A sinusoidal wave traveling to the left is shown in the figure. If the speed of this wave is 20 m/s, find the following:

(a) The wavelength of this wave

From the figure
\[ \lambda = 6 \text{ cm} \]

(b) The maximum displacement

From the figure
\[ y_m = 1 \text{ cm} \]

(c) The angular frequency of this wave

\[
\begin{align*}
\nu &= \frac{\omega}{k} = \frac{\omega}{\frac{2\pi}{\lambda}} = \frac{\lambda \omega}{2\pi} \\
&= \frac{2\pi \nu}{\lambda} = \frac{2\pi \times 20}{0.06} \\
&= 666.7 \pi \text{ rad/s}
\end{align*}
\]

(d) The phase constant

From the figure
\[ y(0,0) = 1 \text{ cm} \]

and in general:
\[ y(x,t) = y_m \sin(2\pi x + \omega t + \phi) \]

\[ y(0,0) = y_m \sin(\phi) = (1 \text{ cm}) \sin \phi = 1 \text{ cm} \]

\[ \Rightarrow \sin \phi = 1 \Rightarrow \phi = \frac{\pi}{2} \]

(e) Write the expression for the displacement as a function of position and time

\[
\begin{align*}
\kappa &= \frac{2\pi}{\lambda} = \frac{2\pi}{0.06} = 33.3 \pi \text{ rad/m} \\
\Rightarrow \quad y_f(x,t) &= (0.01 \text{ m}) \sin \left( 33.3 \pi x + 666.7 \pi t + \frac{\pi}{2} \right)
\end{align*}
\]
The expression of a traveling wave is given by \( y(x,t) = (0.2\, m) \cos \pi (2x - 50t + \frac{1}{4}) \), where \( x \) is in meters and \( t \) in seconds.

Find the following including the correct units:

(a) The wavelength of this wave

\[
\lambda = \frac{2\pi}{k} = \frac{2\pi}{2\pi} = 1\, \text{m}
\]

(b) The speed of the wave

\[
\nu = \frac{\omega}{k} = \frac{50\pi}{2\pi} = 25\, \text{m/s}
\]

(c) The maximum transverse speed

\[
u_{\text{max}} = \omega |y_m| = 50\pi \times 0.2 = 10\pi\, \text{rad/s}
\]

(d) The displacement of the particle at \( x = 0.2\, m \) and \( t = 1\, s \).

\[
y(0.2, 1) = 0.2 \cos \pi (2 \times 0.2 - 50 \times 1 + \frac{1}{4})
\]

\[
= -0.09\, \text{m}
\]
Physics 102.15
Quiz#1
Chapters 16

Instructor: Dr. A. Mekki

Name: ___________________________  Key: ______  Id: ______  Sect: ______

1. A stretched string of mass 30 g and length 1.0 m vibrates in its second harmonic when driven by an oscillator of frequency \( f = 120 \) Hz. What is the tension in the string?

\[
\begin{align*}
\frac{f_2}{2} &= \frac{2v}{2L} = 120 \implies v = 120 \times L = 120 \times 1 = 120 \text{ m/s} \\
\nu &= \sqrt{\frac{T}{\mu}} \implies T = \nu^2 \mu \\
\mu &= \frac{m}{L} = \frac{30 \times 10^{-3}}{1} = 30 \times 10^{-3} \text{ kg/m} \\
\implies T &= (120)^2 \times 30 \times 10^{-3} = 143.2 \text{ N}
\end{align*}
\]

2. The resultant wave of two equal waves moving in the same direction is given by:

\[ y(x,t) = (0.5m)\sin[\pi(2x - 200t + \frac{1}{6})] \]

Write the displacement wave for the two original waves.

\[
\text{In general: } y = 2y_m \cos \frac{\phi}{2} \sin \left( kx - \omega t + \frac{\phi}{2} \right) \\
\implies \frac{\phi}{2} = \frac{\pi}{6} \implies \phi = \frac{\pi}{3} \text{ rad} \\
2y_m \cos \frac{\phi}{2} = 0.5 \implies 2y_m \cos \frac{\pi}{6} \implies y_m = \frac{0.5}{1.73} \\
\implies y_m = 0.29 \text{ m} \\
\implies y_1 = (0.29m) \sin \left[ \pi(2x - 200t) \right] \\
y_2 = (0.29m) \sin \left[ \pi(2x - 200t + \frac{\pi}{3}) \right]
\]