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Effect of structural defects on the strength and damping properties of a solid material

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Abstract

In material sciences solid materials are known to be more or less dispersive, i.e., the Modulus of Elasticity, MOE, and the loss factor η are frequency dependent. Furthermore, these two parameters are not totally independent of each other as their frequency variations exhibit some interrelationship. Early studies have revealed that the elasticity and damping properties of wood, unlike many of other solid materials, start to show some frequency dependence already at a few KHz and that these variations depend among others on the species, the drying process, and the size of the specimen. In the present study the variations of the MOE and the loss factor are studied in terms of the number of defects in a wooden element. To this end an increasing number of holes is drilled in a wooden beam, and the major resonance frequencies for the longitudinal mode of vibration are localised on the frequency response curve permitting the determination of the MOE and η. The loss factor is evaluated by means of a room acoustical technique using the concept of reverberation time. A refined procedure permits to evaluate in an efficient manner the reverberation time from a single measurement of the impulse response. This latter is also shown to be easily assessed through the use of a cross-correlation operation between the response signal of the system and the input signal to it, this latter being taken as a random broadband noise. As an application, these concepts are used for the study of a wooden beam, and the results obtained for the longitudinal mode of vibration are presented and discussed. It is found that the MOE and η values are dependent on the number of defects present in the test sample, and that for an increasing number of these defects the MOE's value decreases steadily whereas the loss factor increases, although to a lesser degree. Some possible explanations of the phenomena underlying such behaviour are addressed and discussed.

Keywords: MOE; Damping; Defects; Solid material; Impulse response; Vibration; Acoustical technique

1. Introduction

It is well known that all solid materials may be characterised by both elastic and damping properties. The elasticity of a material is exhibited by the fact that the action of a stress is accompanied by a strain in a sample of the material. In the limit of linear behaviour, elasticity is defined in terms of stress σ and strain ε which are related to each other through Hooke's law by means of the modulus of elasticity, MOE, denoted by E.

$$\sigma = E\varepsilon. \tag{1}$$

Damping on the other hand is the ability of a material to dissipate vibration energy into heat. Usually, damping is characterised by the loss factor, often denoted by η, and which is defined as the ratio of lost energy to the vibratory reversible energy during one cycle of vibration, i.e.:

$$\eta = \frac{1}{2\pi}\frac{W_l}{W_R}, \tag{2}$$

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where W_l and W_R are the lost (heat) and the mechanical energies respectively (Cremer and Heckl, 1988). Due to its importance in noise and vibration control, and in the prevention of fatigue in structural elements, the study of damping in materials has been given some attention, and several mathematical models have been proposed to describe it (Bert, 1973; Crandall, 1973; Lazan, 1968; Nashif et al., 1985). For some solid materials, like polymers, the dynamic properties are frequency and temperature dependent, and experimental studies reveal moreover that the variation with frequency of the MOE and η are in fact dependent on each other. For other solid materials like for instance steel, concrete and plexiglass, the frequency variation of the MOE and η is not very obvious, for the simple reason that the changes in the dynamical properties of these materials start to exhibit themselves rather in the ultrasonic frequency range as compared to the audio frequencies for polymers (von Nöll, 1971).

2. Theoretical background

Both the elastic and the damping properties of all solid materials are to some extent dependent on frequency, and a common feature of the dynamic MOE is that its value increases with frequency. Moreover, experimental data on organic polymeric materials show that the curve of the frequency variation of the MOE passes through an inflection point at about the same frequency when the loss factor goes through a maximum (Ferry, 1980). These frequency dependencies are not specific to only organic viscoelastic materials but are in general valid for any solid real material, regardless of the actual damping mechanism. The linear dynamic elastic and damping properties of materials can be together characterised in the notion of complex MOE. Hence if E_d and E_l are respectively the real and imaginary parts of the complex modulus of elasticity, and ω is the angular frequency, then:

$$E = \frac{\sigma(\omega)}{\varepsilon(\omega)} = E_d(\omega) + jE_l(\omega) = E_d(\omega)\big(1 + j\eta(\omega)\big) \quad \text{and} \quad \eta(\omega) = \frac{E_l(\omega)}{E_d(\omega)}. \tag{3}$$

