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SOUND SCATTERING BY A HARD HALF-PLANE: EXPERIMENTAL EVIDENCE OF THE EDGE-DIFFRACTED WAVE

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In this short note, some experimental results are presented on the diffraction of a spherical way by a hard half-plane. This study was conducted with the aim to give evidence to the existence of the edge-diffracted wave. The sound source used in this experimental study is a condenser microphone operating in a reverse way. The wave emitted by a sound source propagates in space and hits a thin aluminium sheet with a straight edge, considered as an idealization of the hard half-plane. The resulting impulse response includes among others a wave diffracted by the edge of the half-plane, which is compared to its theoretical prediction. This latter is calculated from the exact Biot and Tolstoy solution to the problem of diffraction of a spherical wave by a hard wedge. Relatively satisfactory agreement is found between theory and experiment.

1. INTRODUCTION

The problem of interaction of waves with scattering objects delimited by straight edges has been known for a long time. When light propagates near the edge of an opaque screen, the classical geometrical optics fails to explain the cause of appearance of diffraction bands visible in the vicinity of the geometrical propagation boundaries. Earlier, Young attempted to explain this phenomenon by attributing to the diffracting screen the ability of emitting an edge wave when an incident wave falls upon it. This idea was taken up later by Fresnel who used Huygens' construction of the wave envelopes in combination with Young's principle of interference of waves to present for the first time a quite rigorous treatment of the phenomenon of light diffraction by a sharp straight edge [1]. Due to its importance and relative simplicity, the case of the half-plane has since then been given special attention in the theory of diffraction. Hence, in the light of the Huygens–Fresnel principle, the appearance of the light and dark diffraction fringes may then be simply attributed to either the constructive or destructive interference between the directly incident wave and the wave diffracted by the edge of the obstructing object. However, for a long time after the elaboration of the Huygens–Fresnel principle of diffraction, it had been assumed that the edge wave was simply a mathematical artifice used to ensure the smooth continuity of the total field at the transition between the geometrical boundaries. Consequently, it had to wait for Sommerlfeld's exact theory of diffraction to put Young's ideas on a sound basis.

On the other hand, the solutions to the presumably simple problem of the half-plane are few, and the exact ones among them resort often to the use of quite elaborate mathematics. To take again Sommerfeld's approach to the problem, the original two-dimensional

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solution to the problem of plane wave incidence makes use of the concept of branched solutions [2]. Hence, it is only at quite high frequencies, ideally in the limit of a vanishing wavelength, that the total field may explicitly be split into the sum of two contributions of different natures; the classical geometrical component and the edge-diffracted wave. In this case, the edge wave has some characteristics that remind one much of a cylindrical wave. It behaves as if it were originating at the edge of the diffracting screen, and its amplitude in the far field decreases in proportionality to the square root of the distance from the edge. It is this concept of the edge-diffracted wave which again has contributed to the elaboration of the Geometrical Theory of Diffraction [3], and its various subsequent refinements [4]. Earlier, Rubinowicz had also been able to isolate the edge wave from his reformulation of Kirchhoff's diffraction integral formula, which is the mathematical formulation of the Huygens–Fresnel principle. In this case too, the total field scattered by an aperture in a screen may be expressed as the sum of a geometrical part and of a wave considered as the build-up of contributions from fictive sources seeming to radiate from the rim of the aperture [5].

It would also be of some interest to note that most of the solutions to the problems of diffraction of the kind at hand start at the Helmholtz equation, and therefore are formulated in the frequency domain. However, a little more than 40 years ago, Biot and Tolstoy succeeded in presenting a new solution, this time in the time domain, to the more general problem of scattering of a spherical wave by a hard wedge [6]. The attractiveness of this approach is that it is formulated for a pulse excitation which could have interesting practical applications in the calculation of impulse responses, and that it uses rather simple mathematics. Hence, for a spherical pulse, an observer would possibly receive the geometrical components of the scattered field, depending on the geometry of the problem, but would always sense a wave diffracted from the edge of the wedge, the expression of which is given in an explicit closed form. The edge-diffracted wave then no longer remains a mathematical artefact used to ensure the smooth transition of the total field between the various geometrical zones in space, but it has a real physical existence. It is therefore the aim of this study to make a further contribution to the experimental evidence of the edge wave.

