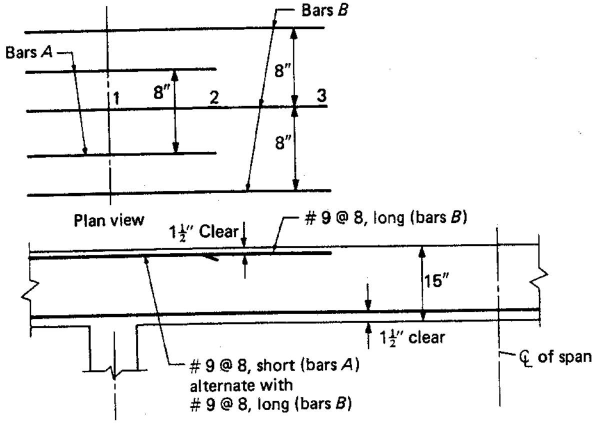


**Figure 6.8.1** Top bars for Example 6.8.1.

to be used for bars $A$ is the spacing of the closest bars that terminate at the same point. In other words, the spacing for both bars $A$ and $B$ is 8 in.

**(b)** Modification $\alpha$ for top bars. Since the negative moment region bars are cast with more than 12 in. of fresh concrete below them, they are top bars according to ACI-12.2.4; thus $\alpha = 1.3$.

**(c)** Modification $\beta$ for epoxy-coated bars. Check clear cover,

$$\text{Clear cover} = \frac{1.5}{d_b} = \frac{1.5}{1.128} = 1.3d_b < 3d_b$$

Since clear cover is less than $3d_b$, $\beta = 1.5$. The maximum value of $\alpha\beta = 1.7$.

**(d)** Modification $\lambda$ for lightweight aggregate concrete. The lightweight aggregate concrete multiplier $\lambda = 1.3$.

**(e)** Final development length $L_d$.

$$L_d = 53.5\alpha\beta\lambda = 53.5(1.7)1.3 = 118 \text{ in}.$$

**(f)** Compute development length $L_d$. Using the general equation, Eq. (6.7.1).

$$\frac{L_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c + K_{tr}}{d_b}\right)} \qquad \textbf{[6.7.1]}$$

There are no stirrups; thus $K_{tr} = 0$. The value of $c$ for Eq. (6.7.1) is the smaller of the cover (i.e., the distance from the center of the bar to the nearest concrete face) or one-half the center-to-center spacing of the bars being developed. In this case,

$$\text{Cover} = 1.5 + 1.128/2 = 2.06 \text{ in.}$$

$$(\text{Center-to-center spacing})/2 = 8/2 = 4 \text{ in.}$$

2.06

Thus, $\left(\dfrac{c + K_{tr}}{d_b} = \dfrac{2.06 + 0}{1.128} = 1.83\right) \leq 2.5$ **[6.7.3]**

$$\frac{L_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c + K_{tr}}{d_b}\right)}$$

$$= \frac{3}{40}\frac{60{,}000}{\sqrt{4000}}\frac{(1.7)(1.0)1.3}{1.83} = 71.1\frac{(1.7)(1.0)1.3}{1.83} = 85.9$$

In the above calculation, $\alpha\beta = 1.7$, the upper limit of that product, which exceeds the actual $\alpha\beta = 1.3(1.5) = 1.95$. The bar size factor $\lambda = 1.0$ for #7 bars and larger. Thus,

$$L_d = 85.9d_b = 85.9(1.128) = 96.9 \text{ in.}$$

The simplified method gave $L_d = 118$ in. That formula used $(c + K_{tr})/d_b = 1.5$, whereas Eq. (6.7.3) gave 1.83, thus giving the more accurate $L_d$ as 82% of the value from the simplified equation. ■

## 6.9 Development Length for Compression Reinforcement

Relatively less is known about the development length for compression bars than for tension bars, except that the weakening effect of flexural tension cracks is not present and there is beneficial effect of the end bearing of the bars on the concrete. ACI-12.3 gives as the basic development length $L_{db}$,

$$L_{db} = 0.02d_b\frac{f_y}{\sqrt{f'_c}} \qquad \textbf{(6.9.1)*}$$

which is basically two-thirds of the minimum development length for tension reinforcement to prevent a "pullout" mode of failure. ACI-12.3 also states that $L_{db}$ must not be less than

$$L_{db} \geq 0.0003d_bf_y \qquad \textbf{(6.9.2)*}$$

which means that only $f'_c$ up to about 4400 psi may be counted upon. Thus the basic development length $L_{db}$ is to be taken as the larger of Eqs. (6.9.1) and (6.9.2).

When excess bar area is provided such that provided $A_s$ exceeds required $A_s$, Eqs. (6.9.1) or (6.9.2), whichever controls, may be reduced by applying the multiplier (required $A_s$/provided $A_s$).