Solid materials are also known to satisfy the causality principle, which states that no response can be expected before the application of any excitation. It follows then from linear system theory that the real and imaginary parts of the frequency response function of a system are interrelated (Papoulis, 1962), and these interrelations are often referred to as the Kramers–Kronig dispersion relations (Kramers, 1927; Kronig, 1926). Mathematically speaking, for a causal function $h(t)$ having a frequency form $H(\omega) = R(\omega) + jX(\omega)$, and with no singularities at the origin, $R(\omega)$ and $X(\omega)$ are interrelated through a Hilbert transform operation (Papoulis, 1962, p. 198). Hence, considering a material specimen subject to an excitation represented by the stress σ and responding with a strain ε, the MOE according to Fig. 1 and Eq. (3) may then be considered as the frequency response of the material specimen. Therefore, any change in the value of either the MOE or η parameters resulting from some treatment of the material is expected to lead necessarily to a change of the other parameter.

In the mathematical context, the dispersion relations are formulated with the help of Hilbert transforms, and are of general nature, finding applications in several branches of physics, including acoustics, electromagnetism and optics. Several forms of such pairs of relations have been formulated for viscoelastic materials, and for instance a simplified form of such a set which includes the static modulus E_0 is (Tshoegl, 1989; Pritz, 1998):

$$E_d(\omega) = E_0 + \frac{2\omega^2}{\pi} \mathrm{P}\int_0^\infty \frac{E_l(x)/x}{\omega^2 - x^2}\,\mathrm{d}x, \qquad E_l(\omega) = -\frac{2\omega}{\pi} \mathrm{P}\int_0^\infty \frac{E_d(x)}{\omega^2 - x^2}\,\mathrm{d}x, \tag{4}$$

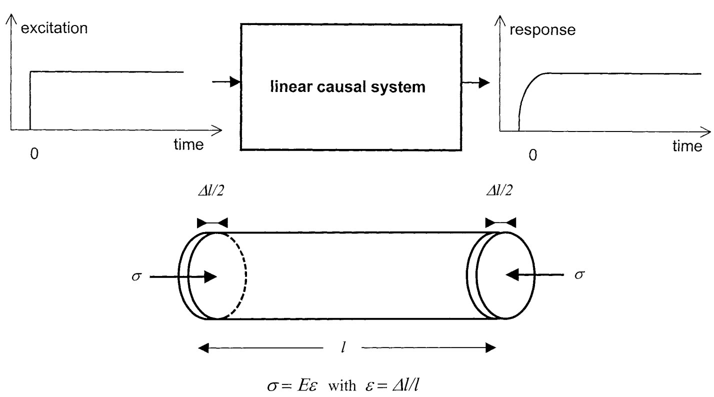

Fig. 1. Representation of the MOE of a material as the frequency response of a sample of this material.

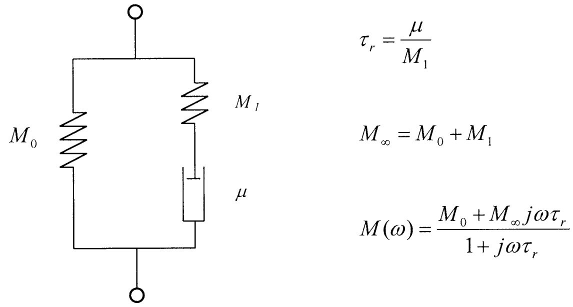

Fig. 2. The Zener model, also known as the standard viscoelastic body.

where x is an integration variable and P stands for the principal value of the integrals. These last formulas express the fact that the knowledge of the frequency behaviour of one of the moduli permits the determination of the other modulus at any frequency.

The investigation of viscoelastic materials often necessitates efficient modeling for describing viscoelastic behaviour, and the relatively simple model developed by Zener (1948), has sometimes been referred to in the literature. Zener's model, also known as the viscoelastic standard body, consists of a spring coupled in parallel with a Maxwell element, Fig. 2.

However, this model has been known to be limited in its performance, and lately, Pritz (1999) proposed a slight modification of it through making appeal to the concept of fractional derivates (Pritz, 1996; Torvik and Bagley, 1984).