2. THEORY: THE BIOT–TOLSTOY THEORY OF DIFFRACTION APPLIED TO THE HARD HALF-PLANE

Consider in Figure 1 an infinitely rigid half-plane in a fluid with density ρ, a point source at S and a receiver at R. Then, according to the Biot and Tolstoy, B–T, diffraction theory, the total field at the receiver position would be made of a geometrical optics component and a diffraction component. In its original formulation, the B–T diffraction theory treats the problem of a source emitting an instantaneous explosion, but Medwin with slight mathematical changes presented the B–T solution to the diffraction problem of a concentrated Dirac impulse source [7]. The total field is equal to the sum of the geometrical components and the edge-diffracted field, that is $u_{tot} = u_i + u_r + u_d$. The geometrical optics contributions are u_i and u_r, respectively, the incident, direct wave from the source, and the reflected one, seeming to emanate from the source image of the real source through one face, or both faces, of the wedge. These two components are expressed as $u_i = \varepsilon_i \rho S \delta(t - d_i/c)/(4\pi d_i)$ and $u_r = \varepsilon_r \rho S \delta(t - d_r/c)/(4\pi d_r)$ for, respectively, the direct and reflected waves, d_i and d_r being the distances to the field point from the real sound source and its image. The coefficient ε_i (ε_r) is equal to one whenever the field point "sees" the real sound source (its image through the face of wedge), and zero otherwise. Moreover, the B–T theory considers the problem of a wedge with an arbitrary exterior angle, and the half-plane

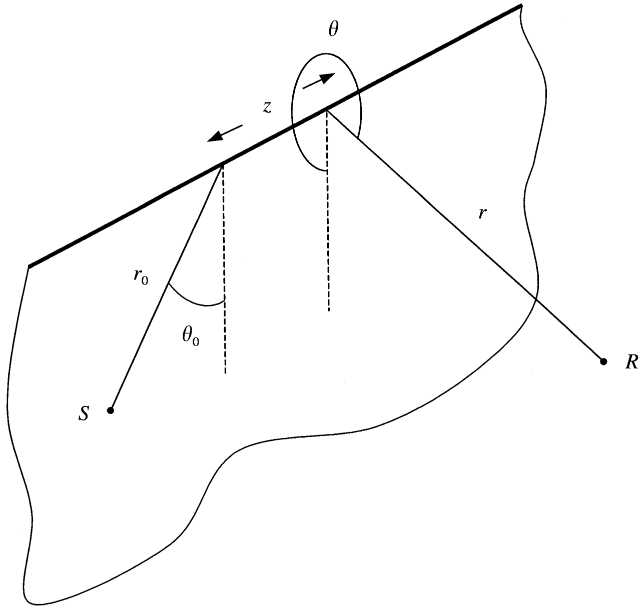

Figure 1. Geometry of the problem of diffraction of a spherical wave by a hard half-plane.

may simply be considered as a special case when letting the wedge exterior angle approach 2π. Hence, let S radiate a delta pulse of pressure

$$u_\delta = A\,\delta(t - d/c)/d, \tag{1}$$

in which A is a quantity proportional to the strength of the source, c the speed of sound propagation, d the distance from the source, and δ the Dirac delta function. After the source emits its divergent pulse, and at a time longer than τ_0 afterwards appears the diffracted wave due to the tip of the half-plane. The time τ_0 is the time needed by the wave to travel its shortest path from the point source to the field point via the crest line of the half-plane, and it is given by