Reduction in development length is permitted when reinforcement is enclosed by spirals or closely spaced ties (typically in columns; see Chapter 13) which are not less than $\frac{1}{4}$-in. diameter for spirals (ACI-7.10.4), or #4 bars for ties (ACI-7.10.5), and having a pitch (for spirals) or center-to-center spacing (for ties) not exceeding 4 in. Under these confinement conditions, $L_{db}$ may be reduced 25%.

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*For SI, ACI 318–95M, for $L_{db}$ and $d_b$ in mm, and $f'_c$ and $f_y$ in MPa, gives

$$L_{db} = 24d_b\frac{f_y}{\sqrt{f'_c}} \qquad \textbf{(6.9.1)}$$

$$L_{db} = 0.044\ d_bf_y \qquad \textbf{(6.9.2)}$$

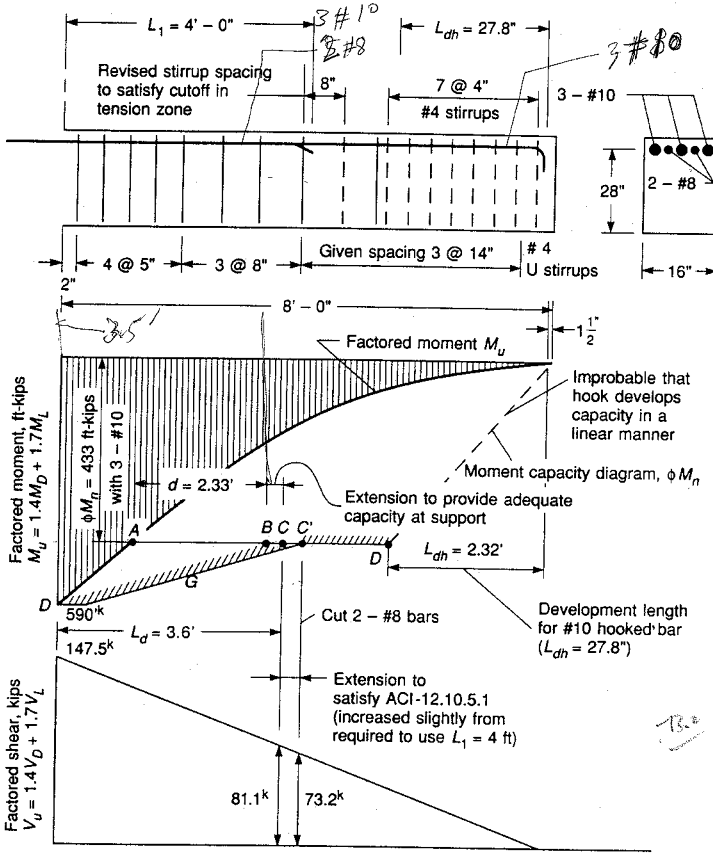



**Figure 6.14.2** Beam of Example 6.14.1.

$$M_n = 323[28 - 0.5(7.92)]\tfrac{1}{12} = 647 \text{ ft-kips}$$

$$\phi M_n = 0.90(647) = 582 \text{ ft-kips} \approx M_u = 590 \text{ ft-kips} \qquad \textbf{OK}$$

**(b)** Determine the theoretical cutoff point for 2-#8 bars. The moment capacity $\phi M_n$ remaining with 3-#10 bars is

$$C = 40.8a$$

$$T = 3.81(60) = 229 \text{ kips}$$

$$a = \frac{229}{40.8} = 5.61 \text{ in.}$$

$$\phi M_n = 0.90(229)[28 - 0.5(5.61)]\tfrac{1}{12} = 433 \text{ ft-kips}$$

Plot on the factored moment $M_u$ diagram and locate the theoretical cutoff point $A$. Extend to the right 12 bar diameters (of the #8 bars that are to be cut) or the effective depth of the member, whichever is greater, to arrive at point $B$.

$$d = 28 \text{ in.} (2.33 \text{ ft}) > [12d_b = 12(1.0) = 12 \text{ in.}]$$

to see whether A or B.

**(c)** Use the simplified equations to determine the development length $L_d$ for #8 bars. Can Category A, the more favorable one, be used? Check the clear spacing of bars. Assuming the bars, though unequal in size, are uniformly spaced, the clear spacing between them is

$$\text{clear spacing} = \frac{16 - 2(1.5) - 2(0.5) - 3(1.27) - 2(1.0)}{4} = 1.55 \text{ in}$$

Since only the 2-#8 bars are being developed, and the 3-#10 are presumed to continue beyond the #8 cutoff location, it is the spacing between the two #8 that determines the Category. The failure mode would have splitting from a #8 bar to the side or top face of the member, or between the two #8 bars. The ACI Code rules consider a bar (or bars) as essentially inert when it is not being developed within the development region of other bars. Thus, when the #10 bars of this example have a development length from their termination near the free end of the cantilever that is less than the distance to the #8 bar cut, the #10 bars are considered to have no influence on $L_d$ for the #8 bars. It is a matter of opinion whether or not the #10 itself should be treated as concrete. That is, in this case whether to use the full spacing between the #8 bars, 2(1.55) + 1.27 diam. of #10 = 4.37 in. The authors believe it appropriate in this case to consider the spacing of the #8 to be 4.37 in. for the purpose of satisfying a Category A requirement, *assuming $L_d$ for the #10 does not overlap the $L_d$ for the #8 bars.*

Even if the concrete width between #8 bars were taken as 2(1.55) = 3.10 in., it still exceeds the $2d_b$ for the #8 bar to satisfy Category A, item 2(a), given in Section 6.7 (ACI-12.2.2), as well as item 2(b), because cover to the top face of the beam is 2 in., which exceeds $d_b$ needed for that item.