Hence, let the αth fractional derivative of a function $f(t)$ of the time t be defined by:

$$\frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{\mathrm{d}}{\mathrm{d}t} \int_0^t \frac{f(\tau)}{(t-\tau)^\alpha}\, \mathrm{d}\tau, \tag{5}$$

Γ being the gamma function, and τ a dummy variable. With reference to Fig. 2, the complex elastic modulus becomes then:

$$E(\omega) = \frac{E_0 + E_\infty (j\omega\tau_r)^\alpha}{1 + (j\omega\tau_r)^\alpha}, \tag{6}$$

E_∞ being the value taken by the MOE in the limit of infinite frequency. An identification of the real and imaginary parts as respectively the dynamic and loss moduli gives in normalised form:

$$\frac{E_d(\omega)}{E_0} = \frac{1 + (c+1)\cos(\alpha\pi/2)\omega_n^\alpha + c\omega_n^{2\alpha}}{1 + 2\cos(\alpha\pi/2)\omega_n^\alpha + \omega_n^{2\alpha}}, \tag{7a}$$

$$\frac{E_l(\omega)}{E_0} = \frac{(c-1)\sin(\alpha\pi/2)\omega_n^\alpha}{1 + 2\cos(\alpha\pi/2)\omega_n^\alpha + \omega_n^{2\alpha}}. \tag{7b}$$

The loss factor is simply the ratio of these two expressions, and is therefore given by:

$$\eta(\omega) = \frac{(c-1)\sin(\alpha\pi/2)\omega_n^\alpha}{1 + (c+1)\cos(\alpha\pi/2)\omega_n^\alpha + c\omega_n^{2\alpha}}, \tag{8}$$

where $c = E_\infty/E_0$. The quantity $\omega_n = \omega\tau_r$ is the normalised frequency with τ_r being the relaxation time. It may be noted that the application of fractional calculus to viscoelasticity is not in itself a novelty, but dates back in time to about a century ago with early contributions due to Volterra. The fractional calculus models are used to describe the viscoelasticity of materials for the reasons that they are in harmony with the molecular theories describing these materials, that they fit better with the experimental and further that they permit a material modeling using fewer parameters than those based on classical differential calculus (Beyer and Kempfle, 1995; Koeller, 1984).

3. Acquisition of the impulse response of a system

The study of any system in acoustics, mechanics or electromagnetism requires often the knowledge of its impulse response which in a way is the signature of the system, and from which the transfer function may be processed through a usual Fourier transform. Ideally, the impulse response would be obtained through submitting the system to a Dirac pulse excitation. However, due to technical difficulties, it is in practice impossible to realise a signal having the time and frequency characteristics of such

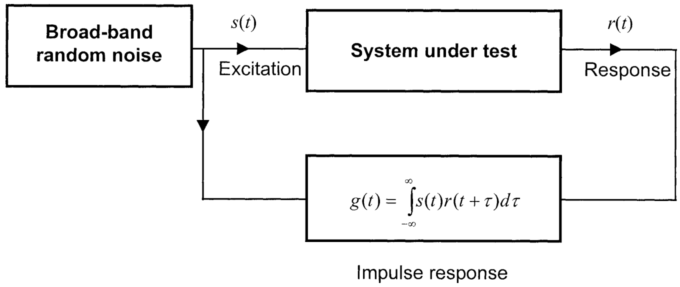

Fig. 3. Measurement of the impulse response of a system by a method based on a cross-correlation principle.

a pulse. Instead, one can resort to a technique consisting of performing a cross-correlation operation of the input signal and the response of the system to it, Fig. 3. The result approximates the true impulse response if the signal is random.

Nowadays, use is often made of Maximum-Length Sequences, MLS, signals which are periodic signals though having properties similar to those of random signals (Kuttruff, 1991).

4. Determination of the loss factor from the Impulse Response via the Reverberation Time

The Reverberation Time, RT, and denoted T_{60}, is a room acoustical quantity used to assess the amount of sound absorption in a room. It is defined as the time in seconds made by the sound level to drop by 60 dB from the time a sound source in the room has been switched off. The use of the concept of T_{60} may be used to any vibrating system for evaluating the amount of damping present in its material. Hence, the relationship between η and T_{60} is given by (see, for instance, Cremer and Heckl, 1988):

$$\eta = \frac{\ln 10^6}{\omega T_{60}} \approx \frac{2.2}{f T_{60}}, \quad \omega = 2\pi f. \tag{9}$$

However, the classical way of determining the RT presents difficulties in finding the best fit for the curve of sound level decay with time. These difficulties are nowadays circumvented through using Schroeder's elegant "Method of Integrated Impulse Response" where the impulse response is first squared and then integrated backwards to yield the Energy Decay Curve from which the RT can easily be calculated (Schroeder, 1965).

