Chapter 20

The Kinetic Theory of Gases
Kinetic theory of gases relates the macroscopic property of gases (pressure – temperature – volume - internal energy) to the motion of atoms or molecules.
20-2 Avogadro’s Number

Avogadro’s Number
The number of atoms in a 12 g sample of carbon 12 (1 mole)
From experiments
\[ N_A = 6.02 \times 10^{23} \text{ mol}^{-1} \]

Number of molecules or atoms

Avogadro’s number

mass of the sample

Number of moles

\[ n = \frac{N}{N_A} \]

Molar mass (mass of one mole)

\[ n = \frac{M_{\text{Sam}}}{M} \]

Molecular mass (mass of one molecule)

\[ n = \frac{mN_A}{M_{\text{Sam}}} \]
20-3 Ideal Gases

At low enough densities, all gases tend to obey the ideal gas law:

\[ pV = nRT \]

- **Gas Constant**: \( R = 8.31 \text{ J/mole} \cdot \text{K} \)
- **Temperature**: In Kelvins
- **Volume**: \( V \)
- **Number of moles**: \( n \)
- **Absolute pressure**: \( p \)

Although, there is no such thing in nature as a truly ideal gas, all real gases approach the ideal state at low enough densities.
20-3 Ideal Gases

Ideal gas Law

\[ pV = nRT \]

Gas Constant
\[ = 8.31 \text{ J/mole} \cdot \text{K} \]

Boltzmann constant
\[ = 1.38 \times 10^{-23} \text{ J/K} \]

Number of moles

\[ pV = NkT \]

Number of molecules

\[ k = \frac{nR}{N} = \frac{R}{N_A} = \frac{8.31 \text{ J/mole} \cdot \text{K}}{6.02 \times 10^{23} \text{ mol}^{-1}} = 1.38 \times 10^{-23} \text{ J/K} \]
20-3 Ideal Gases

A process in which the temperature is kept constant is called **isothermal process**.

On a P-V diagram, a curve that connects points that have the same temperature is called **isotherm**.

Isothermal expansion

Isothermal compression
20-3 Ideal Gases

Work done by an ideal gas at constant temperature

\[ W = \int_{V_i}^{V_f} p\,dV = \int_{V_i}^{V_f} \frac{nRT}{V} \,dV \]

\[ W = nRT \int_{V_i}^{V_f} \frac{dV}{V} \]

\[ W = nRT \ln\left( \frac{V_f}{V_i} \right) \]

Ideal gas isothermal process
20-3 Ideal Gases

Work done by an ideal gas at constant volume

\[ W = \int_{V_i}^{V_f} p \, dV = 0 \]

Constant-volume process
isochoric process

Work done by an ideal gas at constant pressure

\[ W = \int_{V_i}^{V_f} p \, dV = p(V_f - V_i) \]

Constant-pressure process
isobaric process
20-3 Ideal Gases

Checkpoint 1
20-3 Ideal Gases
Sample Problem 20-1

Oxygen in a cylinder
Assume that it is ideal

Initial State
\( P_i = 15 \text{ Atm} \)
\( V_i = 12 \text{ L} \)
\( T_i = 20^\circ\text{C} \)

Final State
\( P_f = ? \)
\( V_f = 8.5 \text{ L} \)
\( T_f = 35^\circ\text{C} \)

\[
\frac{p_f V_f}{p_i V_i} = \frac{n R T_f}{n R T_i}
\]

\[
p_f = p_i \frac{V_i T_f}{V_f T_i}
\]

\[
p_f = (15 \text{ atm}) \frac{(12 \text{ L})(35 + 273)\text{K}}{(8.5 \text{ L})(20 + 273)\text{K}} = 22\text{ atm}
\]

Need to convert to Kelvins
Sample Problem 20-2

1 mole of oxygen expands at a constant temperature
Assume that it is ideal

Initial State
\( V_i = 12 \text{ L} \)
\( T_i = 310 \text{ K} \)