Thus, Category A applies! Using simplified Eq. (6.7.5) for #7 and larger bars

$$\frac{L_d}{d_b} = \frac{f_y}{20\sqrt{f'_c}}\alpha\beta\lambda \qquad \textbf{[6.7.5]}$$

For the modification factors $\alpha\beta\lambda$, only the top bar factor $\alpha = 1.3$ applies. The epoxy-coated bar factor $\beta$ and the lightweight aggregate concrete factor $\lambda$ are both 1.0 because those factors do not apply. Thus, Eq. (6.7.5) gives

$$L_d = \frac{d_b f_y}{20\sqrt{f'_c}}\alpha\beta\lambda = \frac{1.0(60{,}000)}{20\sqrt{3000}}\alpha\beta\lambda$$

$$= 54.8\alpha\beta\lambda = 54.8(1.3)(1.0)1.0 = 71.2 \text{ in.}$$

The 54.8 in. can be verified from Table 6.7.1. Thus,

$$L_d(\text{for \#8}) = 71.2 \text{ in. } (5.9 \text{ ft})$$

This development length is 58% longer than the $L_d$ of 3.75 ft obtained under the 1989 ACI Code, where the most favorable conditions applied.

**(d)** Use the general equation, Eq. (6.7.1), to determine the development length $L_d$ for #8 bars. That equation is

$$\frac{L_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c + K_{tr}}{d_b}\right)} \qquad [6.7.1]$$

The cover or spacing dimension $c$ is the smaller of (1) distance from center of bar being developed to nearest concrete surface, and (2) one-half center-to-center spacing (*clear* spacing computed as 1.55 in. in part a) of bars being developed. The distance $c$ is the smaller of the following two values:

top and side cover = 1.5(i.e., clear) + 0.5(i.e., stirrup) + 0.5(i.e., bar radius) = 2.5 in.

one-half center-to-center spacing = 1.55 + 0.5(i.e., bar radius) = 2.05 in.

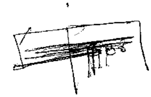
Thus, $c = 2.05$ in.

For the stirrups in the development region, use the 8 in. spacing for computation. Use Eq. (6.7.2),

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500sn} \qquad [6.7.2]$$

The number $n$ of bars being developed is 2, and $A_{tr}$ is the total area of stirrups surrounding the bars being developed, in this case, for #4 stirrups it is 2(0.2) times 8 stirrups. Thus, evaluation of Eq. (6.7.2) gives

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500sn} = \frac{8(2)(0.20)60{,}000}{1500(8)2} = 8$$



Evaluating Eq. (6.7.3),

$$\left[\frac{c + K_{tr}}{d_b} = \frac{2.06 + 8.0}{1.0} = 10.1\right] > 2.5 \text{ max}$$

Thus, $(c + K_{tr})/d_b = 2.5$.

Evaluate Eq. (6.7.1),

$$L_d = \frac{3}{40}\frac{d_b f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c + K_{tr}}{d_b}\right)}$$

$$= \frac{3}{40}\frac{(1.0)60{,}000}{\sqrt{3000}}\frac{\alpha\beta\gamma\lambda}{2.5} = 82.2\frac{1.3(1.0)(1.0)1.0}{2.5} = 42.7 \text{ in. } (3.6 \text{ ft})$$

The 42.7 in. compares favorably with the 1989 ACI Code value of 45.0 in., and is significantly lower than the 71.2 in. from the 1995 simplified equation. Use $L_d = 42.7$ in. for the moment capacity diagram in Fig. 6.14.2.

Since point $B$, the proposed cutoff point, lies only about 3.5 ft from the support, the #8 bars would not have full capacity at the support. Therefore, extend the proposed cutoff to point $C$, which is located at $L_d$ (for #8) = 3.6 ft from the support.