A schematic representation of all the steps included in the measurement procedure is summarised in Fig. 4. It may be noted that there is available on the market a measurement package, the MLSSA Acoustical Measurement System from the DRA Laboratories, and which consists of a card to be slotted in a PC with the accompanying software (MLSSA, 2001).

5. Study on a wood bar with artificial defects

As an application, a study is made of the variation of the loss factor in a piece of wood resulting from the introduction of artificial defects. The experiment consisted of drilling holes in the bar of Norway spruce with size $70 \times 7 \times 7$ cm^3, and to follow the variation of the MOE and the loss factor for an increasing number of holes. The holes had all the same size, 10 mm in diameter, drilled across the bar in its transversal direction. These were evenly distributed on the body of the bar, which was resting on soft supports. A schematic representation of the experimental set-up for the longitudinal mode of vibration is shown in Fig. 5.

For a bar with length l, resonance occurs at frequencies such that the round-trip of the wave down the bar (distance from excitation driver to the other extremity of the bar and back to the driver, admitting ideal elastic reflections at the extremities) is equal to a multiple of the wavelength λ, that is $2l = n\lambda$ where n is an integer. The wavelength is calculated according to $\lambda = c/f$, c being the speed of longitudinal wave propagation and f the frequency of the excitation. Furthermore, the propagation speed c for longitudinal waves in a material with a MOE E and a material density ρ is given by: $c = \sqrt{E/\rho}$. Hence, the expression of the resonance frequencies is formulated according to:

$$f_n = \frac{n}{2l}\sqrt{\frac{E}{\rho}}. \tag{10}$$

An example of a measured frequency response of the wood bar is shown in Fig. 6. The frequency peaks are quite well distinguished in this curve with their typical harmonic pattern.

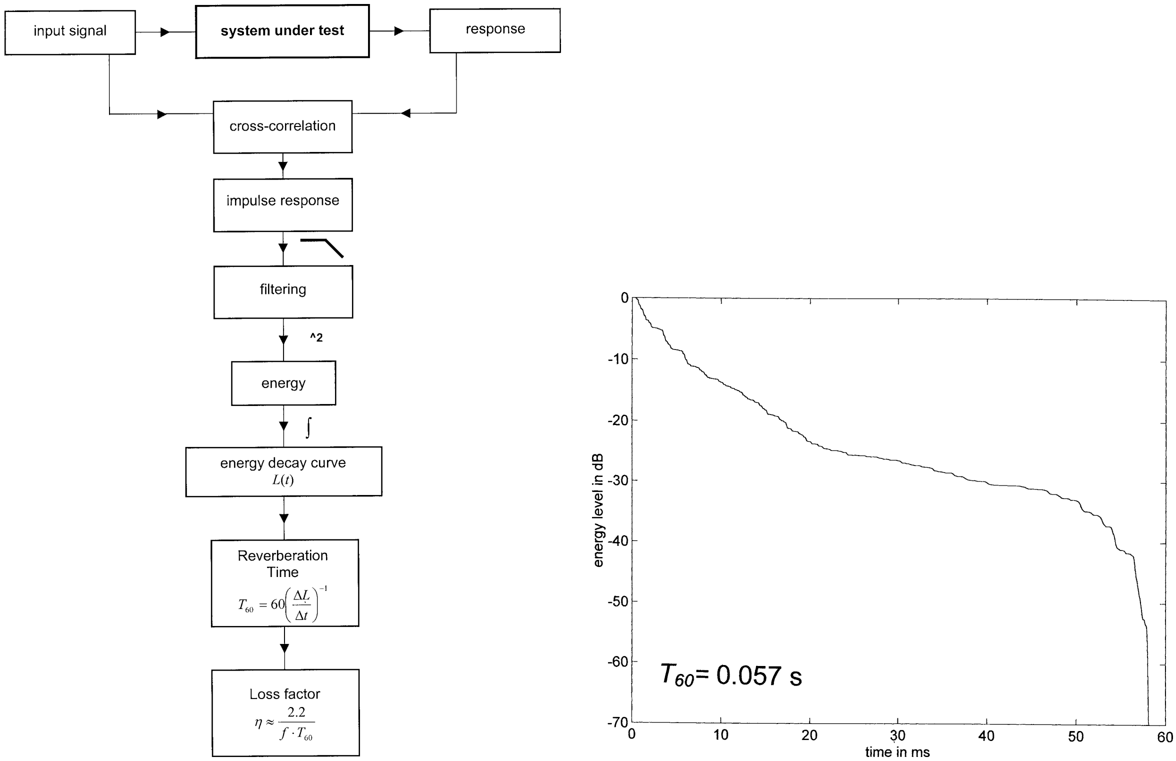

Fig. 4. Left: representation of a procedure for the measurement of the loss factor of a material specimen. Right: a typical energy decay curve resulting from the application of the integrated impulse response method.

