Chapter 3: Kinematics in Two Dimensions; Vectors

3.1 Vectors and Scalars
3.2 Addition of Vectors—Graphical Methods
3.3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar
3.4 Adding Vectors by Components
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Addition of vectors

Two displacements
\[ \vec{D}_1 = 8 \text{ km east} \]
\[ \vec{D}_2 = 6 \text{ km north} \]

- Addition
  - graphically: tail-to-head (tail to tip)
  - generalize to any 2, 3 etc vectors.

- Subtraction

- Multiplication

Analytical methods

1) Resolving vector into components

\[ \vec{V} = \vec{V}_x + \vec{V}_y \]
\[ \vec{V} = 5.0 \text{ m} \quad @ \quad 36.87^\circ \]

there are mutually perpendicular

\[ V = \text{ the length of } \vec{V} \]
\[ V_x = \text{ the length of } \vec{V}_x \]
\[ V_y = -\text{ of } \vec{V}_y \]

\[ V_x = V \cos \theta = 5 \cdot \cos (36.87^\circ) = 4.0 \text{ m} \]
\[ V_y = V \sin \theta = 5 \cdot \sin (36.87^\circ) = 3.0 \text{ m} \]

2) Add two vectors

\[ \vec{V}_1 = V_{1x} + V_{1y} \]
\[ \vec{V}_2 = V_{2x} + V_{2y} \]

EXAMPLES: see projectile motion
Three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are as shown. Which vector is $\vec{S} = \vec{A} + \vec{B} - \vec{C}$?

Purple: None of these!
The position vector of a particle moving with constant velocity is shown below at two different times, an earlier time $t_1$ and a later time $t_2$. Which arrow shows the direction of the velocity vector?

Note: The grid is for positions, not speeds!

Purple: None of these!
The velocity vector of a particle moving with constant acceleration is shown below at two different times, an earlier time $t_1$ and a later time $t_2$. What is the direction of the acceleration vector?

![Diagram of velocity vectors $v_1$ and $v_2$]

- Pink
- Blue
- Green
- Yellow

Purple: None of these!
Both hit the ground at the same time.

Galileo pointed out that one can consider horizontal and vertical motion separately.

\[ v_y = v_{0y} + a_y t \]
\[ y = v_{0y} t + \frac{1}{2} a_y t^2 \]
and
\[ a_y = -9.81 \text{ m/s}^2 \]

**Examples**

**Problem 2/2**

Bird 50 m tall, hands 40 m away.

1) Find initial velocity \( v_0 \).

\[ x = 0 \]
\[ v_{0x} = v_0 \rightarrow x = v_0 t \]
\[ y = 50 \text{ m} \]
\[ v_{0y} = 0 \]
\[ a_y = -9.81 \text{ m/s}^2 \]
\[ t = \frac{50 - 4.9 t^2}{4.9} = 10.2 \]
\[ t = 3.15 \text{ s} \]

Horizontal: \[ 45 = v_x \cdot (3.15) \rightarrow 14.1 \text{ m/s} \]

2) Find final speed.

\[ v_y = u_{0y} - gt \]
\[ v_y = -9.81(3.15) = -31.5 \text{ m/s} \]
\[ v = \sqrt{v_x^2 + v_y^2} = 24.3 \text{ m/s} \]
A barrel of a rifle points straight at a monkey hanging from a branch of a tree. The instant the gun is fired, the monkey releases a grip on the branch and starts falling. The initial speed of the bullet is $v_o$, and the monkey is well within the range of the rifle for this value of $v_o$. What happens?

a) Bullet finds its target.
b) Bullet hits the monkey only if $v_o$ is large enough.
c) Bullet misses.
A particle is moving along the path shown, with constant speed. Its velocity vector at two different times is shown. What is the direction of the acceleration when the particle is at point A?

Purple: the acceleration is zero.
Kicking a football

You kick a football, giving it initial velocity of 20 m/s in the direction of 37° above the ground.
How far does it land? How high does it go?

Let's resolve the initial velocity into components:

\[
\begin{align*}
\text{Horizontal:} & \quad v_{x0} = 20 \text{ m/s} \cdot \cos 37° = 16 \text{ m/s} \\
\text{Vertical:} & \quad v_{y0} = 20 \text{ m/s} \cdot \sin 37° = 12 \text{ m/s}
\end{align*}
\]

Now, horizontal velocity does not change. I show it on the graph.
The vertical velocity at the top is 0, and upon landing is -12 m/s, so the speed upon landing is 20 m/s.

- At the top, \( v_y = 0 \). But \( v_y = 12 \text{ m/s} - 9.8 \text{ m/s}^2 \cdot t_{up} \)
  so \( t_{up} = \frac{12 \text{ m/s}}{9.8 \text{ m/s}^2} = 1.224 \text{ s} \)

- Time of flight is \( 2 \times t_{up} = 2 \times 1.224 \text{ s} = 2.448 \text{ s} \)
- Horizontal distance is \( L = (16 \frac{\text{m}}{3})(2.448 \text{ s}) = 39 \text{ m} \)
- How high?
  a) easy way: Average vertical velocity while rising is \( \frac{12 \text{ m/s} + 0}{2} = 6 \text{ m/s} \)
  so \( H = (6 \text{ m/s})(1.224 \text{ s}) = 7.3 \text{ m} \)
  b) rigorous way: \( y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2 \)
  so \( H = 0 + (12 \text{ m/s})(1.224 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2)(1.224 \text{ s})^2 = 7.3 \text{ m} \)

Note: You will achieve same horizontal distance kicking the football in such a way that it starts at 20 m/s at 53° above horizontal of course, it will go higher, and the time of flight will be longer, but \( L \) will be same as before (Do calculations yourself!)
Two projectiles are simultaneously fired from two different cannons with the same initial speed, and both hit the same target, as shown. For projectile B, the cannon is tilted upward at the angle of 25 degrees above the horizontal.

**Question 1:** At what angle was the cannon tilted for the projectile A?

a) 55 degrees  
b) 60 degrees  
c) 70 degrees  
d) 75 degrees  
e) None of the above

**Question 2:** Which projectile hits the target first?

a) A  
b) B  
c) Both hit at the same time  
d) Not enough information given.
Two projectiles are fired one after another from a cannon. For projectile A, the cannon is tilted upward at an angle larger that of projectile B. (As usual, neglect air resistance.)

Which projectile was in the air longer?

a) A  
b) B  
c) A and B were in the air the same length of time.  
d) Not enough information to answer the question.