Problem 19. \( R_1 = 100 \, \Omega, \quad R_2 = 50 \, \Omega \),

batteries have emfs \( \varepsilon_1 = 6.0 \, V \), \( \varepsilon_2 = 5.0 \, V \), and \( \varepsilon_3 = 4.0 \, V \).

Find (a) the current in resistor 1, (b) the current in resistor 2, and (c) the potential difference between points \( a \) and \( b \).

SSM

Fig. 27-34 Problem 19.

(a) \(- I_1 R_1 + \varepsilon_2 = 0 \Rightarrow I_1 = \frac{\varepsilon_2}{R_1} = \frac{5}{100} = 0.05 \, A.\)

(b) \(- I_2 R_2 + \varepsilon_1 - \varepsilon_2 - \varepsilon_3 = 0 \Rightarrow I_2 = \frac{\varepsilon_1 - \varepsilon_2 - \varepsilon_3}{R_2} = \frac{-3}{50} = -0.06 \, A.\)

(c) \(V_b - V_a = - \varepsilon_2 - \varepsilon_3 = -5 - 4 = -9 \, V\)

or \(V_b - V_a = - I_1 R_1 - \varepsilon_3 = -0.05 \times 100 - 4 = -9 \, V\)

or \(V_b - V_a = - \varepsilon_1 - I_2 R_2 = -6 - 3 = -9 \, V\)
Switch S in Fig. 27-52 is closed at time $t = 0$, to begin charging an initially uncharged capacitor of capacitance $C = 15.0 \ \mu F$ through a resistor of resistance $R = 20.0 \ \Omega$. At what time is the potential across the capacitor equal to that across the resistor?

\[
V_C = E \left( 1 - e^{-\frac{t}{RC}} \right)
\]

\[
V_R = IR = RI_{\text{max}} e^{-\frac{t}{RC}}
\]

\[V_C = V_R \]

\[E \left( 1 - e^{-\frac{t}{RC}} \right) = E e^{-\frac{t}{RC}} \]

\[2 e^{-\frac{t}{RC}} = 1 \]

\[e^{-\frac{t}{RC}} = \frac{1}{2} \implies -\frac{t}{RC} = \ln(\frac{1}{2}) \]

\[t = RC \ln 2 \]

\[t = 0.69 RC = 2.08 \times 10^{-4} \text{ s} \]
In an $RC$ series circuit, $\varepsilon = 12.0$ V, $R = 1.40 \text{ M}\Omega$, and $C = 1.80 \mu\text{F}$. (a) Calculate the time constant. (b) Find the maximum charge that will appear on the capacitor during charging. (c) How long does it take for the charge to build up to 16.0 $\mu\text{C}$?

\[ a) \ T = RC = 2.52 \text{s}. \]

\[ b) \ q_{\text{max}} = CE = 21.6 \mu\text{C} \text{ (after a long time)} \]

\[ c) \ q(t) = q_{\text{max}} \left( 1 - e^{-\frac{t}{RC}} \right) \]

16 $\mu\text{C} = 21.6 \mu\text{C} \left( 1 - e^{-\frac{t}{2.52}} \right)$

\[ 0.74 = 1 - e^{-\frac{t}{2.52}} \]

\[ e^{-\frac{t}{2.52}} = 0.74 \]

\[ -\frac{t}{2.52} = \ln(0.26) = -1.35 \]

\[ t = 3.4 \text{s} \]
A capacitor with initial charge $q_0$ is discharged through a resistor. What multiple of the time constant $\tau$ gives the time the capacitor takes to lose (a) the first one-third of its charge and (b) two-thirds of its charge?

\[ R_C = \tau \]

It has unit of sec.

\[ C \quad q_0 \]

\[ R \]

charge remaining

\[ q(t) = q_0 e^{-t/RC} \]

\[ \frac{2q_0}{3} = q_0 e^{-t/RC} \]

\[ \ln \left( \frac{2}{3} \right) = -\frac{t}{RC} \]

\[ t = RC \ln \left( \frac{2}{3} \right) = 0.41 \frac{RC}{\tau} \]

\[ = 0.41 \tau \]

\[ \frac{1}{3} q_0 = q_0 e^{-t/RC} \]

\[ t = RC \ln 3 = 1.1 RC = 1.1 \tau \]
In Fig. 27-42, $R_1 = 100 \Omega$, $R_2 = R_3 = 50.0 \Omega$, $R_4 = 75.0 \Omega$, and the ideal battery has emf $E = 6.00 \, \text{V}$. (a) What is the equivalent resistance? What is $i$ in (b) resistance 1, (c) resistance 2, (d) resistance 3, and (e) resistance 4?

\[
\begin{align*}
\text{Fig. 27-42 Problems 30 and 36.} \\
\text{a)} & \quad R_{34} = 30.0 \, \Omega \\
\text{b)} & \quad I = \frac{E}{R_{eq}} = 0.05 \, \text{A} \\
\text{c)} & \quad V_2 = I_2 R_2 \Rightarrow I_2 = \frac{V_2}{R_2} = \frac{1}{50} = 0.02 \, \text{A} \\
\text{d)} & \quad I_3 = I_2 = 0.02 \, \text{A} \\
\text{e)} & \quad I_4 = \frac{V_4}{R_4} = \frac{1}{75} = 0.01 \, \text{A}.
\end{align*}
\]
Problem 19. $R_1 = 100 \, \Omega$, $R_2 = 50 \, \Omega$.

The batteries have emfs $E_1 = 6.0 \, \text{V}$, $E_2 = 5.0 \, \text{V}$, and $E_3 = 4.0 \, \text{V}$.

Find (a) the current in resistor 1, (b) the current in resistor 2, and (c) the potential difference between points $a$ and $b$. 

Fig. 27-34  Problem 19.

(a) $- I_1 R_1 + E_2 = 0 \Rightarrow I_1 = \frac{E_2}{R_1} = \frac{5}{100} = 0.05 \, \text{A}.$

(b) $- I_2 R_2 + E_1 - E_2 - E_3 = 0$

$I_2 = \frac{E_1 - E_2 - E_3}{R_2} = \frac{-3}{50} = -0.06 \, \text{A}.$

(c) $V_b - V_a = - E_2 - E_3$

$= -5 - 4 = -9 \, \text{V}$

or $V_b - V_a = - I_1 R_1 - E_3 = -0.05 \times 100 - 4$

$= -9 \, \text{V}$

or $V_b - V_a = - E_1 - I_2 R_2 = -6 - 3 = -9 \, \text{V}$
A capacitor with an initial potential difference of 100 V is discharged through a resistor when a switch between them is closed at $t = 0$. At $t = 10.0 \text{ s}$, the potential difference across the capacitor is 1.00 V. (a) What is the time constant of the circuit? (b) What is the potential difference across the capacitor at $t = 17.0 \text{ s}$?

\[ V_c(t) = \frac{q(t)}{C} = \frac{q_0}{C} e^{-\frac{t}{\tau_{RC}}} = V_0 e^{-\frac{t}{\tau_{RC}}} \]

\[ 1 = 100 e^{-\frac{10}{\tau}} \]

\[ -\frac{10}{\tau} = \ln \left( \frac{1}{100} \right) = -\ln 100 \]

\[ \tau = \frac{10}{\ln 100} = 2.17 \text{ s.} \]

\[ b) \quad V_c = 100 e^{-\frac{17}{2.17}} = 0.04 \text{ V} \]