1 1.3 Quadratic Equations

1. A quadratic equation in $x$ is an equation that can be written in the form $ax^2 + bx + c$, where $a \neq 0$.

2. If the quadratic polynomial in a quadratic equation is factorable over the set of integers, then the equation can be solved by factoring and using the zero product property.

3. If the quadratic equation can be written in the form $(ax + b)^2 = c$, then the equation can be solved by taking square property.

4. Every quadratic equation can be solved by completing the square or by using the quadratic formula.

5. The quadratic formula, for the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

6. The quadratic equation $ax^2 + bx + c = 0$, with real coefficients and $a \neq 0$, has discriminant $b^2 - 4ac$.
   
   (a) If $b^2 - 4ac > 0$, then the quadratic equation has two distinct real roots.
   
   (b) If $b^2 - 4ac = 0$, then the quadratic equation has a real root that is a double root.
   
   (c) If $b^2 - 4ac < 0$, then the quadratic equation has two distinct complex roots that are not real. The roots are conjugates of each other.

2 1.4 Other Types of Equations

1. If the polynomial in an equation is factorable over the set of integers, then the equation can be solved by factoring and using the zero product property.

2. If the equation involves radical expression, then we can use the Power Principle.

3. The power principle states that, if $P$ and $Q$ are algebraic expressions and $n$ is a positive integer, then every solution of $P = Q$ is a solution of $P^n = Q^n$.

4. Any solution of $P^n = Q^n$ that is not a solution of $P = Q$ is called an extraneous solution. Extraneous solutions maybe introduced whenever we raise each side of an equation to an even power.

5. An equation is said to be quadratic in form if it can be written in the form $au^2 + bu + c = 0$, where $a \neq 0$ and $u$ is an algebraic expression.
3 1.5 Inequalities

1. The set of all solutions of an inequality is the solution set of the inequality.

2. Equivalent inequalities have the same solution set.

3. Adding the same real number to each side of an inequality preserves the direction of the inequality symbol.

4. Multiplying each side of an inequality by the same positive real number preserves the direction of the inequality symbol.

5. Multiplying each side of an inequality by the same negative real number changes the direction of the inequality symbol.

6. To solve an inequality, use the properties of an inequality or the critical value method.

7. A compound inequality is formed by joining two inequalities with the connective word and or or.

8. The solution set of a compound inequality with the connective word or is the union of the solution set of the two inequalities.

9. The solution set of a compound inequality with the connective word and is the intersection of the solution set of the two inequalities.

10. For any variable expression $E$ and any nonnegative real number $k$,

    (a) $|E| \leq k$ if and only if $-k \leq E \leq k$.

    (b) $|E| \geq k$ if and only if $E \leq -k$ or $E \geq k$

11. Nonzero polynomials in $x$ have the property that for any value of $x$ between two consecutive real zeros, either all values of the polynomial are positive or all values of the polynomial are negative.

12. The critical values of the polynomial inequality are the real zeros of the polynomial.

13. The critical values of a rational expression are the numbers that cause the numerator of the rational expression to equal zero or the denominator of the rational expression to equal zero.

4 2.1 Two-Dimensional Coordinate System and Graphs

1. The distance $d$ between the points represented by $(x_1, y_1)$ and $(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$
2. The **Midpoint** of the line segment from $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

3. If $(x_1, 0)$ satisfies an equation, then the point $(x_1, 0)$ is called an **x-intercept** of the graph of the equation.

4. If $(0, y_1)$ satisfies an equation, then the point $(0, y_1)$ is called an **y-intercept** of the graph of the equation.

5. The **standard form of the equation of a circle** with center at $(h, k)$ and radius $r$ is $(x-h)^2 + (y-k)^2 = r^2$.

### 5 2.2 Introduction to Functions

1. A **function** is a set of ordered pairs in which no two ordered pairs that have the same first coordinate have different second coordinate.

2. Unless otherwise stated, the **domain** of a function is the set of all real numbers for which the function makes sense and yields real numbers.

3. If $a$ and $b$ are elements of an interval $I$ that is a subset of the domain of a function $f$, then

   (a) $f$ is **increasing** on $I$ if $f(a) < f(b)$ whenever $a < b$.

   (b) $f$ is **decreasing** on $I$ if $f(a) > f(b)$ whenever $a < b$.

