The differential equation for a damped harmonic oscillator is
\[ \dddot{x} + 2\beta \dot{x} + \omega_n^2 x = 0 \]

Given that \( m = 0.05 \text{ kg} \) and \( k = 5.0 \text{ N/m} \), discuss the nature of the oscillations, sketch the graph of \( x \) vs. \( t \) and give the general solution for the initial conditions \( x(0) = 0.1 \text{ m} \) and \( \ddot{x}(0) = 0 \). For the following two cases:

(a) \( \beta = 10 \text{ rad/s} \).
(b) \( \beta = 1 \text{ rad/s} \).

\[ a) \ \omega_n = \sqrt{\frac{k}{m}} = \sqrt{100} = 10 \text{ rad/s} \]

\[ \omega_n^2 = \beta^2 \Rightarrow \text{Critical damping} \]

\[ x(t) = (A + Bt)e^{-\beta t} \]

\[ x(0) = A = 0.1 \text{ m} \]

\[ \dot{x}(t) = B e^{-\beta t} - \beta (A + Bt) e^{-\beta t} \]

\[ \ddot{x}(0) = 0 \Rightarrow B = \beta A = 10 \times 0.1 = 1 \text{ m/s} \]

\[ x(t) = (0.1 + t) e^{-10t} \]

\[ b) \ \beta^2 < \omega_n^2 \Rightarrow \text{underdamping} \]

\[ x(t) = A e^{-\beta t} \cos(\omega_d t - \delta) \]

\[ x(0) = A \ cos \delta = 0.1 \]

\[ \dot{x}(t) = -\beta A e^{-\beta t} \cos(\omega_d t - \delta) - A \omega_d e^{-\beta t} \sin(\omega_d t - \delta) \]

\[ \ddot{x}(0) = -\beta A \cos \delta + A \omega_d \sin \delta = 0 \Rightarrow \tan \delta = \frac{\beta}{\omega_n} = \frac{1}{\sqrt{99}} \]

\[ \delta = 0.1 \text{ rad} \]

\[ x(0) = A \cos(0.1) \Rightarrow A = 0.1 \text{ m} \]

\[ x(t) = 0.1 e^{-t} \cos((\sqrt{99} t - 0.1)) \]