1 Section 1.1 Linear Equations

An equation is a statement about the equality of two expressions. For example, $2x + 1 = 7$ is an equation and it is true for $x = 3$ but false for any number except 3. The number 3 is said to satisfy the equation $2x + 1 = 7$.

Solve an equation means find all values of the variable that satisfy the equation. Solutions or roots of the equation are the values that satisfy the equation. Equivalent equations are equations that have exactly the same solution(s).

Procedure that Produce Equivalent Equations
1) Simplification of an expression on either side

Example 1 $2x + 3 + 5x = −11$ and $7x + 3 = −11$ are equivalent equations

2) Addition or subtraction of the same quantity on both sides.

Example 2 $3x − 7 = 2$ and $3x = 9$ are equivalent equations.

3) Multiplication or division by the same nonzero quantity on both sides.

Example 3 $\frac{5}{6}x = 10$ and $x = 12$ are equivalent equations.

Linear Equation in $x$ is an equation that can be written in the form $ax + b = 0$ where $a$ and $b$ are real numbers and $a \neq 0$.

Example 4 Solve the following equations: 1) $\frac{3}{5}x − 16 = −1$ 2) $\frac{3}{4}x + 2 − \frac{x}{2} = −3$ 3) $(x − 2) (2x + 3) = 2x (x − 1)$

Contradictions, Conditional Equations and Identities
An equation that has no solutions is called a contradiction.

Example 5 $x = x + 1$ is a contradiction.

An equation that is true for some values but not for other values is called a conditional equation.

Example 6 $x + 2 = 8$ is a conditional equation.

An equation that is true for every real number for which all terms of the equation are defined is called an identity.

Example 7 $x + x = 2x$ is an identity.

Example 8 Which one of the following statements is TRUE?

1) $(x + 4)^2 = x^2 + 16$ is an identity. 2) $x − 3 = 0$ and $x^2 = 9$ are equivalent equations. 3) $x (5 + x) = x^2 + 5 (x + 1)$ is a contradiction. 4) $\frac{5x + 1}{3} = 2x + \frac{1}{3}$ is a conditional equation. 5) $(x − 3) (x + 4) = x^2 − x − 12$ is an identity.
Example 9  Verify the identity \( \frac{3(x^3-8)}{x-2} = 3x^2 + 6x + 12, \ x \neq 2 \).

Solve Equations that Have Restrictions

If each side of an equation is multiplied by an expression that involves a variable, then we restrict the variable so that the expression is not equal to zero.

Example 10  Find the solution set of the following equations:
1) \( \frac{x}{x-3} = \frac{9}{x-3} - 5 \)
2) \( 1 + \frac{x}{x-3} = \frac{5}{x-3} \)
3) \( \frac{8}{2m+1} - \frac{1}{m-2} = \frac{5}{2m+1} \)

Absolute Value Equations: For any variable expression \( E \) and any non-negative real number \( k \), \( |E| = k \) if and only if \( E = k \) or \( E = -k \).

Example 11  Find the solution set of the following equations:
1) \( |2x - 5| = 21 \)
2) \( |2x + 5| = 0 \)
3) \( |2x + 5| = -8 \)
4) \( |x - 3| = -(x - 3) \)
5) \( 2|x + 3| + 4 = 34 \)

2  Section 1.2 Formulas

Solve a formula for a specified variable in terms of the other variable(s).

Example 12  Solve \( 2l + 2w = p \) for \( l \).

Example 13  Solve \( xy - z = yz \) for \( y \).

Example 14  Solve \( z = y \left(1 + \frac{m}{x}\right) \) for \( x \).