$$\tau_0 = [(r + r_0)^2 + z^2]^{1/2}/c. \tag{2}$$

With some minor changes, the diffracted field $u_d(t)$ is given by [6, 7]

$$u_d = \frac{-Ac}{4\pi}\frac{\{\beta\}}{rr_0\sinh(y)}, \tag{3}$$

where

$$y = \operatorname{arccosh}\frac{c^2t^2 - (r^2 + r_0^2 + z^2)}{2rr_0} \tag{4}$$

and the expression

$$\{\beta\} = \frac{\cos(\pm\theta/2 \pm \theta_0/2)}{\cosh(y/2) + \sin(\pm\theta/2 \pm \theta_0/2)} \tag{5}$$

is the sum of four terms resulting from the four possible combinations of the signs in $(\pm\theta/2 \pm \theta_0/2)$.

For $z = 0$, equation (3) leads to the simplified form

$$u_d(t) = \frac{-A}{c}\sqrt{\frac{t_+^2 - t_-^2}{t^2 - t_+^2}}\left\{\frac{\cos(\theta + \theta_0)/2}{t^2 - t_+^2 + (t_+^2 - t_-^2)\cos^2(\theta + \theta_0)/2} + \frac{\cos(\theta - \theta_0)/2}{t^2 - t_+^2 + (t_+^2 - t_-^2)\cos^2(\theta - \theta_0)/2}\right\} \qquad (6)$$

with $t_\pm = (r \pm r_0)/c$. It may be noted that in our case $t_+ = \tau_0$, because the source and the receiver lie in the same plane that is perpendicular to the half-plane. Upon introducing the time variable $\tau = t - \tau_0$, equation (6) may again be expressed as

$$u_d(t) = \frac{-A}{c}\frac{\sqrt{t_+^2 - t_-^2}}{\tau\sqrt{1 + 2t_+/\tau}}\left\{\frac{\cos(\theta + \theta_0)/2}{\tau^2(1 + 2t_+/\tau) + (t_+^2 - t_-^2)\cos^2(\theta + \theta_0)/2} + \frac{\cos(\theta - \theta_0)/2}{\tau^2(1 + 2t_+/\tau) + (t_+^2 - t_-^2)\cos^2(\theta - \theta_0)/2}\right\}, \qquad (7)$$

which lends itself to easy and quite simple formulation in the frequency domain [8].

3. EXPERIMENTAL RESULTS

The most reliable quantitative theoretical investigations of the problem of diffraction by a half-plane have extended over a period of time of more than a hundred and fifty years. But due to the delicacy of the problem in acoustics, it is only during the few past decades, with the development of more powerful measurement techniques and better performing instruments, that successful experiments could be conducted. Some earlier experiments having direct relevance to the present work may be found in Maekawa's classical paper on the charts for noise reduction by screens [9], as well as those published by several other authors mainly on the validation of different theories of diffraction [10–14]. Related experimental work includes also, to name only a few, the diffraction by a thick half-plane [15] or the cases of noise reduction by thin barriers on the ground [16–22].