**(e)** Check ACI-12.10.5 for cutting bars at point $C$ in the tension zone. The shear strength, including contribution of stirrups, is first computed. Using the simplified method of constant $V_c$,

$$V_c = 2\sqrt{f_c'}\,b_w d = 2\sqrt{3000}(16)(28)\frac{1}{1000} = 49.1 \text{ kips}$$

For the 14-in. spaced #4 stirrups in the vicinity of the potential cut point $C$,

$$V_s = \frac{A_v f_y d}{s} = \frac{2(0.20)(60)28}{14} = 48.0 \text{ kips}$$

The shear strength $\phi V_n$ at point $C$ is

$$\phi V_n = \phi(V_c + V_s) = 0.85(49.1 + 48.0) = 82.5 \text{ kips}$$

$$\text{percent stressed in shear} = \frac{V_u}{\phi V_n} = \frac{81.1}{82.5} = 98\% > 75\% \qquad \textbf{NG}$$

Even when only 50% of the moment strength $\phi M_n$ is used by $M_u$, the percent stressed in shear cannot exceed 75% (see Condition 3, Eqs. 6.12.3 and 6.12.4). Try using one more 8-in. stirrup spacing to cover the potential cut at point $C$, and see whether or not Condition 1, Eq. (6.12.1), is satisfied.

$$V_s = 48.0\left(\frac{14}{8}\right) = 84.0 \text{ kips}$$

$$\text{percent stressed in shear} = \frac{81.1}{0.85(49.1 + 84)} = 71\%$$

This is borderline to satisfy the two-thirds limit of ACI-12.10.5.1 (Eq. 6.12.1). Extend the #8 bars to point $C'$ 4 ft from face of support.

**(f)** Check whether the continuing #10 bars have adequate development length to the right of point C. The clear spacing between the continuing three #10 bars is

$$\text{clear spacing} = \frac{16 - 2(1.5) - 2(0.5) - 3(1.27)}{2} = 4.1 \text{ in.}$$

which exceeds the $2d_b$ of 2.54 in. required for Category A, item 2(a). Top cover of 2.64 in. [i.e., $1.5 + 0.5 + 1.27/2) = 2.64$ in.] to the center of the #10 bars exceeds the $d_b$ requirement of Category A, item 2(b). Thus, the simplified equation, Eq. (6.7.5) for #7 and larger bars,

$$L_d(\text{for \#10}) = \frac{d_b f_y}{20\sqrt{f_c'}}\alpha\beta\lambda = \frac{1.27(60{,}000)}{20\sqrt{3000}}\alpha\beta\lambda$$

$$= 69.6\alpha\beta\lambda = 69.6(1.3)(1.0)1.0 = 90.5 \text{ in. (7.5 ft)}$$

For the modification factors $\alpha\beta\lambda$, only the top bar factor $\alpha = 1.3$ applies.

Calculate the development length $L_d$ based on the general equation, Eq. (6.7.1). The distance $c$ is the smaller of the following two values:

top and side cover = 1.5(i.e., clear) + 0.5(i.e., stirrup) + 0.635(i.e., bar radius) = 2.6 in.

one-half center-to-center spacing = 4.1/2 + 0.635(i.e., bar radius) = 2.7 in.

Thus $c = 2.6$ in.

For the stirrups in the development region, use the given 14 in. spacing near the free end of the cantilever for computation. The number $n$ of bars being developed is 3, and $A_{tr}$ is the total area of stirrups surrounding the bars being developed, in this case, for #4 stirrups it is 2(0.2) times 3 stirrups. Use Eq. (6.7.2),

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500sn} = \frac{3(2)(0.20)60{,}000}{1500(14)3} = 1.1$$

Evaluating Eq. (6.7.3),

$$\left[\frac{c + K_{tr}}{d_b} = \frac{2.6 + 1.0}{1.27} = 2.8\right] > 2.5 \text{ max}$$

Thus, $(c + K_{tr})/d_b = 2.5$.

Evaluate Eq. (6.7.1),

$$L_d = \frac{3}{40} \frac{d_b f_y}{\sqrt{f'_c}} \frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c + K_{tr}}{d_b}\right)}$$

$$= \frac{3}{40} \frac{(1.27)60{,}000}{\sqrt{3000}} \frac{\alpha\beta\gamma\lambda}{2.5} = 104.3 \frac{1.3(1.0)(1.0)1.0}{2.5} = 54.2 \text{ in. } (4.5 \text{ ft})$$

This embedment of 4.5 ft measured from the end of straight #10 bars would overlap the development length region of the #8 bars, possibly requiring longer development length $L_d$ for the #8 bars because the center-to-center spacing then would be the reduced value based on five bars in the 16 in. width. However, in this case because $K_{tr}$ is 8.0 [see part (d)] the value of $(c + K_{tr})/d_b$ remains at 2.5 and the $L_d$ of #8 bars stands at 3.6 ft in part (d). The #10 bars would satisfy literally the statement of ACI-12.10.4, which requires "Continuing reinforcement shall have an embedment length not less than the development length $L_d$ beyond the point where bent or terminated tension reinforcement is no longer required to resist flexure." In other words, the distance from point $A$ to the free end of the cantilever must be at least $L_d$ (for #10). The authors believe in a somewhat more conservative approach, requiring the moment capacity $\phi M_n$ diagram to have an offset from the factored moment $M_u$ diagram, except at or near a simple support or the free end of a cantilever, equal to 12 bar diameters of the effective length $d$, whichever is greater.