   (c) $f$ is **constant** on $I$ if $f(a) = f(b)$ whenever $a < b$.

4. A graph is the **graph of a function** if and only if no vertical line intersects the graph at more than one point.

5. If every horizontal line intersects the graph of a function at most once, then the graph is the graph of a **one-to-one function**.


### 6 2.3 Linear Functions

1. A function is a **linear function** of $x$ if it can be written in the form $f(x) = mx + b$, where $m$ and $b$ are real numbers and $m \neq 0$.

2. The **slope** $m$ of the line passing through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ with $x_1 \neq x_2$ is given by $m = \frac{y_2-y_1}{x_2-x_1}$.

3. The graph of the equation $f(x) = mx + b$ has slope $m$ and y-intercept $(0,b)$.

4. The graph of $y - y_1 = m(x - x_1)$ is a line that has slope $m$ and passes through $(x_1, y_1)$.
5. Let $l_1$ be the graph of $f_1(x) = m_1x + b$ and let $l_2$ be the graph of $f_2(x) = m_2x + b$. Then

(a) $l_1$ and $l_2$ are parallel if and only if $m_1 = m_2$.

(b) $l_1$ and $l_2$ are perpendicular if and only if $m_1 = -\frac{1}{m_2}$.

7 2.4 Quadratic Functions

1. A **quadratic function** of $x$ is a function that can be represented by an equation of the form $f(x) = ax^2 + bx + c$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$.

2. Every quadratic function $f(x) = ax^2 + bx + c$ can be written in the **standard form** $f(x) = a(x - h)^2 + k$, $a \neq 0$.

   (a) The graph of a parabola with **vertex** $(h, k)$.

   (b) The parabola is symmetric with respect to the vertical line $x = h$, which is called the **axis of symmetry**.

   (c) If $a > 0$, then

   (d) the parabola opens up

   (e) the vertex is the **lowest point** on the graph of the parabola

   (f) the $y$-coordinate $k$ of the vertex is the **minimum** value of the function

   (g) the **range** is $\{y|y \geq k\}$.

   (h) If $a > 0$, then

   (i) the parabola opens down

   (j) the vertex is the **highest point** on the graph of the parabola

   (k) the $y$-coordinate $k$ of the vertex is the **maximum** value of the function

   (l) the **range** is $\{y|y \leq k\}$.

3. The **vertex** of the graph of $f(x) = ax^2 + bx + c$ is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

8 2.5 Properties of Graphs

1. The graph of an equation is **symmetric** with respect to

   (a) the **y-axis** if the replacement of $x$ with $-x$ leaves the equation unaltered.

   (b) the **x-axis** if the replacement of $y$ with $-y$ leaves the equation unaltered.

   (c) the **origin** if the replacement of $x$ with $-x$ and $y$ with $-y$ leaves the equation unaltered.
2. The function $f$ is an **even function** if $f(-x) = f(x)$ for all $x$ in the domain of $f$.

3. The function $f$ is an **odd function** if $f(-x) = -f(x)$ for all $x$ in the domain of $f$.

4. If $f$ is a function and $c$ is a positive constant, then
   
   (a) $y = f(x) + c$ is the graph of $y = f(x)$ shifted **up vertically** $c$ units.
   (b) $y = f(x) - c$ is the graph of $y = f(x)$ shifted **down vertically** $c$ units.
   (c) $y = f(x + c)$ is the graph of $y = f(x)$ shifted **left horizontally** $c$ units.
   (d) $y = f(x - c)$ is the graph of $y = f(x)$ shifted **right horizontally** $c$ units.

5. The graph of
   
   (a) $y = -f(x)$ is the graph of $y = f(x)$ **reflected across the x-axis**.
   (b) $y = f(-x)$ is the graph of $y = f(x)$ **reflected across the y-axis**.

6. (a) If $0 < c < 1$, then the graph of $y = c \cdot f(x)$ is obtained by **shrinking** the graph of $y = f(x)$ **vertically**.
   (b) If $c > 1$, then the graph of $y = c \cdot f(x)$ is obtained by **stretching** the graph of $y = f(x)$ **vertically**.

7. (a) If $a > 1$, then the graph of $y = f(ax)$ is a **horizontal shrinking** of the graph of $y = f(x)$.
   (b) If $0 < a < 1$, then the graph of $y = f(ax)$ is a **horizontal stretching** of the graph of $y = f(x)$.