In an experiment, and in order to be able to isolate the edge-diffracted wave from the other components of the total field, an intense and short pulse, ideally a Dirac pulse, of sound would be the appropriate signal; hence, the idea of measuring the impulse response for a set-up composed of a sound source, a receiver and a diffracting half-plane. However, in the real world sound signals are finite both in amplitude and in frequency content. To overcome these difficulties in the best possible way, the impulse response was accessed in the present experiment through using a technique which during the last years has gained much popularity within the room acoustical community. The principle behind it is to process the cross-correlation function between the signal used to excite the system under investigation and the response of the system to this same signal. According to theory, the best estimate of the impulse response would be achieved for a driving signal that has the characteristics of random noise. The latter, again, may not be realized in practice, and instead, use is often made of the so-called maximum length sequences (MLS), periodic signals, which have similar properties to those of random signals though deterministic. For a short and well-accounted description of the theory underlying these techniques, the reader is referred to chapter VIII of Kuttruff's book [23]. For the purpose of our experiments, use was made of the popular MLSSA system, a ready-made measurement package comprising a measurement circuit board (ready to be slotted in a lap-top computer, for the easy performance of field measurements) and the accompanying software permitting to assess several room acoustical parameters†. Another technical concern is to find a sound source that approaches best a spherical one. The sound source is also expected not to disturb appreciably the acoustical field, and hence be of a suitably small size. A condenser microphone operating in the reverse way was therefore chosen: i.e., the electrical voltage of the signal is fed between the cathodes of the condenser of the microphone causing its membrane to vibrate, and thereby to radiate sound in the surrounding medium. The dynamic range of the obtained sound signal is rather small and therefore for obtaining reliable results, both the sound source and the field point must be selected at the greatest possible proximity of the diffracting edge (this is dictated by the time duration of the excitation pulse, hence by the bandwidth of working frequency). A similar measurement procedure had been reported earlier by Jebsen and Medwin [12], and for technical details on microphones operating as sound sources, reference is made to the pertaining literature [24].

The measurement set-up comprised a 1·5 mm thick aluminium plate with dimensions 2·0 m × 1·0 m laid on a 2 cm thick plywood board. The microphone used as a sound source was a 1 in Brüel & Kjær condenser microphone of the type B & K 4145, and the receiver microphone was a $\frac{1}{2}$ in, B & K of the type 4133. According to equation (7) and its form in the frequency domain, the edge-diffracted wave exhibits its strongest behaviour in the regions of space near the geometrical boundary zones. The geometrical wave incidence boundary is the plane containing the sound source and the edge of the half-plane away from the sound source. Similar considerations apply to the wave reflection boundary and the image of the sound source through the half-plane. The polar plot in Figure 2 represents the variation of the amplitude of the edge-diffracted field around the edge of the hard half-plane for a sound source at the angle $\theta_0 = 91^\circ$. Two properties of the edge-diffracted field may be noted from this figure. For the first, its angular symmetry (with a phase opposition) with respect to the half-plane (which may be proved through changing θ by $2\pi - \theta$ in equation (7)), and second, the relatively slow variation of its amplitude as one moves at a constant distance in the shadow zone (or its image through the half-plane).

Turning towards the experimental results, the pressure in the time domain for some configuration of the source and receiver positions is presented in Figure 3, where the presence of the wave diffracted by the edge of the metal plate is clearly manifested. The receiver microphone is in this case positioned near the reflection boundary. Some traces resulting from the scattering by other remote edges of the plate are also persistent in the whole signal time history, but in the signal analysis procedure these disturbances were smoothed out by a tapering window. Figure 4 shows a comparison between the theoretically predicted and the experimentally measured edge-diffracted pressures both in the time domain and in the frequency domain. The theoretically predicted time history of the pressure is evaluated simply through performing a convolution operation of the freefield pressure on the expression of the edge-diffracted field as expressed in equations (3–6), with the appropriate amplitude scaling and time translation. As may be seen, a quite satisfactory agreement is found between the two curves. In the lower range of the working frequency band, the poor performance of the sound source may be the cause of the large discrepancies between the theoretical and the experimental curves. On the other hand, due to the

†See for instance the website *www.mlssa.com* for more information on the MLSSA measurement system and related techniques.