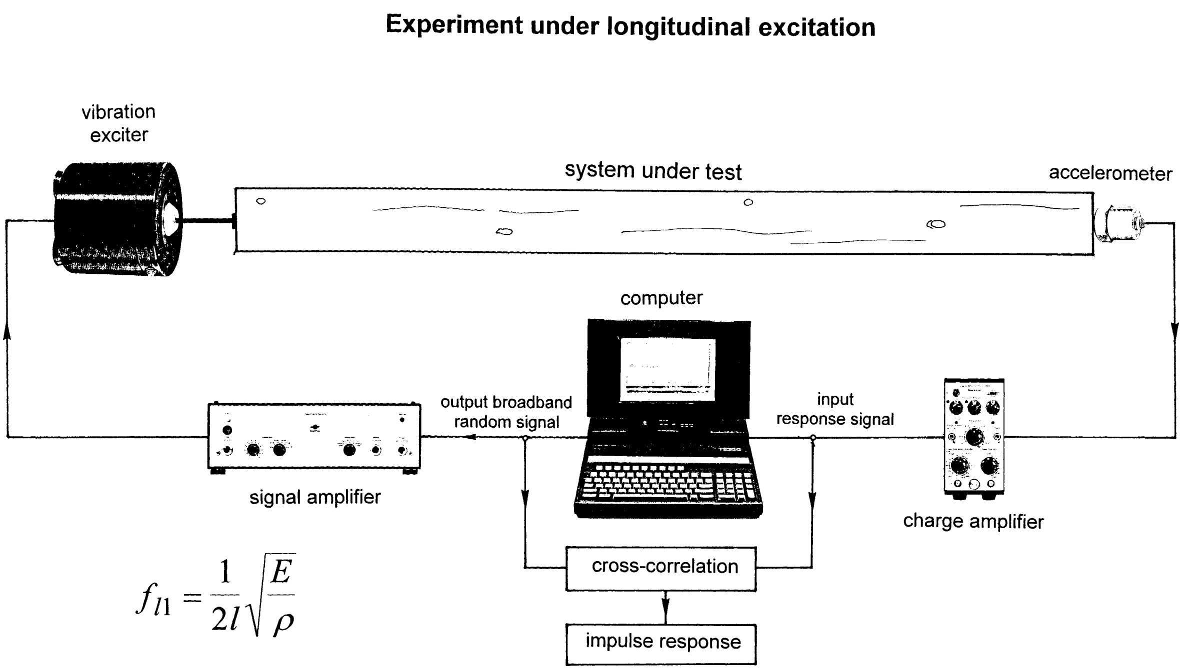

Fig. 5. Schematic representation of the experimental set-up for assessing the vibration impulse response of a test specimen in the longitudinal mode of vibration.

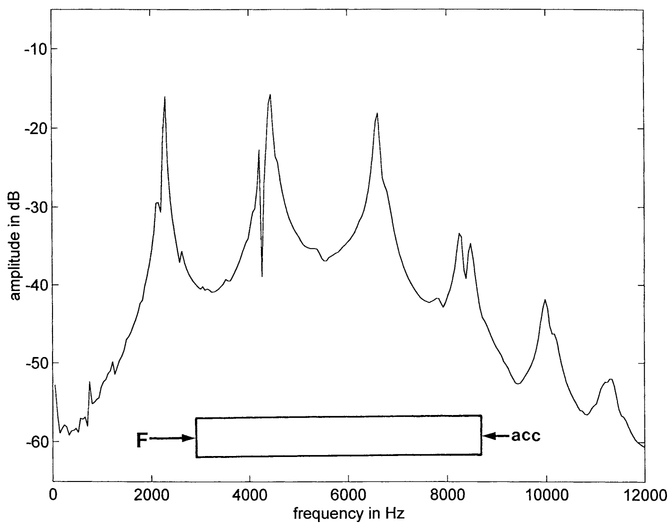

Fig. 6. Frequency response of the wooden bar submitted to a longitudinal vibration excitation.