Final State
\( V_f = 19 \text{ L} \)
\( T_f = 310 \text{ K} \)

Work done by the gas?

\[ W = nRT \ln \left( \frac{V_f}{V_i} \right) \]

\[ W = (1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(310 \text{ K}) \ln \left( \frac{19 \text{ L}}{12 \text{ L}} \right) = 1190 \text{ J} \]
Root Mean Square (RMS) speed

For 4 atoms

\[ V_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2}{4}} \]

For N atoms

\[ V_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + \ldots + v_N^2}{N}} \]
Root Mean Square (RMS) speed

\[ v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \]

Gas constant = 8.31 J/mol·K

Temperature in Kelvins

Molar mass (mass of one mole)

Using ideal gas law

\[ v_{\text{rms}} = \sqrt{\frac{3pV}{nM}} \]
20-4 Pressure, Temperature, and RMS speed

\[ v_{rms} = \sqrt{\frac{3RT}{M}} \]

<table>
<thead>
<tr>
<th>Gas</th>
<th>Molar Mass (g/mol)</th>
<th>( v_{rms} ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>2</td>
<td>1920</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>28</td>
<td>517</td>
</tr>
<tr>
<td>Oxygen</td>
<td>32</td>
<td>483</td>
</tr>
</tbody>
</table>
20-4 Pressure, Temperature, and RMS speed

Derivation of $v_{rms} = \sqrt{\frac{3RT}{M}}$
Sample Problem 20-3

You have five numbers: 5, 11, 32, 67, and 89.

Find the average values of these numbers:

\[
\text{average value} = \frac{5 + 11 + 32 + 67 + 89}{5} = 50.8
\]

Find the rms values of these numbers:

\[
\text{rms value} = \sqrt{\frac{5^2 + 11^2 + 32^2 + 67^2 + 89^2}{5}} = 52.1
\]
20-5 Translational Kinetic Energy

The average translational kinetic energy per molecule of an ideal gas

\[ K_{\text{avg}} = \frac{3}{2} kT \]

Boltzmann constant = 1.38 \times 10^{-23} \text{ J/K}

Temperature in Kelvins

At a given temperature, all ideal gas molecules – no matter what their masses – have the same average translational kinetic energy.
20-5 Translational Kinetic Energy

Checkpoint 2
20-5 Translational Kinetic Energy

Derivation of

\[
K_{\text{avg}} = \frac{3}{2} kT
\]

\[
K_{\text{avg}} = \frac{1}{2} m \left( v_1^2 + v_2^2 + \ldots + v_N^2 \right) / N
\]

\[
K_{\text{avg}} = \frac{1}{2} m v_{\text{rms}}^2
\]

\[
v_{\text{rms}} = \sqrt{\frac{3RT}{M}}
\]

\[
K_{\text{avg}} = \frac{1}{2} m \left( \frac{3RT}{M} \right)
\]

\[
K_{\text{avg}} = \frac{3}{2} \left( \frac{m}{M} \right) RT
\]

\[
\frac{m}{M} = \frac{1}{N_A}
\]

\[
K_{\text{avg}} = \frac{3}{2} \left( \frac{1}{N_A} \right) RT
\]

\[
\frac{R}{N_A} = k
\]

\[
K_{\text{avg}} = \frac{3}{2} kT
\]
20-8 The Molar Specific Heat of an Ideal Gas

For a monatomic gas (which has individual atoms rather than molecules), the internal energy $E_{\text{int}}$ is the sum of the translational kinetic energies of the atoms

$$E_{\text{int}} = N K_{\text{avg}}$$

$$E_{\text{int}} = N \left( \frac{3}{2} kT \right) = \frac{3}{2} N kT = \frac{3}{2} nRT$$

Internal energy $E_{\text{int}}$ of a monatomic gas

Gas constant = 8.31 J/mole·K

Temperature in Kelvins

Number of moles
The internal energy $E_{\text{int}}$ of an ideal gas is a function of the gas temperature only, it does not depend on any other variable.

$$E_{\text{int}} = \frac{3}{2} nRT$$
20-8 The Molar Specific Heat of an Ideal Gas

Review form Ch. 19

Heat capacity

Heat

\[ Q = C \Delta T \]

Heat capacity = (Molar specific heat) (Number of moles)

Molar specific heat is the heat needed to raise one mole of a substance by one unit temperature
Molar specific heat is the heat needed to raise one mole of a substance by one unit temperature.

