NOTE: Give the solution of any SIX questions.

1. Show that the real line $\mathbb{R}$ is not compact. (7 Marks)
2. Prove that a subspace $Y$ of a complete normed space $X$ is complete if and only if $Y$ is closed in $X$. (6 Marks)
3. If $X$ is a finite dimensional normed space, then prove that every linear operator defined on $X$ is bounded. (6 Marks)
4. Show that the continuity and boundedness are equivalent for a linear operator $T$ defined on a normed space $X$ into another normed space $Y$. (7 Marks)
5. State uniform boundedness principle and show that it is not valid if the space $X$ is only a normed space. (6 Marks)
6. If $T(x) = T(y)$ for every bounded linear functional $T$ defined on a normed space $X$, then prove that $x = y$. (6 Marks)
7. Give the definition of a closed operator $T$ defined on a normed space $X$ into another normed space $Y$. Also, give the characterization of closed operator. (7 Marks)
8. Is every closed operator continuous and vice-versa? Justify your answer. (7 Marks)

Solution.