Chapter 2: Describing Motion: Kinematics in One Dimension

2-1 Reference Frame and Displacement
2-2 Average Velocity
2-3 Instantaneous Velocity
2-4 Acceleration
2-5 Motion at Constant Acceleration
2-6 Solving Problems
2-7 Falling Objects

EX.1
Consider a walk \( A \rightarrow C \rightarrow B \)

\[ t_1 = 0, \quad t_2 = 4.0 \text{ h}, \quad 3.0 \text{ milo}, \quad 10.0 \text{ milo} \]

- Distance travelled = length of the path = 7.0 milo + 3.0 milo = 10.0 milo
- Duration of the trip = 4.0 hours

Average speed \( \bar{v} \) = \( \frac{\text{Distance}}{\text{Time}} \) = \( \frac{10.0 \text{ miles}}{4.0 \text{ hours}} = 2.5 \text{ miles/hour} = 2.5 \text{ mph} \)

Also, Distance = (Average speed) x Time = \( 2.5 \text{ miles/hour} \times 4.0 \text{ hours} = 10.0 \text{ miles} \)

- Displacement - we need a reference frame. Let's choose a reference frame by drawing a x-axis and selecting an origin (this is up to you to choose).

In this reference frame, \( x_A = 2.0 \text{ miles}, \quad x_B = 6.0 \text{ miles} \)

- Displacement: (Net change of position) \( \Delta x \equiv x_{\text{final}} - x_{\text{initial}} \)
  Here \( \Delta x = 6.0 \text{ miles} - 2.0 \text{ miles} = 4.0 \text{ miles} \)
  \( \uparrow \) towards the "+" end of the axis

Average velocity \( \bar{v}_x \) = \( \frac{\text{Displacement}}{\text{Time}} = \frac{\Delta x}{\Delta t} \)

Here \( \bar{v}_x = \frac{4.0 \text{ miles}}{4.0 \text{ h}} = 1.0 \text{ miles/hour} \)
  \( \uparrow \) means toward the "+" end of the x-axis

Note: \( \Delta x = \bar{v}_x \cdot \Delta t \)

If you run at 8.0 m/s for 11 s, then \( \Delta x = (8.0 \text{ m/s})(11 \text{ s}) = 88 \text{ m} \)
A person starts in A, drives to B, 50 km away, in 1 hour, stays in B for 1 hour, then speeds back to A in 30 minutes.

Q1. What is the average speed of this round trip?

a) zero  
b) 40 km/hr  
c) 67 km/hr  
d) 75 km/hr  
e) None of these.

Q2. What is the average velocity of this round trip?

a) zero  
b) 40 km/hr  
c) 67 km/hr  
d) 75 km/hr  
e) None of these.
A motorist wishes to travel 40 kilometers at an average speed of 40 km/h. During the first 20 kilometers he maintains an average speed of 40 km/h. No problem here. During the next 10 kilometers, however, the motorist goofs off and averages only 20 km/h.

With what speed must the motorist drive the last 10 km in order to achieve the average 40 km/h for the whole trip?

60 km/h  
80 km/h  
90 km/h  
Faster than the speed of light  
None of the above
Since $v_x = \frac{\Delta x}{\Delta t}$

Then $\Delta x = v_x \cdot \Delta t$

**Example 2**

A jogger runs her first 100 m at 5 m/s and the second 100 m at 4 m/s. Find $\overline{v}_x$.

$$\overline{v}_x = \frac{\text{Displacement}}{\text{time}}$$

Displacement $= 100 \text{ m} + 100 \text{ m} = 200 \text{ m}$

Time $= \Delta t_1 + \Delta t_2 = 20 \text{ s} + 25 \text{ s} = 45 \text{ s}$

$$\overline{v}_x = \frac{200 \text{ m}}{45 \text{ s}} = 4.44 \text{ m/s}$$

**Instantaneous Velocity**

What the speedometer shows

A car starts from rest, speeds up to 50 km/h, keeps going for a while, slows down to 20 km/h and finally gets to a final destination and stops. It travelled 15 km in 30 min.

$$\overline{v} = \frac{15 \text{ km}}{0.5 \text{ h}} = 30 \text{ km/h}$$

$$v(t) \equiv \begin{cases} \text{Initially at the time } t \text{ the object is at } x \\
\text{after a short period of time } \Delta t \text{ the object is now at } x + \Delta x 
\end{cases}$$

$$v(t) = \lim_{\Delta t \to 0} \frac{(x + \Delta x) - x}{\Delta t} = \frac{\Delta x}{\Delta t}$$

Measuring $\Delta t$ takes $\Delta t$.

$$\Delta t = 1 \text{ s}$$

$$\Delta t = 2 \text{ s}$$

$$\Delta t = \frac{1}{10} \text{ s}$$
\[ a = \frac{u_f - u_i}{t_f - t_i} = \frac{\Delta u}{\Delta t} \]

\[ a = \lim_{\Delta t \to 0} \frac{\Delta u}{\Delta t} \quad \text{instantaneous} \]

**Ex. 3**

A car accelerates along a straight road from rest to 60 km/h in 5 s. Find average acceleration.

\[ a = \frac{60 \text{ km/h} - 0}{5 \text{ s}} = 12 \text{ km/h/s} \]

12 km/h/s is constant, then

\[ 60 \quad \text{(km/h)} \]

\[ 48 \quad \text{km/h} \]

\[ 36 \quad \text{km/h} \]

\[ 24 \quad \text{km/h} \]

\[ 12 \quad \text{km/h} \]

\[ \begin{array}{c}
0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\
\end{array} \]

