1. Solve the following boundary-value problems using a finite difference method

2. \( y'' + 9y = 0, \quad y(0) = 4, \quad y(2) = 1; \quad h = 1/8 \)

3. \( y'' - y = x^2, \quad y(0) = 0, \quad y(1) = 0; \quad h = 1/4 \)

4. \( y'' + 2y' + y = 5x, \quad y(0) = 0, \quad y(1) = 0; \quad h = 1/5. \)

5. \( y'' - 10y' + 25y = 1, \quad y(0) = 1, \quad y(1) = 0; \quad h = 1/5 \)

6. \( y'' - 4y' + 4y = (x + 1)e^{2x}, \quad y(0) = 3, \quad y(1) = 0; \quad h = 1/5 \)

7. \( y'' + 5y' = 4\sqrt{x}, \quad y(1) = 1, \quad y(2) = -1; \quad h = 1/5 \)

8. \( x^2y'' + 3xy' + 3y = 0, \quad y(1) = 5, \quad y(2) = 0; \quad h = 1/8 \)

9. \( x^2y'' - xy' + y = \ln x, \quad y(1) = 0, \quad y(2) = -2; \quad h = 1/10. \)

10. \( y'' + (1 - x)y' + xy = x, \quad y(0) = 0, \quad y(1) = 2; \quad h = 1/10 \)

11. \( y'' + xy' + y = x, \quad y(0) = 1, \quad y(1) = 0; \quad h = 1/10 \)

12. \( y'' + 6.55(1 + x)y = 1, \quad y(0) = 0, \quad y(1) = 0; \quad h = 1/10. \)

13. The electrostatic potential \( u \) between two concentric spheres of radius \( r = 1 \) and \( r = 4 \) is determined from

\[
\frac{d^2u}{dr^2} + \frac{2}{r} \frac{du}{dr} = 0, \quad u(1) = 50, \quad u(4) = 100.
\]

Use the finite difference method described in section 10.1 with \( n = 12 \) to approximate the solution of the BVP.

14. Consider the BVP \( y'' + xy = 0, \ y(0), \ y(1) = -1 \). Find the difference equation corresponding to the differential equation.

Solve the following boundary-value problems using the shooting method

15. \( y'' = y' - \sin(xy), \quad y(0) = 1, \quad y(1) = 1.5; \quad h = 1/10 \)

16. \( y'' = \frac{1}{2}y - \frac{2(y')^2}{y}, \quad y(0) = 1, \quad y(1) = 1.5; \quad h = 1/10 \)

17. \( y'' = yy' + e^x, \quad y(0) = 1, \quad y(1) = -1; \quad h = 1/5 \)

18. \( y'' + xy' + y = x, \quad y(0) = 1, \quad y(1) = 0; \quad h = 1/10 \)

19. \( y'' = 2 - \frac{4u^2}{\sin^2 x}, \quad y(1) = 0.70807, \quad y(2) = 0.82682; \quad h = 1/10 \)

20. \( y'' = 2y^3, \quad y(2) = \frac{1}{5}, \quad y(4) = \frac{1}{7}; \quad h = 1/10 \)

21. \( y'' = -(y')^2 - y + \ln x, \quad y(2) = \ln 2, \quad y(4) = 2 \ln 2; \quad h = 1/10 \)

**Computer Assignments**
1. Use the MATLAB function `finitediff.m` to approximate the solution of Exercise 6.

2. Use the MATLAB function `lshoot.m` to approximate the solution of Exercise 8.

3. Use the MATLAB function `lshoot.m` to approximate the solution of the BVP

\[ x^2 y'' = -4xy' + 2y - 2 \ln x, \quad y(1) = -\frac{1}{2}, \quad y(2) = \ln 2, \quad 1 \leq x \leq 2 \]

with \( h = 0.01 \)