But heat depends on the path!
To find the specific heat we need to specify the path.

Molar specific heat at constant volume $C_V$

Molar specific heat at constant pressure $C_p$
20-8 The Molar Specific Heat of an Ideal Gas

For solids and liquids, the difference between $C_V$ and $C_p$ is small. Usually we do not need to distinguish $C_V$ from $C_p$. We just use one value for specific heat or molar specific heat.

For gases, the difference between $C_V$ and $C_p$ is big. We need to distinguish $C_V$ from $C_p$. 
20-8 The Molar Specific Heat of an Ideal Gas

Molar specific heat at constant volume

\[ C_v = \frac{3}{2} R \]

monatomic ideal gas

(Gas consists of single atoms)

Gas constant = 8.31 J/mol·K

Molar specific heat at constant pressure

\[ C_p = \frac{5}{2} R \]

monatomic ideal gas
20-8 The Molar Specific Heat of an Ideal Gas

Derivation of $C_V$

Monatomic gas

$$\Delta E_{\text{int}} = \frac{3}{2} nR \Delta T$$

First law of thermodynamics

$$W = 0$$

$$Q = nC_v \Delta T$$

$$\frac{3}{2} nR \Delta T = nC_v \Delta T$$

$$\frac{3}{2} R = C_v$$
20-8 The Molar Specific Heat of an Ideal Gas

Derivation of $C_P$

First law of thermodynamics

monatomic gas

$$\Delta E_{int} = \frac{3}{2} nR \Delta T$$

$$\Delta E_{int} = Q - W$$

$$W = P \Delta V$$

$$PV = nRT$$

$$P \Delta V = nR \Delta T$$

$$Q = nC_P \Delta T$$

$$\frac{3}{2} nR \Delta T = nC_P \Delta T - nR \Delta T$$

$$\frac{3}{2} R = C_P - R$$

$$\frac{5}{2} R = C_P$$
20-8 The Molar Specific Heat of an Ideal Gas

Extra: Not for Phys102

Monatomic gas
Gas consists of single atoms

Diatomonic gas
Each molecule has two atoms

Polyatomic gas
Each molecule has more than two atoms
## The Molar Specific Heat of an Ideal Gas

### Extra: Not for Phys102

### Table: Molar Specific Heats of Different Gases

<table>
<thead>
<tr>
<th>Gas Type</th>
<th>$C_v$</th>
<th>$E_{\text{int}} = nC_vT$</th>
<th>$C_p = C_v + R$</th>
<th>Degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monatomic gas</td>
<td>$\frac{3}{2} R$</td>
<td>$\frac{3}{2} nRT$</td>
<td>$\frac{5}{2} R$</td>
<td>3</td>
</tr>
<tr>
<td>Gas consists of single atoms</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diatomic gas</td>
<td>$\frac{5}{2} R$</td>
<td>$\frac{5}{2} nRT$</td>
<td>$\frac{7}{2} R$</td>
<td>5</td>
</tr>
<tr>
<td>Each molecule has two atoms</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polyatomic gas</td>
<td>$\frac{6}{2} R$</td>
<td>$\frac{6}{2} nRT$</td>
<td>$\frac{8}{2} R$</td>
<td>6</td>
</tr>
<tr>
<td>Each molecule has more than two atoms</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$E_{\text{int}} = nC_vT$$
20-8 The Molar Specific Heat of an Ideal Gas

\[ \Delta E_{\text{int}} = nC_V \Delta T \]

A change in the internal energy \( E_{\text{int}} \) of a confined ideal gas depends on the change of the gas temperature only; it does not depend on what type of process produces the change in the temperature.