In this case, try standard 90° hooks (see Fig. 6.11.1) on the ends of the #10 bars. Since the beam has the usual 1.5-in. clear cover and #4 stirrups, the cover to the hooked bars is 2 in., which is less than the $2\frac{1}{2}$ in. required by ACI-12.5.4; thus, the special provisions of that Code section must be satisfied.

The development length $L_{dh}$ for the #10 hooked bar is the basic value $L_{hb}$ (i.e., no modification to $L_{hb}$ applies) given by Eq. (6.11.1) and Table 6.11.2. Thus, for #10 hooked bar,

$$L_{dh} = L_{hb} = \frac{1200 d_b}{\sqrt{f'_c}} = \frac{1200(1.27)}{\sqrt{3000}} = 27.8 \text{ in.}$$

After all modifications, the development length $L_d$ is not permitted to be less than 8 in. (200 mm). Thus, in general, for compression reinforcement.

$$L_d = \begin{bmatrix} \text{Eqs. (6.9.1)} \\ \text{or (6.9.2)} \end{bmatrix} \begin{bmatrix} \dfrac{\text{required } A_s}{\text{provided } A_s} \end{bmatrix} \begin{bmatrix} \text{0.75 for enclosure} \\ \text{by spirals or ties} \end{bmatrix} \geq 8 \text{ in.} \quad \textbf{(6.9.3)}$$

■ **EXAMPLE 6.14.1** For the cantilever beam shown in Fig. 6.14.2 determine the distance $L_1$ from the support to the point where 2-#8 bars may be cut off. Assume the #4 stirrups shown (solid, not the dashed ones) have been preliminarily designed. Assume there will be at least $L_d$ embedment of the bars into the support. Draw the resulting moment capacity $\phi M_n$ diagram for the entire beam. Use $f_c' =$ 3000 psi and $f_y = 60{,}000$ psi.

***Solution:*** **(a)** Compute the maximum moment capacity $\phi M_n$ of the section.

$$0.75\rho_b \text{(Table 3.6.1)} = 0.0160$$

$$\rho = \frac{3(1.27) + 2(0.79)}{16(28)} = 0.0120 < 0.75\rho_b \qquad \textbf{OK}$$

$$C = 0.85(3)16a = 40.8a$$

$$T = [3(1.27) + 2(0.79)]60 = (3.81 + 1.58)60 = 323 \text{ kips}$$

$$a = \frac{323}{40.8} = 7.92 \text{ in.}$$

# Foundation Systems

Soil is often classified by its constituents. The table below provides some common names classified based on characteristics and behavior rather than upon size.

| Particle Type | Approx. Diameter (in) | |
|---|---|---|
| Cobble | > 3.0 | |
| Gravel | 0.25 – 3.0 | ↑ Coarse |
| Sand | 0.003 – 0.25 | |
| | | # 200 Sieve |
| Silt | 0.0001 – 0.003 | |
| Clay | < 0.0001 | ↓ Fine |

* Coarse aggregate if 50% or more retained in # 200 Sieve
* Fine aggregate if 50% or more passes through # 200 Sieve

* Sieve # 200 is a 200 X 200 openings in one square inch

* The most important property for soil is its bearing capacity which can be determined by

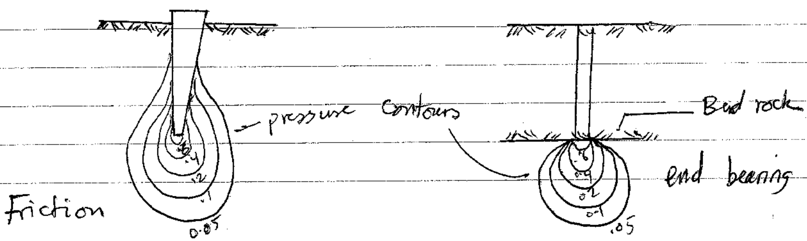
  - drilling to determine type of soil
  - Field test such as the "standard penetration"
  - lab test to determine soil characteristics

Soils also classified as cohesionless and cohesive. The coarse - grained soil tend to be cohesionless whereas fine-grained soil tend to be cohesive. A cohesionless soil falls apart when dry (sand) wheras a cohesive soil tends to stick togather when dry (clay). The cohesionless soil carry loads by the development of frictional forces between the particles. The cohesive soil carry loads by development of shear and tensile stresses. Cohesionless soil are less affected by the presence of moisture whereas cohesive soils are usually sensetive to water and their properties heavily depend on water.