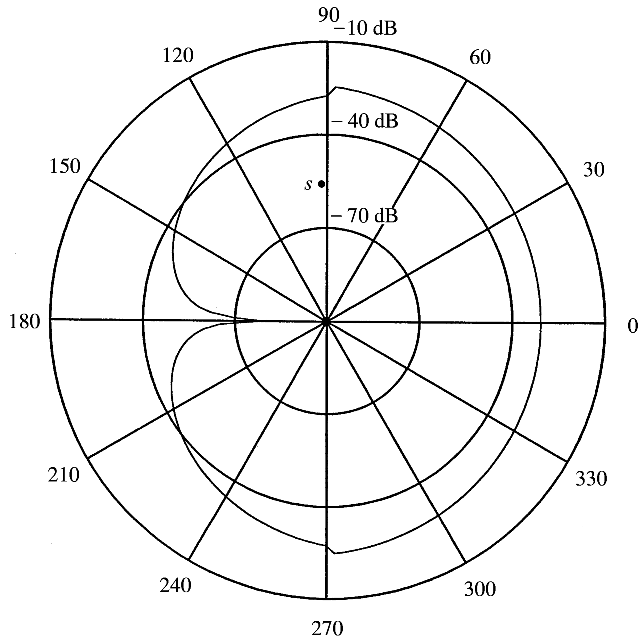

Figure 2. Amplitude of the diffracted field for a constant distance of the receiver from the edge of the half-plane; plot of the theoretical angular variation. $r_0 = \lambda$, $r = 1{\cdot}5\lambda$, $\theta_0 = 91°$.

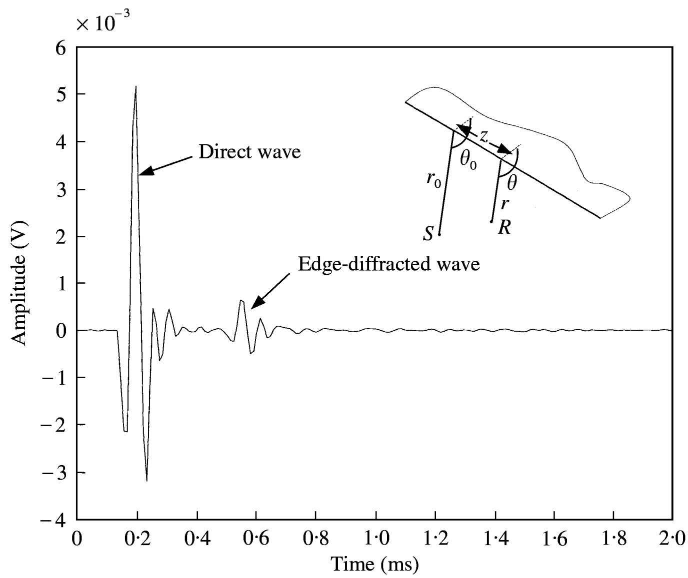

Figure 3. Experimental determination of the impulse response for a thin metallic plane in the incidence zone. $r_0 = 8{\cdot}9$ cm, $\theta_0 = 92°$, $r = 6{\cdot}9$ cm, $\theta = 91{\cdot}6°$, $z = 2{\cdot}6$ cm.

relatively short wavelengths in the upper frequency range, the disagreements between the curves may be attributed to either the mutual scattering of the field between the two microphones, or the scattering around the receiver microphone itself. Therefore, another experiment was conducted, this time the receiver was positioned away from the sound

