2. \( V_0 = 14 \text{ m/s} \) at an angle \( \theta = 41^\circ \) above the horizontal

\[ \text{a)} \quad \text{Work done by the Gravitational force when the ball goes from A to B:} \]

We see that the displacement \( \vec{d} = -12.5 \hat{j} \) (vertical drop of 12.5 m from its starting position) \( \vec{F} = mg \hat{j} \)

\[ W = \vec{mg} \cdot \vec{d} = m \vec{g} \cdot \vec{d} = m g d = (1.5 \text{ kg})(9.8 \text{ m/s}^2)(12.5 \text{ m}) \]

\[ = 184 \text{ J} \]

\[ \text{b)} \quad \text{We know that} \quad \Delta U = -W = -184 \text{ J} \]

\[ \text{c)} \quad \text{If the gravitational potential energy is taken to be zero at the top of the cliff (starting position), then the potential energy of the system on reaching the ground will be 184 J less than at the top of the cliff. Hence} \quad U(\text{at B}) = -184 \text{ J} \]

You can look at it as follows:

\[ \Delta U = \text{(change in potential energy)} = U_f - U_i \]

\( \Delta U \) is negative when the final position is lower than the initial position and vice versa.

\[ \therefore \quad \Delta U = U_B - U_A = -W = -184 \text{ J} \]

\[ U_B = U_A - 184 \text{ J} = 0 - 184 \text{ J} = -184 \text{ J} \]
21: 

i) \( m = 12 \, \text{kg} \)

ii) The spring can be compressed by 2.0 cm by a force of 270 N. This statement gives you the value of the spring constant indirectly, i.e.

\[
K = \frac{F}{\Delta x} = \frac{270 \, \text{N}}{0.02 \, \text{m}} = 1.35 \times 10^4 \, \text{N/m}
\]

a) 

iii) Using \( \Delta K + \Delta U_g + \Delta U_s = 0 \)

\[
\Delta K = K_B - K_A = 0 \quad \text{(The block is at rest at A)}
\]

\[
\Delta U_g = U_B - U_A = -mg \cdot h = -mg (d + x) \sin 30^\circ
\]

\[
\left[ \frac{h}{d + x} = \sin 30^\circ \Rightarrow h = (d + x) \sin 30^\circ \right]
\]

\[
\Delta U_s = (U_s)_B - (U_s)_A = \frac{1}{2} K x^2 = 0 \quad \text{\{block at A\}}
\]

\[
= \frac{1}{2} K x^2 \quad (x = \text{Compression of the spring})
\]

\[
= \frac{1}{2} (1.35 \times 10^4) (0.055)^2 = 0.055 \, \text{m}
\]

\[
0 - mg (d + x) \sin 30^\circ + \frac{1}{2} K x^2 = 0 \Rightarrow -mg (d + x) \sin 30^\circ = -\frac{1}{2} K x^2
\]

\[
mgd \times 0.5 + mgx \times 0.5 = \frac{1}{2} (1.35 \times 10^4) (0.055)^2
\]

\[
58.8 \, d + 3.23 = 20.419 \Rightarrow d = 0.292 \, \text{m}
\]

Total distance covered down the incline is 0.35 m

b) To find the speed of the block just before it hits the spring, we again use

\[\text{(NEXT SHEET)}\]
21. Continued

\[ \Delta K + \Delta U_f + \Delta U_s = 0 \]

In this case

\[ \Delta K = K_B - K_A = \frac{1}{2} m v^2 - 0 \]

\[ \Delta U_f = -mgd \sin 30^\circ \]

\[ \Delta U_s = 0 \] (the spring is still in its normal state)

\[ \therefore \frac{1}{2} m v^2 - mgd \sin 30^\circ = 0 \]

\[ v^2 = 2gd \sin 30^\circ = 2 \times 9.8 \times 0.293 \times 0.5 \]

\[ v = 1.69 \text{ m/s} = 1.7 \text{ m/s} \]

31. The rod of length L is fixed at point O to form a pendulum.

a) The ball is initially at point A, finally at point B. Using for the ball:

\[ \Delta K + \Delta U_f + \Delta U_s = W_U \]

\[ \Delta K = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 \]

\[ \Delta K + \Delta U_f = 0 \]

\[ \Delta U_f = -mg(2L) \]

\[ \therefore \frac{1}{2} m v_B^2 = mg (2L) \]

\[ \boxed{v_B = 2 \sqrt{gL}} \]

b) At the bottom of the swing, the free-body diagram looks like this:

\[ T - mg = \frac{m v_B^2}{L} \implies T = mg + \frac{m v_B^2}{L} \]

\[ \therefore T = mg + \frac{m 4gL}{k} = (5mg) \]
31: (Continued):

(c) The ball is released from horizontal position as shown.

Initial position is A, final position is C.

\( \theta \) is the angle with the vertical.

\[
T - mg \cos \theta = \frac{mV_c^2}{L}
\]

\[
T = mg \cos \theta + \frac{mV_c^2}{L}
\]

Now we already find \( V_c \)?

\[ A \rightarrow C: \Delta K + \Delta U = 0 \]

\[
\frac{1}{2}mV_c^2 = mgL \cos \theta
\]

\[
V_c^2 = 2gL \cos \theta
\]

Then

\[
T = mg \cos \theta + 2mg \cos \theta = 3mg \cos \theta
\]

According to the information given \( T = mg \)

\[
mg = \frac{3}{1} mg \cos \theta
\]

\[
\cos \theta = \frac{1}{3} \Rightarrow \theta = \cos^{-1}(\frac{1}{3})
\]

\[
\theta = 70.5 \approx 71^\circ
\]
54: \[ \Delta K + \Delta U = W_{nc} \]
\[ \Delta K = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = -128 \text{J} \]

\[ \Delta U = +mg \Delta h = mg d \sin 30^\circ \]

\[ W_{nc} = -\int f_k \, d = -K mg \cos 30^\circ \cdot d \]

\[ -128 + mg d \sin 30^\circ = -K mg \cos 30^\circ \cdot d \]

\[ d \left[ mg \sin 30^\circ + K mg \cos 30^\circ \right] = 128 \]

\[ d \left[ 19.6 + 10.18 \right] = 128 \implies d = 4.3 \text{m} \]

59: \[ v_0 = 6.0 \text{m/s} \]
\[ h = 1.1 \text{m} \]

Going from A to B:
\[ \Delta K + \Delta U + \Delta U = W_{nc} \]

\[ -\frac{1}{2} m v_0^2 + mg h = -K mg d \]

\[ d = \frac{\frac{1}{2} v_0^2}{K mg} + gh \]

\[ d = \frac{\frac{1}{2} (6.0)^2 - 9.8 \times 1.1}{0.5 + 9.8} = 1.22 \text{m} \]

\[ d = 1.2 \text{m} \]