## Types of Foundations

1. Shallow foundation system
   - isolated footing
   - strip footing
   - Combined footing
   - mat foundation

2. Deep foundation system (Pile)



# Footing Design

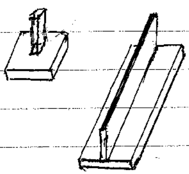
Footing design is basically composed of three major steps:

1) The determination of the overall required bearing area such that the allowable soil bearing capacity is not exceeded

2) Computation of the necessary footing thickness such that the concrete will not fail in shear

3) Computation of steel or reinforcement to carry the bending moment

In determining the overall plan dimension of the footing, one needs to take into account all loads which contribute to the soil pressure. These loads may include:



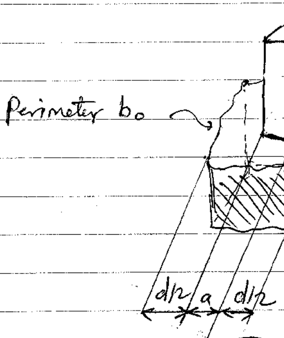
(a) The actual dead and live loads applied to the footing via wall or column

(b) The weight of footing itself

(c) The weight of the soil above footing

(d) The weight of the slab-on-grade (if any) and the loads acting on it

For isolated column footing the thickness of the footing is governed by column punching which create four inclined faces at 45° and therefore the area which resist shear will be around the column and at a distance d/2.

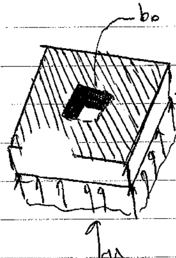



$$b_o = 4 \,(\text{width of Column} + d)$$

The total shear force resisted by concrete
$$V_c = \phi \, 4 \sqrt{f'_c} \; b_o \, d$$

if $P_u = 1.4\,DL + 1.7\,LL$, then the reaction from soil on footing will be

$$q_u = \frac{P_u}{A_{footing}},$$

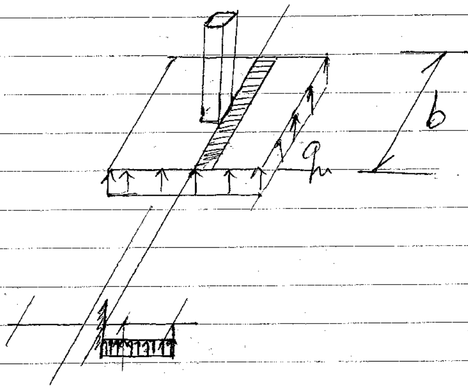



The reaction force which contribute to punching is the square "donut" (shaded area)

$$V_u = q_u \,(\text{shaded area})$$

$\boxed{V_u = V_c}$ $\Longrightarrow$ solve for d

The total thickness $h = d + \text{cover}$




The amount of steel will be calculated based on cantilever bending due to a distributed upward pressure $q_u$ (kips/ft).

$$M_u = \frac{w_u l^2}{2}$$

$$\phi M_n = M_u$$

$$\phi b d^2 R_n = M_u$$

get $R_n$ and find $\rho$ and $A_s = \rho b d$. and from table find the required steel.

**Example 13.1** Design the square column pad of Figure 13.4. The dead load, including the column itself, is 100 kips and the live load is 85 kips. The

***Solution:*** Assume a trial thickness of 18 in; depending upon the size of the reinforcing bars, this means a minimum $d$ distance of about 13 in. An effective allowable bearing pressure $q_e$ can be obtained by subtracting from the given 3500 psf all the loads present which are not part of the column load, in this case $1\frac{1}{2}$ ft of concrete and $2\frac{1}{2}$ ft of soil.

$$q_e = 3500 - 1.5(150) - 2.5(100)$$
$$= 3025 \text{ psf} \quad \text{or } 3.025 \text{ ksf}$$

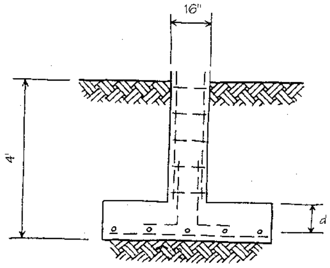



**FIGURE 13.4**

The required area of footing is

$$A_r = \frac{P}{q_e}$$
$$= \frac{100 + 85}{3.025}$$
$$= 61.1 \text{ ft}^2$$

Plan dimensions are often done in 3-in increments, in this case resulting in an even 8-ft dimension so the provided area will be 64 ft$^2$.

Now the punching shear can be checked to see if the trial thickness of 18 in is enough. The factored column load is

$$P_u = 1.4(100) + 1.7(85)$$
$$= 285 \text{ kips}$$

(Note that the column load is the only one that will cause shear and moment in the pad; the weight of the footing and the soil above will not.)

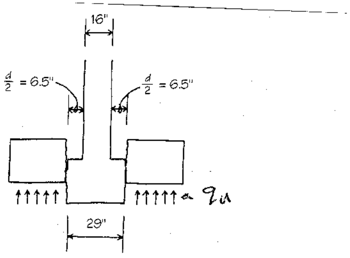
The upward pressure on the base of the footing due to the factored load is

$$q_u = \frac{P_u}{A}$$

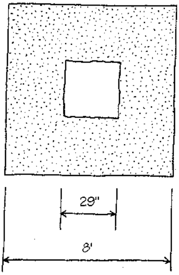
$$= \frac{285}{64}$$

$$= 4.45 \text{ ksf} \quad \text{or } 4450 \text{ psf}$$

The punching shear force due to this load can be




(a) section




(b) plan

**FIGURE 13.5**

calculated with the aid of Figure 13.5. Only the pressure on the shaded area (the "donut") causes punching shear:

$$V_u = 4450\left[8^2 - \left(\frac{29}{12}\right)^2\right]$$

$$= 259\ 000 \text{ lb}$$

The punching shear strength is, according to the Code,

$$V_c = \phi 4\sqrt{f_c'}\, b_o d \qquad (13\text{-}1)$$

The perimeter of the hole is 4 × 29, or 116 in:

