KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
Physics Department

PHYS 301
Second Major Exam (2nd May, 2001) – Term 002

Instructor’s Name: Dr. A. Mekki

Student’s Name: ________________________________

I.D. No: ________________________

Exam Time: 90 minutes
Show the details of your work and circle your answer

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Q1. (40 points)
Consider an RLC circuit connected in series to an alternating source $\varepsilon = \varepsilon_o \cos (\omega t)$ as shown in the figure. The switch is closed at $t = 0$.
(a) Show that the differential equation governing the charge in the capacitor is given by:

$$\dot{q} + \frac{R}{L} \dot{q} + \frac{1}{LC} q = \left(\frac{\varepsilon_o}{L}\right) \cos \omega t$$  \hspace{1cm} (1)

(b) Write the mechanical equivalent to the above equation and deduce the solution to equation (1) in the steady state mode in terms of $R$, $L$, $C$, $\omega$ and $t$.
(c) Find the voltage across the resistance as a function of time.
(d) What is the time averaged energy dissipated in the resistor?
(e) What is the time averaged energy stored in the capacitor?

(recall $\langle x \rangle = \frac{1}{T} \int_0^T x dt; \quad T = \frac{2\pi}{\omega}$)
Q2. (30 points)
Consider a non-uniform spherical mass distribution of density \( \rho (r) = Ar \) (A is a constant) and radius \( R \). Calculate the gravitational field \( g(r) \) and potential \( \phi(r) \):
(a) inside the sphere \( (r < R) \)
(b) outside the sphere \( (r > R) \)
(c) Draw few field lines and the corresponding equipotentials outside the sphere.
(d) What is the energy required to bring a point mass \( m \) from infinity to the surface of the sphere?
Q3. (30 points)

(a) Prove (in cartesian coordinates) that the curve that gives the shortest path between two fixed points \( P_1(x_1, y_1) \) and \( P_2(x_2, y_2) \) in the xy plane is given by a straight line that joins these two points.

(b) Consider two arbitrary points \( P_1(a, Z_1, \phi_1) \) and \( P_2(a, Z_2, \phi_2) \) lying on the surface of a right cylinder of radius \( a \).

(i) Use cylindrical coordinates to show that the path between the two points can be written in the form

\[
S = \int_{\phi_1}^{\phi_2} \sqrt{a^2 + \dot{z}^2} \, d\phi
\]

(ii) Show that in order to minimize the integral, the \( z \) coordinates should vary as

\[ z = A \phi \quad \text{where } A \text{ is a constant} \]

with the initial condition that \( Z(\phi = 0) = 0 \)

(iii) Using the fact that in cylindrical coordinates, \( x = a \cos \phi \) and \( y = a \sin \phi \), what would be the shortest path between the points \( P_1 \) and \( P_2 \)?