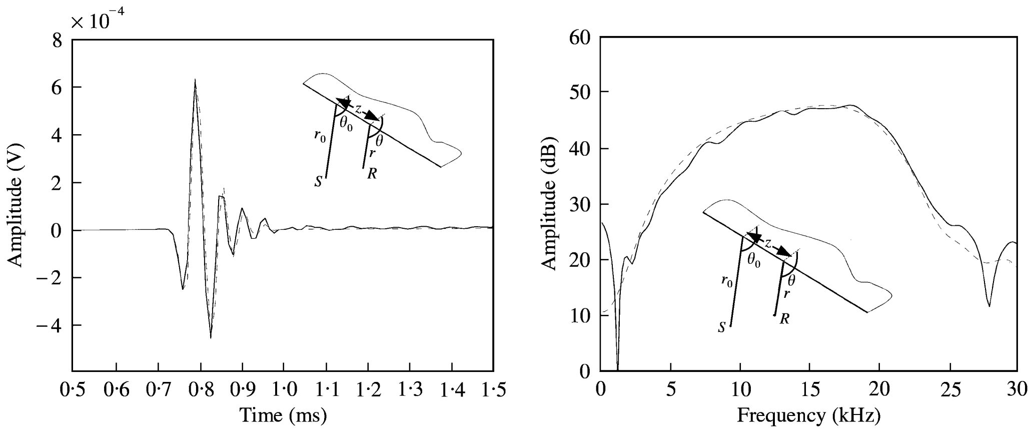

Figure 4. Edge-diffracted field: comparison between theory and experiment. $r_0 = 8{\cdot}9$ cm, $\theta_0 = 91{\cdot}6°$, $r = 6{\cdot}9$ cm, $\theta = 91{\cdot}6°$, $z = 2{\cdot}6$ cm. (a) Time domain. (b) Frequency domain. ——, predicted; ------, measured.

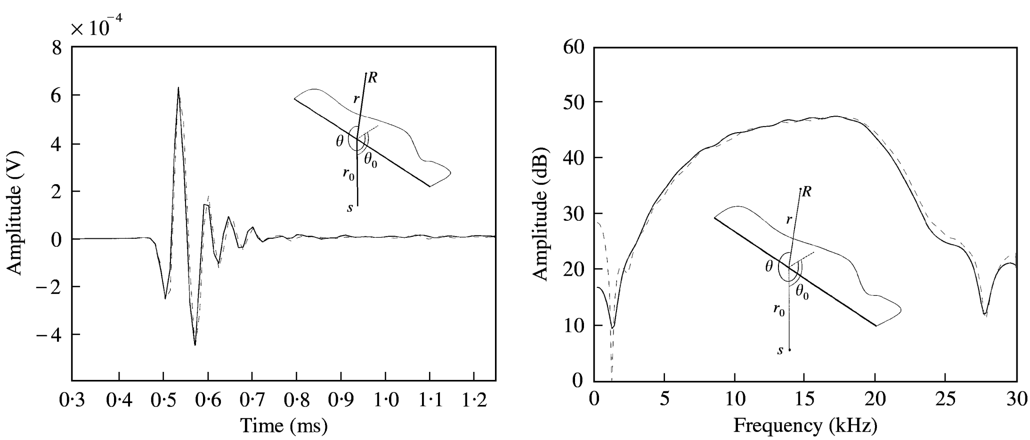

Figure 5. Same as Figure 4 but for $r_0 = 10{\cdot}0$ cm, $\theta_0 = 88{\cdot}5°$, $r = 13{\cdot}9$ cm, $\theta = 271{\cdot}5°$.

source in the shadow of the direct sound field, and at an angular position of just a few degrees away from the geometrical incidence boundary. The results are presented in Figure 5 where this time the convolution of the time history of the pressure was performed by means of equation (6). In this latter case, better agreement is found between theory and experiment.

4. CONCLUSIONS

In this short note, some experimental results on the diffraction of a sound wave by a hard half-plane have been presented. The investigation emphasized on the wave diffracted by the straight edge of the half-plane. Previously published work on this topic considered mostly the diffraction at specific frequencies whereas the present study highlights the edge diffraction phenomenon in a wide frequency range. To this end, a simple experimental set-up was mounted, consisting of a thin metallic plate representing the half-plane, and a microphone operating as a sound source. For some positions of the sound source and the receiver, it is possible to isolate the contribution of the edge wave from the total signal and it may be compared to its theoretical prediction. Two positions of the receiver were chosen therefore: one position near the source in the incidence zone (where the total field is composed of the incident field and the diffracted field), and another position in the shadow zone where the total field comprises only the edge-diffracted field. Comparisons were made with theoretical predictions using the Biot and Tolstoy exact theory of diffraction and the results show reasonably satisfactory agreement between experiment and theory.