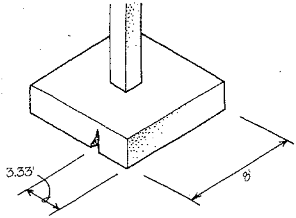
$$V_c = 0.85(4)\sqrt{3000}\,(116)(13)$$
$$= 281\ 000\ \text{lb}$$

Since $V_c > V_u$, the thickness is adequate and is slightly overdesigned.

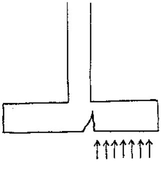
Now determine the amount of steel needed for the moment. The length of the cantilever is $(8 - \frac{16}{12})/2$, or 3.33 ft, and its width is 8 ft (Figure 13.6). The design load per foot will equal the pressure times the width:

$$w_u = q_u(8)$$
$$= 4.45(8)$$
$$= 35.6\ \text{klf}$$

**FIGURE 13.6**




(a)




(b)

So, the moment at the back end of the cantilever is

$$M_u = \frac{w_u L^2}{2}$$

$$= \frac{35.6(3.33)^2}{2}$$

$$= 198 \text{ kip-ft}$$

We must provide at least this much resisting moment:

$$\frac{M_r}{\phi b d^2} = R \qquad (7\text{-}2a)$$

$$\frac{198(12)}{0.9(96)(13)^2} = 0.163$$

Using Table B.1(3), we find that $\rho_{min}$ will control:

$$A_s = \rho bd \qquad (6\text{-}7)$$

$$= 0.0033(96)(13)$$

$$= 4.12 \text{ in}^2$$

Table A.1 indicates that seven #7 bars will provide 4.20 in². The footing will need this much steel in each direction, of course.

## 13.3 WALL FOOTINGS

The approach to design for a wall footing is very similar to that for a column pad except, of course, that the four-sided punching shear failure cannot take place. A one-foot length

## 13.3 WALL FOOTINGS

The approach to design for a wall footing is very similar to that for a column pad except, of course, that the four-sided punching shear failure cannot take place. A one-foot length of wall is usually analyzed and the previously mentioned one-way or beam shear governs the thickness if the Code minimum requirements do not. Transverse steel is needed for moment, except in lightly loaded residential footings, and temperature/shrinkage steel in the direction parallel to the wall is always necessary. The critical section for moment depends upon the relative stiffness of the wall and is illustrated in Figure 13.7.

**Example 13.2** Design the wall footing of Figure 13.8. The dead load, including the wall weight,

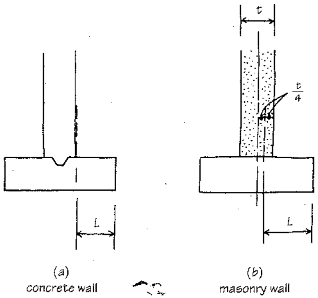
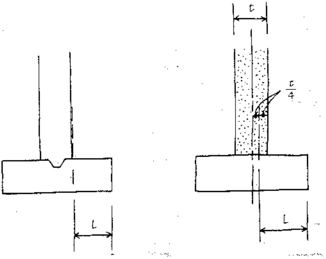



**FIGURE 13.7** Critical sections for moment.

is 4 kips/ft and the live load is 3 kips/ft. The soil weight is 110 pcf and its bearing capacity is 2750 psf. Use $f'_c = 3000$ psi and $f_y = 40$ ksi.




(a) concrete wall (b) masonry wall

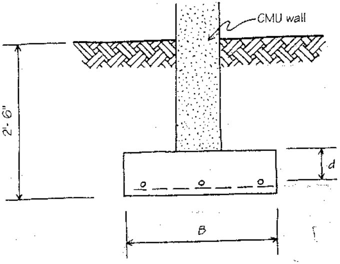
**FIGURE 13.7** Critical sections for moment.

is 4 kips/ft and the live load is 3 kips/ft. The soil weight is 110 pcf and its bearing capacity is 2750 psf. Use $f'_c = 3000$ psi and $f_y = 40$ ksi.

