1 Section 2.5 Properties of Graphs

Symmetry
The graph of an equation is symmetric with respect to
The y-axis if the replacement of x with -x leaves the equation unaltered.
The x-axis if the replacement of y with -y leaves the equation unaltered.

Example 1 Determine whether the graph of the given equations has symmetry
with respect to either the x- or the y-axis. 1) \(x + y^2 = 4\) 2) \(|x| + y = 4\)

A graph is symmetric with respect to a point \(Q\) if for each point \(P\) on the
graph, there is a point \(P'\) on the graph such that \(Q\) is the midpoint of the line
segment \(PP'\).

The graph of an equation is symmetric with respect to the origin if the
replacement of x with -x and of y with -y leaves the equation unaltered.

Example 2 Determine whether the graph of each equation has symmetry with
respect to the origin? 1) \(xy = 4\) 2) \(y = x^3 + 1\) 3) \(|x| + |y| = 2\)

Example 3 Plot the image of the point \(Q(-4,1)\) with respect to the 1)y-axis
2)x-axis 3)origin.

Even and Odd Functions
The function \(f\) is an even function if \(f(-x) = f(x)\) for all \(x\) in the domain.
The function \(f\) is an odd function if \(f(-x) = -f(x)\) for all \(x\) in the domain.

Example 4 Determine whether each function is even, odd or neither. 1) \(f(x) =
4x^5 + 5x\) 2) \(f(x) = |3x|\) 3) \(f(x) = x^3 + 2x^2\)

Note that the graph of an even function is symmetric with respect to the
y-axis and the graph of an odd function is symmetric with respect to the origin.

Translations of Graphs Vertical Translations
If \(f\) is a function and \(c\) is a positive constant, then the graph of \(y = f(x) + c\)
is the graph of \(y = f(x)\) shifted up vertically \(c\) units.
The graph of \(y = f(x) - c\) is the graph of \(y = f(x)\) shifted down \(c\) units.

Horizontal Translations
If \(f\) is a function and \(c\) is a positive constant, then the graph of \(y = f(x + c)\)
is the graph of \(y = f(x)\) shifted left horizontally \(c\) units.
The graph of \(y = f(x - c)\) is the graph of \(y = f(x)\) shifted right \(c\) units.

Example 5 Graph each of the following using vertical and horizontal transla-
tions of the graph of \(f(x) = x^2\).

1) \(g(x) = x^2 + 1\) 2) \(h(x) = (x - 2)^2\) 3) \(p(x) = (x - 2)^2 + 1\)

Reflections of graphs
The graph of \(y = -f(x)\) is the graph of \(y = f(x)\) reflected across the x-axis.
The graph of \(y = f(-x)\) is the graph of \(y = f(x)\) reflected across the y-axis.
Example 6 Use reflections of the graph of \( f(x) = |x-2| + 3 \) to graph 1) \( g(x) = -(|x-2| + 3) \) 2) \( h(x) = |-x+2| + 3 \)

Shrinking and stretching of graphs

Vertical shrinking and stretching
If \( 0 < c < 1 \), then the graph of \( y = cf(x) \) is obtained by shrinking \( y = f(x) \). If \( c > 1 \), then the graph of \( y = cf(x) \) is obtained by stretching \( y = f(x) \).

Example 7 Graph 1) \( H(x) = \frac{1}{2}|x| + 2 \) 2) \( G(x) = 3|x| + 2 \)

Horizontal shrinking and stretching
If \( a > 0 \) and the graph of \( y = f(x) \) contains the point \((x, y)\), then the graph of \( y = f(ax) \) contains the point \((\frac{1}{a}x, y)\).
If \( a > 1 \), then the graph of \( y = f(ax) \) is a horizontal shrinking of \( y = f(x) \).
If \( 0 < a < 1 \), then the graph of \( y = f(ax) \) is a horizontal stretching of \( y = f(x) \).

Example 8 Use the graph of \( y = f(x) \) to graph 1) \( y = f(2x) \) 2) \( y = f\left(\frac{1}{3}x\right) \)

Exercise 9 Determine whether the graph of each equation has a symmetry with respect to the x-axis, the y-axis or the origin.

1) \( |xy| + |x|y = 1 \) 2) \( |y| = \frac{|x+2|}{x^2} \) 3) \( |x| = |x-y| \) 4) \( y^2 = |x+1| - 3x^2 \) 5) \( (xy)^2 - 2xy = 3 \) 6) \( x^2 = |x-y^3| \) 7) \( y = \frac{x^2-3}{2x} \)

Exercise 10 Determine whether each function is even, odd or neither.

1) \( f(x) = x|x| \) 2) \( f(x) = 5 \) 3) \( g(x) = \frac{1}{2}[f(x) + f(-x)] \) for any nonzero function \( f \). 4) \( f(x) = \sqrt{3-x^2} \) 5) \( g(x) = \frac{x^2}{x^2 + 1} \) 6) \( h(x) = 2 + x + x^2 \)

Exercise 11 Which translations should be applied to obtain the graph of the equation \( y = |x+3| - 2 \) from the graph of \( y = |x-1| + 3 \).

Exercise 12 Which translations should be applied to obtain the graph of the equation \( y^2 - 2y - 4x - 7 = 0 \) from the graph of \( y^2 = 4x \).

Exercise 13 What is the resulting function of shifting the graph of the equation \( x = y^2 + y \) one unit to the left and two units upward.

Exercise 14 What is the resulting function of shifting the graph of the equation \( y = \frac{x+2}{x+1} \) one unit to the right and three units downward.

Exercise 15 How can we obtain the graph of the equation \( g(x) = \sqrt{-x-1} + 2 \) from the graph of \( f(x) = \sqrt{x} \).

Exercise 16 The graph of a function \( y = f(x) \) is shown in the adjacent figure. Find the Domain and the Range of the function \( y = -f(x+1) + 2 \).