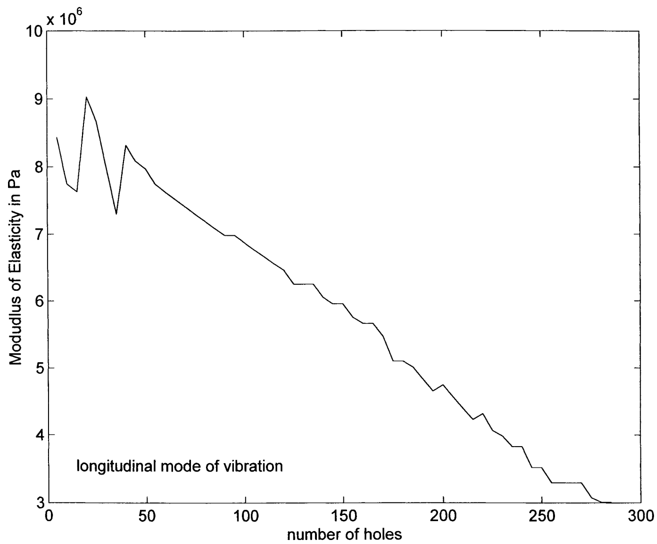

Fig. 7. Variation of the MOE as function of the number of holes drilled in the wooden bar.

The variation of the MOE as function of the number of defects introduced in the wooden bar is shown in Fig. 7.

The loss factor was determined as explained in the theory above at the fundamental frequency of vibration corresponding in the longitudinal mode of vibration to the frequency at which the length of the bar equals half a wavelength. A further consideration in the determination of the loss factor is to select a suitable bandwidth of the frequency filter. This latter being centred at the frequency of interest is due to have a frequency range broad enough in order that the half power bandwidth to be totally included in the frequency analysis (according to Eq. (9) $\Delta f \sim 1/T_{60}$). This is presented in Fig. 8.

The curve of Fig. 7 shows a clear monotonical decrease of the value of the longitudinal MOE with the number of holes bored in the bar of wood. The peculiar behaviour of the curve at the lowest number of defects may be due to the periodical pattern of the holes in the wood bar setting more resonances in its vibration and where the distance between consecutive holes is between one quarter and one half a wavelength. At the same time, the loss factor is seen to increase to some degree with the introduction of an increasing number of defects. It may be pointed out the RT from which the loss factor has been determined was not T_{60} as formulated in Eq. (9). Instead, another variant, the Early Decay Time, EDT, was used for the calculations of η. With the use of weak excitation signals or in highly damped systems (e.g., a sample of wood at an advanced stage of rot), it may happen that the dynamics of the decay curve has not the required 60 dBs for evaluating T_{60}. Hence, acousticians refer instead to the EDT

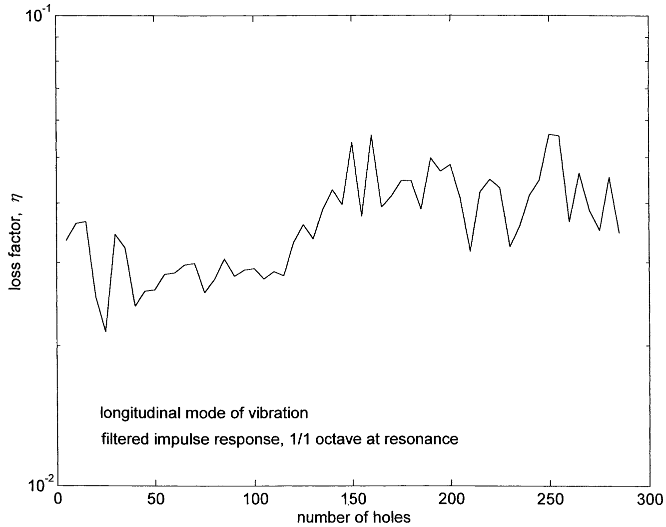

Fig. 8. Variation of the loss factor with the number of defects in the wooden bar.

which is defined as the RT but evaluated from the interval [0 dB, −10 dB] of the energy decay curve. In fact, the EDT has been found to be a better room acoustical descriptor of the human subjective sensation of reverberation than the classical T_{60}.

