Q1. (10 Points - Suggested time: 5 minutes) State if each of the following statements is true or false:

1. Any Cauchy-Euler equation can be transformed with one with constant coefficients.

2. $y_1 = |x|$ and $y_2 = x$ are fundamental solutions for some homogenous 2nd-order linear ODE.

3. The general form for the particular solution of $y'' - y = \sin(x)$ is $y_p = A\sin(x)$.

4. Any linear combination of particular solutions of a linear ODE is also a solution of that equation.

5. The substitution $u = e^x$ turns any linear ODE with constant coefficients into a Cauchy-Euler equation.

Q2. (10 Points - Suggested time: 10 minutes) Find the general solution of the following homogenous ODE, given that $y_1 = 3$ is a solution

$$xy'' + y' = 0$$
Q3. (20 Points - Suggested time: 15 minutes) Solve the following IVP:

\[ y^{'''} - 3y' + 2y = 0; \]
\[ y(0) = 0; \quad y'(0) = 1; \quad y''(0) = 2 \]
Q4. (20 Points - Suggested time: 20 minutes) Solve the following ODE (showing all details).

\[ 2x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = x \]
Q5. (20 Points - Suggested time: 20 minutes) Consider the following ODE:

\[ y''' - 3y'' + y' - 3y = e^{3x} + \cos x. \]

Showing all details:

1. Find the complementary solution.

\[ y_c = \]

2. Find the general form for the particular solution

\[ y_p = \]
Q6. (20 Points - Suggested time: 20 minutes)

A tank containing 200 liters of fluid in which 30 grams of salt is dissolved. Brine with concentration 1 gram/litre is then pumped into the tank at a rate of 4 litre/minute; the well-mixed solution is then pumped out at the same rate. Find the number of grams of salt in the tank after 20 minutes.