An approximate model for wave propagation in piezoelectric materials. I. Laminated composites

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A continuum mixture theory with microstructure is developed for guided wave propagation in bilaminated periodic composites of piezoelectric materials. The theory leads to the simple governing coupled equations for the actual composites which retain the integrity of the propagation process in each constituent but allow them to coexist under analytically derived interaction parameters. As a consequence of the analysis, effective mixture properties of the composite are obtained in the zero-frequency (static) limit. The accuracy of the approximation is demonstrated by direct comparison with the exact solution for the propagation of harmonic waves in the system. © *1999 American Institute of Physics.* [S0021-8979(99)03704-4]

I. INTRODUCTION

There has recently been an increased interest in determining the effective bulk properties of piezoelectric composite materials. This is evidenced from the large volume of current literature devoted to the subject. The state of the art can be found in some of the most recent publications.[1–7] This has been perhaps prompted by the fact that, since the late 1970's piezoelectric composites have been used in the manufacturing of high-tech components such as ultrasonic transducers and actuators. A detailed exposition of advantages offered by piezoelectric composites in such applications is given in a recent review by Smith.[8]

Prior to the current interest in piezoelectric effects, increased demand for the use of composite materials (both laminated and fibrous) in structural and various device applications has led to extensive studies of their modeling, manufacturing, testing, and analysis. Considering the diversity of their applications, researchers in different fields have concentrated their efforts on the study of that property of the composite which is most attractive to their specific need. For most realistic composites, however, the variability in constituent properties, geometrical arrangements, and their interactions render exact analytical or computational investigations rather difficult. As an alternative, investigators have sought to develop all sorts of effective approximate models capable of predicting in some sense the gross behavior of the composite.

In some idealized situations, and for simple systems such as the present one, however, exact solutions might be obtained. These can serve as a check on the accuracy of the approximate results and also on the applicability and utility of the modeling techniques of more complicated systems. Even for determining effective mechanical properties, for example, several schemes are now available. For simple deterministic geometries, limited success has been realized in calculating some of the properties; this is based primarily on solving appropriate boundary value problems. For most situations, however, properties are calculated or estimated by using bounds schemes or by using various theories of mixture depending upon available information about the variability in constituent properties, geometrical arrangements, and interactions (see, for example, Ref. 9 for detailed references). The analyses presented in Refs. 1–8 constitute for the most part, adaptations and, or, extensions of uncoupled case techniques to include piezoelectric coupling.

In the case of dynamic applications and, in particular those involving wave propagation, the applicability of the effective modules theories is somewhat restrictive. In particular, these theories are incapable of reproducing the dispersion and pronounced alteration (spreading and attenuation) of propagation pulses in these composites; such effects are, in the main, a result of the composite's microstructure (see, for example, Ref. 10). The necessity to simulate such effects on the mechanical, thermal, and electromagnetic response of composites has led to development and applications of several theories reflecting the influence of the microstructure. Up to quite recently, most available treatments dealt with the mechanical behaviors of composites, and to a lesser degree, extend their thermal and electromagnetic responses.

In this article, we extend the continuum mixture theory of guided wave propagation in the purely elastic bilaminated[10] to include piezoelectric effects. The theory leads to four coupled simple equations for the propagation process in each constituent subject to some interactions. This system exploits the fiber–matrix microstructural contribution to the propagation process. In particular information as to the distribution of field quantities in both the fiber and matrix constituents will be readily available. As a particular application, the derived system of equations is readily adaptable to the study of harmonic excitation in the system. The utility and range of applicability of this simple theory is established by comparison with an exact treatment of the proposed model as will be presented in Appendix A. For laminated piezoelectric layers there are several recent publications that deal with various aspects of their exact analysis.[11–15] For further literature on exact treatments we refer to Ref. 9.