The holes in the wood bar were intended to simulate rot pockets in the body of the bar and which weakens considerably the strength of the affected parts. Wood being a highly anisotropic material, its properties are affected differently in different directions relative to the grain. At the frequencies of some kHz, the loss of energy in wood is predominantly due to fluid friction, and this phenomenon is somehow enhanced in the present experiments by the loss of energy accompanying the reflection of elastic waves along the fibres and at the boundaries of the holes. The attack of wood by rot results in a destruction of the lignin which serves as a cement binding the wood cells together, the result being a substance of granular-like consistency. In this respect, rot has some similarity with sand which is quite often used as an efficient means for increasing damping in building structures. The holes introduced in the mass of the test wood beam are equivalent to a substance presenting a much lower value of the MOE. The present experimental results support hereby earlier qualitative findings stating that rot increases the damping properties of wood and lowers its strength (Dunlop, 1981, 1983).

It is also important to differentiate between different kinds of damping arising when waves propagate in beams. The most important form of damping with which the present study is concerned is of course material damping. However, energy can also dissipate at various boundaries of the beam, either within it at reflection on inhomogeneities or through sound radiation at the outer surfaces. For longitudinal wave propagation, the latter form of energy loss is proportional to the radiating area which in the case of the beam is its the cross-section. The radiative energy loss is therefore considerably small as compared to the case of bending mode excitation. It is thus sometimes more attractive to evaluate the damping properties of materials by means of longitudinal wave excitations. Regarding wave reflections during propagation in a beam, these are never ideally elastic, and there always occurs some loss of energy whenever the wave bounces at the interface between two different media. In fact, numerous and extensive metallurgical and solid state studies have shown the advantages of the loss factor for investigating relatively small structural changes due to some treatment of a material or following the introduction of foreign objects in it (Cremer and Heckl, 1988, p. 233). Note the fundamental difference here that the loss factor in metals is much lower than in wood (about two orders of magnitude).

Viewed from another perspective, some similarity may be drawn between the propagation of longitudinal elastic waves in the beam and the propagation of sound waves within a room containing obstacles. The process of reverberation in room acoustics can be described in terms of the average loss of energy at the boundaries and the mean free path, denoted respectively by α and l_m. The mean free path is defined as the average distance between two successive reflections within the room, and it is roughly equal to four times the quote between the volume of the room and its total area. Hence, according to Norris–Eyring theory, the reverberation time in the room may be evaluated using the formula (Kuttruff, 1991):

$$T_{60} = -\frac{6\ln 10}{c} \cdot \frac{l_m}{\ln(1-\alpha)}, \tag{11}$$

where c is the speed of wave propagation. According to this last equation the reverberation time, which is inversely proportional to the loss factor, decreases for shorter l_m (more intervening obstacles in the wave path) and/or higher values of α (though not exceeding 1). However, this simple interpretation may be considered only as a qualitative description of the phenomenon of wave attenuation in the bar containing hollow defects. This is due to the fact that the derivation of Eq. (11) is made under statistical assumptions, and this usually applies best to large rooms whose sizes are substantially larger than the wavelength. A more realistic insight would be to consider the phenomenon of wave reflection at the boundary of two different media of being not ideally elastic and that unavoidable losses are always to be expected. A further technical precautionary measure to be taken into consideration when using the Reverberation Time for evaluating the loss factor is that for highly damped systems the product $B \cdot T_{60}$ must be larger than 16, with B being the bandwidth of the filter in Hz (Jacobsen, 1986).

6. Conclusions

This work presented the results of an experimental study on the effects of structural defects on the MOE and loss factor of a solid material. The example of a wooden beam has been taken for experimental practicality. The defects consisted of holes drilled in the beam which was set into longitudinal vibration through excitation along the grain. The longitudinal excitation was considered instead of for instance the bending mode of vibration for the reason that the experimental results may be affected by some coupling between the different modes of vibration due to the high anisotropy of wood.

The MOE and loss factor were evaluated at the fundamental resonance frequency, i.e., the frequency at which the length of the bar corresponds to half a wavelength. The loss factor was evaluated by means of an acoustical technique conceived originally for making measurements of sound absorption in room acoustical applications. The technique is attractive in that the measurements are perfectly repeatable and that there are available affordable measurement packages for conducting measurements in the audio frequency range. This however does not prevent the technique to be applied at the ultrasonic frequency range. The results from the present study reveal that the MOE decreases steadily with an increasing number of defects in the wood element and that at the same time the loss factor increases, though at a somehow slower rate.

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