- Simple harmonic motion
  - displacement
  - velocity
  - acceleration
  - Force law
  - energy

- Angular simple harmonic motion.

- Pendulums
  - Simple pendulum.
  - Physical pendulum.

- Simple harmonic motion and uniform circular motion.
Simple harmonic motion: 

The motion of an object is a simple harmonic motion (SHM) if the object's displacement is given by:

\[ x(t) = x_m \cos(\omega t + \phi) \]

- **Amplitude** (maximum displacement)
- **Angular frequency**
- **Phase constant** (phase angle)
- **Displacement at time t**
- **Time**
- **Phase**
- **Angular frequency** (rad/s)
- **Period**
- **Frequency** (Hertz = Hz = Oscillation per second)

\[ \omega = \frac{2\pi}{T} = 2\pi f \]

\[ T = \frac{1}{f} \]
Dec 02, 01

\[ \alpha_m = 1.5 \]
\[ \alpha_m = 0.5 \]

\[ \alpha \]
\[ T_1 \]
\[ T_2 \]

Decreasing \( \phi \) shifts the curve rightward.

**Velocity**

\[ V(t) = \frac{dx(t)}{dt} = -\omega \alpha_m \sin(\omega t + \phi) \]

\[ V(t) = -V_m \sin(\omega t + \phi) \]

\[ V_m = \omega \alpha_m \]

Velocity amplitude.

**Acceleration**

\[ a(t) = \frac{dV(t)}{dt} = -\omega^2 \alpha_m \cos(\omega t + \phi) \]

\[ a(t) = -a_m \cos(\omega t + \phi) \]

\[ a_m = \omega \alpha_m \]

\[ a(t) = -\omega^2 \alpha(t) \]

In simple harmonic motion,

\[ a(t) = -\omega^2 \alpha(t) \]
Force law:

For simple harmonic motion, \( a(t) = -\omega^2 x(t) \)

From Newton's second law, \( F = ma \)

\[ F = m(-\omega^2 x) \]

Hooke's law, \( F = -\frac{m \omega^2}{k} x \)

Angular frequency, \( \omega = \sqrt{\frac{k}{m}} \)

Period, \( T = 2\pi \sqrt{\frac{m}{k}} \)
Example \( k = 65 \text{ N/m} \)

\[ \text{equilibrium position} \quad n = 0 \]

\[ m = 680 \text{ g} \]

\[ \text{frictionless} \]

\[ \text{at } t = 0 \]

**Q** What is the angular frequency, the frequency and the period of the motion?

\[ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65}{0.680}} = 9.8 \text{ rad/sec} \]

\[ f = \frac{\omega}{2\pi} = 1.6 \text{ Hz} \]

\[ T = \frac{1}{f} = 0.64 \text{ s} \]

**Q** What is the amplitude of the motion?

\[ \chi_m = 11 \text{ cm} \]

**Q** What is the maximum speed of the block and where is the block when it occurs?

\[ v_m = \omega \chi_m = (9.8 \text{ rad/}s) (0.11 \text{ m}) = 1.1 \text{ m/s} \]

This happens when \( \chi = 0 \)

**Q** What is the maximum acceleration of the block?

\[ a_m = \omega^2 \chi_m = (9.8 \text{ rad/}s)^2 (0.11 \text{ m}) = 11 \text{ m/s}^2 \]
Q. what is the phase constant of the motion?

A. at \( t = 0 \), \( x = 11 \text{ cm} \)

\[ x(t) = x_m \cos(\omega t + \phi) \]

\[ 0.11 = 0.11 \cos(\phi) \]

\[ 1 = \cos(\phi) \implies \phi = 0 \]

---

**Example**

```
\[ \begin{align*}
\text{At } t &= 0 \\
\text{x} &= -8.50 \text{ cm} \\
\text{v} &= -9.20 \text{ m/s} \\
\text{a} &= 47.0 \text{ m/s}^2 \\
\end{align*} \]
```

Q. what is the angular frequency of the system?

A. \( x(0) = x_m \cos \phi \)

\( v(0) = -\omega x_m \sin \phi \)