Note:

\[ 60 \text{ km} = 60 \times \frac{1000 \text{ m}}{3600 \text{ s}} = 16.7 \text{ m/s} \]

\[ a = \frac{16.7 \text{ m/s} - 0}{5 \text{ s}} = 3.33 \text{ m/s}^2 = 3.33 \text{ m/s} \times 5 = 3.33 \text{ m/s}^2 \]

**Ex. 4**

You are going 15.0 m/s, then you slam on brakes and in 5.0 s your velocity drops to 5.0 m/s. What is your avg. a?

\[ a = \frac{5.0 \text{ m/s} - 15.0 \text{ m/s}}{5 \text{ s}} = -2.0 \text{ m/s}^2 = -2.0 \text{ m/s}^2 \]

\[ \begin{array}{c}
15 \quad \text{m/s} \\
10 \quad \text{m/s} \\
5 \quad \text{m/s} \\
\end{array} \]

\[ \begin{array}{c}
0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\
\end{array} \]

Run = 2 - 1 = 1

\[ \text{Rise} = 11 - 13 = -2 \quad (\text{drop}) \]

Slope = \[ \frac{\text{Rise}}{\text{Run}} = \frac{-2}{1} = -2 \text{ m/s} \]


Name:  
ID#:  

\[
\begin{align*}
\text{t } = & \quad 0 \text{ s} \\
\bullet & \quad 4 \text{ m/s} \\
\text{t } = & \quad 1 \text{ s} \\
\bullet & \quad 7 \text{ m/s} \\
\end{align*}
\]
\[a = \text{ m/s}^2\]

\[
\begin{align*}
\text{t } = & \quad 0 \text{ s} \\
\bullet & \quad 4 \text{ m/s} \\
\text{t } = & \quad 1 \text{ s} \\
\bullet & \quad 2 \text{ m/s} \\
\end{align*}
\]
\[a = \text{ m/s}^2\]

\[
\begin{align*}
\text{t } = & \quad 1 \text{ s} \\
\bullet & \quad 5 \text{ m/s} \\
\text{t } = & \quad 0 \text{ s} \\
\bullet & \quad 8 \text{ m/s} \\
\end{align*}
\]
\[a = \text{ m/s}^2\]

\[
\begin{align*}
\text{t } = & \quad 1 \text{ s} \\
\bullet & \quad 10 \text{ m/s} \\
\text{t } = & \quad 0 \text{ s} \\
\bullet & \quad 8 \text{ m/s} \\
\end{align*}
\]
\[a = \text{ m/s}^2\]
In general:

Object at \( t=0 \) is at \( x_0 \), has a velocity \( v_0 \), and accelerates at a rate \( a \).

\( \Delta x = \frac{v_0 + v(t)}{2} \cdot t \)

So we obtain:

\( x(t) = x_0 + v_0 t + \frac{1}{2} at^2 \)

Note: calculate \( t \) from first equation: \( t = \frac{v - v_0}{a} \)

and plug it into second equation

\( x = x_0 + v_0 \frac{v - v_0}{a} + \frac{1}{2} a \left( \frac{v - v_0}{a} \right)^2 = x_0 + \frac{v^2 - v_0^2}{2a} \)

Hence

\( v^2 = v_0^2 + 2a (x-x_0) \) < this equation does not involve time.
Consider the following trip:

![Graph showing velocity over time.]

Q1. The total distance traveled by the object was…
   
   a) 50 m  
   b) 100 m  
   c) 125 m  
   d) 150 m  
   e) 200 m

Q2. And the average velocity of the object during the entire trip was…

   a) 5.00 m/s  
   b) 6.25 m/s  
   c) 6.67 m/s  
   d) 7.50 m/s  
   e) none of the above
Examples:

1. How long does it take a car to travel 60.0 m if it starts from rest and accelerates at a rate $a = 2.00 \text{ m/s}^2$?

\[
\begin{array}{c}
\text{t=0} \\
0 \quad \Rightarrow \ a = 2.00 \text{ m/s}^2 \quad \text{50.0 m}
\end{array}
\]

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x-x_0$</td>
<td></td>
</tr>
<tr>
<td>$v_0$</td>
<td></td>
</tr>
<tr>
<td>$v$</td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td></td>
</tr>
</tbody>
</table>
An objects moves along the x-axis with this velocity vs. time:

Q1: What is the displacement after 10 seconds?
   a) 6m  
   b) 8m  
   c) 10m 
   d) 20 m 
   e) None of these.

Q2: What is the distance traveled after 10 seconds?
   a) 6m  
   b) 8m  
   c) 10m 
   d) 20 m 
   e) None of these.
2.10 Falling bodies

In absence of air resistance all bodies fall with \( g = 3.0 \text{ m/s}^2 \) downwark

\[ a = -g = -9.80 \text{ m/s}^2 \]

**EX 1:** Ball thrown down with \( v = 3.0 \text{ m/s} \) from a cliff

- (a) What is its velocity when it hits the ground?
- (b) How long takes to hit the ground?
EX 2. Ball thrown up with \( v = 19.6 \text{ m/s} \).

1) How high does it go? 
2) How long in the air before coming to hand 
3) Its speed then

\[ \begin{align*}
\ddot{y} &= 0 \\
\dot{y} &= 19.6 \text{ m/s} \\
\ddot{a} &= -9.8 \text{ m/s}^2
\end{align*} \]
On planet X, a cannon ball is fired straight upward. The position and velocity of the ball at many times are listed below. Note that we have chosen up as the positive direction.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Height (m)</th>
<th>Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>17.5</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>37.5</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>37.5</td>
<td>-5</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>-10</td>
</tr>
<tr>
<td>7</td>
<td>17.5</td>
<td>-15</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>-20</td>
</tr>
</tbody>
</table>

What is the acceleration due to gravity on Planet X?

a) -5m/s²  
b) -10m/s²  
c) -15m/s²  
d) -20m/s²  
e) None of these.