The change in the internal energy of the gas is the same for all three processes.
20-8 The Molar Specific Heat of an Ideal Gas

Checkpoint 4
Sample Problem 20-8

Heat added to the helium?

\[ Q = n \ C_p \ \Delta T \]
\[ = n \left( \frac{5}{2} \ R \right) \Delta T \]
\[ = (5 \ \text{mol}) \left( \frac{5}{2} \ 8.13 \ \frac{\text{J}}{\text{mol K}} \right)(20^\circ \text{C}) \]
\[ = 2077.5 \ \text{J} \]

Change in the internal energy of the helium?

\[ \Delta E_{\text{int}} = n \ C_v \ \Delta T \]
\[ = n \left( \frac{3}{2} \ R \right) \Delta T \]
\[ = (5 \ \text{mol}) \left( \frac{3}{2} \ 8.13 \ \frac{\text{J}}{\text{mol K}} \right)(20^\circ \text{C}) \]
\[ = 1246.5 \ \text{J} \]

Work done by the helium?

\[ \Delta E_{\text{int}} = Q - W \]
\[ W = Q - \Delta E_{\text{int}} = 2077.5 \ \text{J} - 1246.5 \ \text{J} = 831 \ \text{J} \]
20-11 Adiabatic Expansion

Review form Ch. 19

Adiabatic processes

No heat transfer to the system

\[ Q = 0 \]

Either

System is well insulated

Or

Process occurs so rapidly that no heat transferred to the system (sound waves)
20-11 Adiabatic Expansion

Adiabatic Expansion

No heat transfer

Adiabat \((Q=0)\)

Initial and final states:
- Initial state: \(P_i, V_i, T_i\)
- Final state: \(P_f, V_f, T_f\)

Pressure vs. Volume diagram:
- Pressure axis: \(P_i, P_f\)
- Volume axis: \(V_i, V_f\)
- Temperature axis: \(T_i, T_f\)
20-11 Adiabatic Expansion

Adiabatic Expansion

\[ p_i V_i^\gamma = p_f V_f^\gamma \]

\[ pV^\gamma = \text{Constant} \]

The ratio of the molar specific heats for the gas

\[ \gamma = \frac{C_p}{C_v} \]

monatomic gas

\[ \gamma = \frac{C_p}{C_v} = \frac{\frac{5}{2} R}{\frac{3}{2} R} = \frac{5}{3} \]
Adiabatic Expansion

\[ T_i V_i^{\gamma - 1} = T_f V_f^{\gamma - 1} \]

\[ TV^{\gamma - 1} = \text{Constant} \]

\[ \gamma = \frac{C_p}{C_v} \]

\[ \gamma = \frac{5}{3} \quad \text{monatomic gas} \]
20-11 Adiabatic Expansion

Adiabatic Expansion

\[ pV^\gamma = \text{Constant} \]

\[ TV^{\gamma - 1} = \text{Constant} \]

\[ \gamma = \frac{C_p}{C_v} \quad \text{general} \]

\[ \gamma = \frac{5}{3} \quad \text{monatomic gas} \]
Adiabatic Expansion

Derivation of $PV^\gamma = \text{Constant}$

\[ \Delta E_{\text{int}} = Q - W \quad \text{Small change} \quad \Delta E_{\text{int}} = nC_V \Delta T \]

\[ dE_{\text{int}} = -dW \quad \text{Small change} \quad dE_{\text{int}} = nC_V dT \]

\[ pV = nRT \quad \text{Small change} \quad pdV + Vdp = nRdT \]

\[ pdV + Vdp = nR \quad \frac{-pdV}{nC_V} \]

\[ pdV + Vdp = -\frac{R}{C_V} \quad \text{pdV} \quad (1 + \frac{R}{C_V})pdV + Vdp = 0 \]
20-11 Adiabatic Expansion

\[(1 + \frac{R}{C_v})pdV + Vdp = 0 \rightarrow (\frac{C_v + R}{C_v})pdV + Vdp = 0\]

\[C_v + R = C_p \rightarrow (\frac{C_p}{C_v})pdV + Vdp = 0 \rightarrow \frac{C_p}{C_v} = \gamma\]

\[\rightarrow V^{\gamma-1}(\gamma pdV + Vdp = 0) \rightarrow \gamma V^{\gamma-1}pdV + V^\gamma dp = 0\]

\[\rightarrow d(pV^\gamma) = 0 \rightarrow pV^\gamma = \text{Constant}\]
Adiabatic Expansion

