Analyses of axisymmetric waves in layered piezoelectric rods and their composites

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(Received 25 November 1999; accepted for publication 22 June 2000)

An exact treatment of the propagation of axisymmetric waves in coaxial anisotropic assembly of piezoelectric rod systems is presented. The rod system consists of an arbitrary number of coaxial layers, each possessing transversely isotropic symmetry properties. The treatment, which is based on the transfer matrix technique, is capable of deriving the dispersion relations for a variety of situations. These include the case of a single rod system that is either embedded in an infinitely extended solid or fluid host or kept free. The procedure is also adapted to derive approximate solutions for the cases of a periodic fiber distribution in a matrix material, which model unidirectional fiber-reinforced composites. The results are numerically illustrated for a widely used piezoelectric-polymer composite. It is seen that piezoelectric coupling can significantly change the morphology of the dispersive behavior of the composite. [S0001-4966(00)02410-3]

PACS numbers: 43.25.Cg [HEB]

I. INTRODUCTION

In a recent paper,[1] Nayfeh and Nagy presented a unified study of the propagation of axisymmetric longitudinal waves in coaxial anisotropic elastic fibers with application to a variety of their composites. The fiber bundle system consisted of an arbitrary number of coaxial layers, each possessing transverse isotropic symmetry and bonded at their interfaces in accordance with specified fashions. In the form of applications, the bundle was either embedded in an infinitely extended elastic solid or fluid host or kept free. Uniformly distributed bundles in a matrix material which model unidirectional bundle fiber-reinforced composites were also studied.

The solutions in Ref. 1 were obtained using the matrix transfer technique, originally introduced by Thomson[2] and later on enhanced by Haskell,[3] both for the study of wave interaction with layered isotropic flat media. Recently, this technique has been extensively used by a large number of investigators in a variety of wave propagation applications in layered media, often including cases of anisotropy. Most of this recent literature deals with flat layers. It is not the intent of the present work to review this vast literature. However, for the interested reader on this subject, we refer to Nayfeh.[4]

Up to quite recently, much fewer applications of the matrix transfer method to cylindrical layered media have been reported. In its application to the cylindrical systems of Ref. 1, the specific steps taken in constructing the model are summarized as follows: formal solutions in terms of Bessel functions are first obtained for each layer in terms of its displacement amplitudes. By specializing these solutions to the outer and inner faces (radii) of the layer, followed by eliminating the common displacement amplitudes, the local transfer matrix for the layer is constructed. This matrix relates the field variables (the displacements and stress components) of one face of the layer to its other one. Such a matrix relation can be used, in conjunction with satisfying appropriate interface conditions across neighboring layers, to directly relate the field variables at the inner face of one layer to the outer face of its outer neighbor. When this procedure is carried out consecutively for all layers in the bundle, a global transfer matrix, the product of the individual transfer matrices, results, which relates the field variables at the outer face of the bundle to those at its inner face or vice versa.

The global matrix is then used for all of the specific applications outlined above. For cases involving infinite solid or fluid hosts, the formal solution for the host has to be specialized to insure satisfying the radiation conditions far away from the bundle–host interface; this requires the field variables to stay bounded deep in the host. If the inner component of the bundle is solid with no concentric hole, its local transfer matrix has to be adjusted such that its field variables will not encounter the usual singularities. Finally, in simulating the bundle fiber-reinforced composite, the repeating hexagonal unit cell of the composite is isolated, approximated by a concentric cylindrical system and, due to its inherent symmetry, the vanishing of the radial displacement and shear stress components are imposed on its outer boundary.

The aim of the present article is to extend the results of Ref. 1 and present a parallel treatment for rod-bundle systems including piezoelectric effects. This problem has diverse applications in fields such as medical composite transducer design, fiber optics, signal processing, and nondestructive testing. Up to quite recently, it was believed that cylindrical geometry, together with piezoelectric coupling (which is inherently anisotropic), lead to algebraic complications in the attempt of obtaining exact solutions. That is perhaps why exact treatments of problems dealing

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