\( a(0) = -\omega^2 x_m \cos \phi \)

\[
\frac{a(0)}{x(0)} = \frac{-\omega^2 x_m \cos \phi}{x_m \cos \phi} = -\omega^2
\]

\[ \omega = \sqrt{-\frac{a(0)}{x(0)}} = \sqrt{-\frac{47.0}{-8.50}} = 23.5 \text{ rad/s} \]

Q. what is the phase constant and amplitude?

\[
\frac{v(0)}{x(0)} = \frac{-\omega x_m \sin \phi}{x_m \cos \phi} = -\omega \tan \phi
\]

\[ \tan \phi = -\frac{v(0)}{x(0)} = \frac{-0.920}{(23.5)(-0.085)} = -0.461 \]
\[
\phi = \tan^{-1}(-0.461) \\
\phi = -25^\circ \quad \text{or} \quad \phi = 180^\circ + (-25) = 155^\circ \\
\text{Which one is correct?}
\]

We know that
\[
\begin{align*}
\alpha(0) &= \alpha_m \cos \phi \\
\alpha_m &= \frac{\alpha(0)}{\cos \phi}
\end{align*}
\]

For \( \phi = -25^\circ \)
\[
\alpha_m = -0.094 \text{ m}
\]
not acceptable

For \( \phi = 155^\circ \)
\[
\alpha_m \approx 0.094 \text{ m}
\]
always positive

So \( \phi = 155^\circ \) and \( \alpha_m = 0.094 \text{ m} \)

\[\text{Energy}\]

\[\begin{array}{c}
\text{massless} \\
\includegraphics[width=0.5\textwidth]{mass.png} \\
\text{frictionless}
\end{array}\]

Elastic potential energy
\[
\rightarrow U = \frac{1}{2} k \alpha^2 = \frac{1}{2} k \alpha_m^2 \cos^2(\omega t + \phi)
\]

Kinetic energy
\[
\rightarrow K = \frac{1}{2} m \dot{\alpha}^2 = \frac{1}{2} m (-\omega \alpha_m \sin(\omega t + \phi))^2 \\
= \frac{1}{2} m \omega^2 \alpha_m^2 \sin^2(\omega t + \phi)
\]

Mechanical energy
\[
\rightarrow E = U + K = \frac{1}{2} k \alpha_m^2
\]
Angular simple harmonic motion:

\[ F \rightarrow \omega \]
\[ \pi \rightarrow \theta \]
\[ m \rightarrow I \]

\[ k \rightarrow k \quad \text{spring constant} \quad \text{\# torsion constant} \]

\[ F = -k \chi \quad \Rightarrow \quad \chi = -k \theta \]

\[ T = 2\pi \sqrt{\frac{m}{k}} \quad \Rightarrow \quad T = 2\pi \sqrt{\frac{E}{k}} \]

Example
Pendulums

Simple pendulum

\[ T = 2\pi \sqrt{\frac{L}{g}} \]

- period of oscillation
- length of the wire

- small angle
- massless wire of length \( L \)
- oscillation

- particle of mass \( m \) (the bob)

Physical pendulum

\[ T = 2\pi \sqrt{\frac{I}{mgh}} \]

- period of oscillation
- mass of the solid object

- mass from the solid object
- oscillation through small angles

- pivot point
- center of mass
- solid object

- distance between pivot point and the center of mass
- moment of inertia of the solid object about the pivot point
Example

Q: what is the period of oscillation rod

$$T = 2\pi \sqrt{\frac{I}{mg\ell}}$$

$$= 2\pi \sqrt{\frac{\frac{1}{3} mL^2}{mg \ell}}$$

$$= 2\pi \sqrt{\frac{2L}{3g}}$$

Parallel-axis theorem

$$\frac{1}{12} mL^2 + m(\ell/2)^2 = \frac{1}{3} mL^2$$

moment of inertia about center of mass

Simple harmonic motion and uniform circular motion

Simple harmonic motion is the projection of uniform circular motion on a diameter of the circle in which the latter motion occurs.

$$x = x_m \cos(\omega t + \phi)$$