Derivation of $TV^{\gamma^{-1}} = \text{Constant}$

\[ pV = nRT \]

\[ p = \frac{nRT}{V} \]

\[ pV^\gamma = \text{Constant} \]

\[ \frac{nRT}{V} V^\gamma = \text{Constant} \]

\[ nRTV^{\gamma^{-1}} = \text{Constant} \]

\[ TV^{\gamma^{-1}} = \frac{\text{Constant}}{nR} = \text{Another Constant} \]

\[ TV^{\gamma^{-1}} = \text{Constant} \]
Adiabatic Expansion

**Adiabat** \( (Q=0) \)

\[ pV^\gamma = \text{Constant} \]

\[ TV^{\gamma-1} = \text{Constant} \]

Apply when the gas is almost at equilibrium at each moment between the initial and final states.

We know the pressure and volume at each point along the path.

We can draw a line between initial and final point.
20-11 Adiabatic Expansion

Review form Ch. 19

Free expansions

System is insulated $\rightarrow Q = 0$

$W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f} 0dV = 0$

$\Delta E_{\text{int}} = Q - W \rightarrow \Delta E_{\text{int}} = 0$
20-11 Adiabatic Expansion

Free expansions

We do not know the volume and pressure at points between the initial and final states
Although, a free expansion is an adiabatic process,

\[ pV^\gamma \text{ is not constant} \]

\[ TV^{\gamma-1} \text{ is not constant} \]

In free expansion, the system is not in equilibrium at each moment between the initial and final states.

We do not know the pressure and volume at points between initial and final points.

We can not draw a line between initial and final point.
20-11 Adiabatic Expansion

Free expansions

$$\Delta E_{\text{int}} = n \ C_v \ \Delta T = 0 \ \Rightarrow \ \Delta T = 0$$

$$T = \text{Constant}$$

$$T_i = T_f$$

$$pV = nRT \ \Rightarrow \ pV = \text{Constant}$$

$$p_i V_i = p_f V_i$$
1 mole of oxygen expands adiabatically
Assume that it is ideal
Oxygen is diatomic gas ($\gamma = \frac{7}{5} = 1.4$)

Sample Problem 20-10

Initial State
$V_i = 12$ L
$T_i = 310$ K
$P_i = 2$ Pa

Final State
$V_f = 19$ L
$T_f = ?$
$P_f = $

$$T_i V_i^{\gamma - 1} = T_f V_f^{\gamma - 1}$$

$$T_f = T_i \left( \frac{V_i}{V_f} \right)^{\gamma - 1}$$

$$T_f = (310 \text{ K}) \left( \frac{12 \text{ L}}{19 \text{ L}} \right)^{1.4 - 1} = 256 \text{ K}$$
1 mole of oxygen expands freely
Assume that it is ideal
Oxygen is diatomic gas ($\gamma = 7/5 = 1.4$)

**Sample Problem 20-10**

Continue

**Initial State**

- $V_i = 12$ L
- $T_i = 310$ K
- $P_i = 2$ Pa

**Final State**

- $V_f = 19$ L
- $T_f = ?$
- $P_f = ?$

**Equations**

- $T_f = T_i = 310$ K
- $P_f V_f = P_i V_i$
- $P_f = P_i \frac{V_i}{V_f}$
- $P_f = (2 \text{ Pa}) \left( \frac{12 \text{ L}}{19 \text{ L}} \right) = 1.3 \text{ Pa}$
20-11 Adiabatic Expansion

Checkpoint 5