***Solution:*** Assume a trial thickness of 12 in which, with the required 3-in cover, will give us a $d$ value of about 8.5 in. The effective allowable bearing pressure will be

$$\begin{aligned} q_e &= 2750 - 1.0(150) - 1.5(110) \\ &= 2435 \text{ psf} \end{aligned}$$

**FIGURE 13.8**




The required width of footing $B$ will be

$$\begin{aligned} B_r &= \frac{P}{q_e} \\ &= \frac{4000 + 3000}{2435} \\ &= 2.87 \text{ ft} \end{aligned}$$

Using a 3-ft-wide footing, the factored loads will provide an upward pressure on the base of the footing of

$$\begin{aligned} q_u &= \frac{1.4(4) + 1.7(3)}{3.0} \\ &= 3.57 \text{ ksf} \quad \text{or } 3570 \text{ psf} \end{aligned}$$

Now we can compute the shear force to check the adequacy of our assumed thickness. Moving out from the face of the wall a distance d and referring to Figure 13.9, the shear force will be

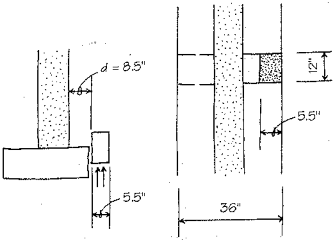
$$\begin{aligned} V_u &= 3570\left(\frac{5.5 \times 12}{144}\right) \\ &= 1640 \text{ lb} \end{aligned}$$

The beam shear strength is, according to the Code,

$$V_c = \phi 2\sqrt{f'_c}\, b\, d \qquad (13\text{-}2)$$

$$\begin{aligned} &= 0.85(2)(\sqrt{3000})\,(12)(8.5) \\ &= 9500 \text{ lb} \end{aligned}$$

**FIGURE 13.9**




(a) section (b) plan

5.5
8.5
14.0

Since $V_c >> V_u$, the thickness is too great. We reduce it and try again. Bearing in mind that 6 in of concrete is required above the steel and 3 in below, the minimum thickness will be about 10 in. (We hold the width at 3 ft as the effective bearing capacity will increase only slightly with the reduction in footing thickness.) The new $d$ will be about 6.5 in and the new $V_u$ becomes

$$V_u = 3570\left(\frac{7.5 \times 12}{144}\right)$$

$$= 2230 \text{ lb}$$

The new $V_c$ then will be

$$V_c = 0.85(2)(\sqrt{3000})\,(12)(6.5)$$

$$= 7260 \text{ lb}$$

So, we are still much more than adequate.

The moment will be checked, but it seems likely that $\rho_{min}$ will govern. The cantilever length, shown in Figure 13.7*b*, is 18 in less 2 in, or 16 in, and its width is, of course, 12 in:

$$M_u = \frac{w_u L^2}{2}$$

$$= \frac{3.57(1.33)^2}{2}$$

$$= 3.16 \text{ kip-ft}$$

This is a one-way slab situation, and using Table B.2(40/3), we can see that a moment of 3.16 kip-ft requires a $\rho$ of only 0.0025. This will be overridden by $\rho_{min}$ at 0.005. If we try #5 bars, we get

$$\boxed{s = \frac{a_s}{\rho d}} \qquad (11\text{-}2)$$

$$= \frac{0.31}{0.005(6.5)}$$

$$= 9.5 \quad \text{or } 9 \text{ in}$$

The longitudinal steel will be that required for temperature and shrinkage; i.e.,

$$A_s = \rho_t bh \qquad (11\text{-}1)$$

$$= 0.0020(36)(10)$$

$$= 0.72 \text{ in}^2$$

This can be provided by either three #5 bars or four #4 bars.

## PROBLEMS

**13.1** Evaluate the adequacy of a 7-ft-square footing for a column of 15 × 15-in section carrying a service live load of 70 kips and a service dead load (including the weight of the column) of 95 kips. The bottom of the footing is 5 ft below the surface of the soil, which weighs 115 pcf and can safely bear 4000 psf. The footing is 16 in thick with an effective depth of 11.75 in and is reinforced for moment with seven #6 bars of 60-ksi steel in each direction. Let $f'_c$ = 3500 psi.

**13.2** An 8-in-thick slab-on-grade with a service live load of 70 psf has been added around the column of Example 13.1 and the footing thickness has been tentatively reduced to 16 in. Will the footing still be adequate in punching shear?

**13.3** Design the footing for a masonry wall that is 15 in thick and carries service loads (including self-weight) of 8 kips/ft (dead) and 7 kips/ft (live). The bottom of the footing should be at a depth of 5 ft. The soil weighs 100 pcf and its allowable bearing capacity is 4000 psf. Use 3000-psi concrete and 60 000-psi steel.

**13.4** A column footing adjacent to a property line has rectangular dimensions of 7 × 13.5 ft. It is 18 in thick, has an effective depth of 13 in, and is reinforced with six #9 bars in each direction; $f'_c$ = 3000 psi and $f_y$ = 60 ksi. The bottom of the footing is 4 ft deep in soil that weighs 100 pcf and has an allowable bearing capacity of 3500 psf. If the 16 × 16-in column carries a service dead load of 85 kips, which includes self-weight, and a service live load of 90 kips, is the design of this footing adequate?

**13.5** Design the footing for an 8-ft-high concrete wall, 9 in thick, that carries a live load