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REFERENCES

1. M. Born and E. Wolf 1986 *Principles of Optics*. Oxford: Pergamon Press; sixth edition.
2. A. Sommerfeld 1954 *Lectures on Theoretical Physics*, Vol. IV: *Optics*. New York: Academic Press.
3. J. B. Keller 1962 *Journal of the Optical Society of America* **52**, 116–130. Geometrical theory of diffraction.
4. R. G. Kouyoumjian and P. H. Pathak 1974 *Proceedings of the IEEE* **62**, 1448–1461. A uniform asymptotic theory of diffraction for an edge in a perfectly conducting surface.
5. A. Rubinowicz 1957 *Polska Akademika Nauka Warszawa*. Die Beugungswelle in der Kirschoffschen Theorie der Beugeung.
6. M. A. Biot and I. Tolstoy 1957 *Journal of the Acoustical Society of America* **29**, 381–391. Formulation of wave propagation in infinite media by normal coordinates with an application to diffraction.
7. H. Medwin 1981 *Journal of the Acoustical Society of America* **69**, 1060–1064. Shadowing by finite noise barriers.
8. D. Ouis 2000 *Applied Acoustics* **59**, 19–66. Scattering of a spherical wave by a thin hard barrier on a reflecting plane.
9. Z. Maekawa 1968 *Applied Acoustics* **1**, 157–173. Noise reduction by screens.
10. P. Ambaud and A. Bergassoli 1972 *Acustica* **27**, 291–298. Le problème du dièdre en acoustique.
11. T. Kawai, K. Fujimoto and T. Itow 1977 *Acustica* **38**, 313–323. Noise propagation around a thin half plane.
12. G. M. Jebsen and H. Medwin 1982 *Journal of the Acoustical Society of America* **72**, 1607–1611. On the failure of Kirchhoff assumption in backscatter.
13. A. I. Papadopoulos and C. G. Don 1991 *Journal of the Acoustical Society of America* **90**, 1011–1018. A study of barrier attenuation by using acoustic impulses.
14. B. B. Narod and M. J. Yeldin 1986 *American Journal of Physics* **54**, 1121–1126. A basic acoustic diffraction experiment for demonstrating the geometrical theory of diffraction.
15. K. Fujiwara, Y. Ando and Z. Maekawa 1977 *Applied Acoustics* **10**, 147–159. Noise reduction by barriers: Part I, noise reduction by a thick barrier.
16. W. E. Scholes, A. C. Salvidge and J. W. Sargent 1971 *Journal of Sound and Vibration* **16**, 627–642. Field performance of a noise barrier.
17. H. G. Jonasson 1972 *Journal of Sound and Vibration* **22**, 113–126. Sound reduction by barriers on the ground.
18. S. I. Thomasson 1978 *Journal of the Acoustical Society of America* **63**, 1768–1781. Diffraction by a screen above an impedance boundary.
19. T. Isei, T. F. W. Embleton and J. E. Piercy 1980 *Journal of the Acoustical Society of America* **67**, 46–58. Noise reduction by barriers on finite impedance ground.
20. J. Nicolas, T. F. W. Embleton and J. E. Piercy 1983 *Journal of the Acoustical Society of America* **73**, 44–54. Precise model measurements versus theoretical prediction of barrier insertion loss in presence of the ground.

21. R. K. PIRINCHIEVA 1991 *Journal of Sound and Vibration* **148**, 183–192. The influence of barrier size on its sound diffraction.
22. D. J. SAUNDERS and R. D. FORD 1993 *Journal of the Acoustical Society of America* **94**, 2859–2875. A study of the reduction of explosive impulses by finite sized barriers.
23. H. KUTTRUFF 1991 *Room Acoustics*. London: Elsevier Applied Science; third edition.
24. Technical Review 1977 *Microphones used as sound sources*. Brüel & Kjær publication, No 3.