The magnitude $F$ of the force required to pull the lid off is $F = (p_o - p_i)A$, where $p_o$ is the pressure outside the box, $p_i$ is the pressure inside, and $A$ is the area of the lid. Recalling that $1 \text{ N/m}^2 = 1 \text{ Pa}$, we obtain

$$p_i = p_o - \frac{F}{A} = 1.0 \times 10^5 \text{ Pa} - \frac{480 \text{ N}}{4 \times 10^{-4} \text{ m}^2} = 3.8 \times 10^4 \text{ Pa}.$$
19. (a) At depth \( y \) the gauge pressure of the water is \( p = \rho gy \), where \( \rho \) is the density of the water. We consider a horizontal strip of width \( W \) at depth \( y \), with (vertical) thickness \( dy \), across the dam. Its area is \( dA = W \, dy \) and the force it exerts on the dam is \( dF = p \, dA = \rho gy \, W \, dy \). The total force of the water on the dam is

\[
F = \int_0^D \rho gy \, W \, dy = \frac{1}{2} \rho g WD^2 .
\]

(b) Again we consider the strip of water at depth \( y \). Its moment arm for the torque it exerts about \( O \) is \( D - y \) so the torque it exerts is \( d\tau = dF(D - y) = \rho gyW(D - y)dy \) and the total torque of the water is

\[
\tau = \int_0^D \rho gyW(D - y) \, dy = \rho gW \left( \frac{1}{2} D^3 - \frac{1}{3} D^3 \right) = \frac{1}{6} \rho g WD^3 .
\]

(c) We write \( \tau = rF \), where \( r \) is the effective moment arm. Then,

\[
r = \frac{\tau}{F} = \frac{\frac{1}{6} \rho g WD^3}{\frac{1}{2} \rho g WD^2} = \frac{D}{3} .
\]
32. (a) Since the lead is not displacing any water (of density $\rho_w$), the lead’s volume is not contributing to the buoyant force $F_b$. If the immersed volume of wood is $V_i$, then

$$F_b = \rho_w V_i g = 0.90 \rho_w V_{\text{wood}} g = 0.90 \rho_w g \left( \frac{m_{\text{wood}}}{\rho_{\text{wood}}} \right),$$

which, when floating, equals the weights of the wood and lead:

$$F_b = 0.90 \rho_w g \left( \frac{m_{\text{wood}}}{\rho_{\text{wood}}} \right) = (m_{\text{wood}} + m_{\text{lead}}) g.$$

Thus,

$$m_{\text{lead}} = 0.90 \rho_w \left( \frac{m_{\text{wood}}}{\rho_{\text{wood}}} \right) - m_{\text{wood}}$$

$$= \frac{(0.90)(1000 \text{ kg/m}^3)(3.67 \text{ kg})}{600 \text{ kg/m}^3} - 3.67 \text{ kg} = 1.84 \text{ kg} \approx 1.8 \text{ kg}.$$

(b) In this case, the volume $V_{\text{lead}} = m_{\text{lead}}/\rho_{\text{lead}}$ also contributes to $F_b$. Consequently,

$$F_b = 0.90 \rho_w g \left( \frac{m_{\text{wood}}}{\rho_{\text{wood}}} \right) + \left( \frac{\rho_w}{\rho_{\text{lead}}} \right) m_{\text{lead}} g = (m_{\text{wood}} + m_{\text{lead}}) g,$$

which leads to

$$m_{\text{lead}} = \frac{0.90(\rho_w/\rho_{\text{wood}})m_{\text{wood}} - m_{\text{wood}}}{1 - \rho_w/\rho_{\text{lead}}}$$

$$= \frac{1.84 \text{ kg}}{1 - \left( 1.00 \times 10^3 \text{ kg/m}^3 / 1.13 \times 10^4 \text{ kg/m}^3 \right)} = 2.0 \text{ kg}.$$
41. Suppose that a mass $\Delta m$ of water is pumped in time $\Delta t$. The pump increases the potential energy of the water by $\Delta mgh$, where $h$ is the vertical distance through which it is lifted, and increases its kinetic energy by $\frac{1}{2}\Delta mv^2$, where $v$ is its final speed. The work it does is $\Delta W = \Delta mgh + \frac{1}{2}\Delta mv^2$ and its power is

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta m}{\Delta t} \left( gh + \frac{1}{2}v^2 \right) .$$

Now the rate of mass flow is $\Delta m/\Delta t = \rho_w Av$, where $\rho_w$ is the density of water and $A$ is the area of the hose. The area of the hose is $A = \pi r^2 = \pi(0.010 \text{ m})^2 = 3.14 \times 10^{-4} \text{ m}^2$ and $\rho_w Av = (1000 \text{ kg/m}^3)(3.14 \times 10^{-4} \text{ m}^2)(5.0 \text{ m/s}) = 1.57 \text{ kg/s}$. Thus,

$$P = \rho Av \left( gh + \frac{1}{2}v^2 \right)$$

$$= (1.57 \text{ kg/s}) \left( (9.8 \text{ m/s}^2)(3.0 \text{ m}) + \frac{(5.0 \text{ m/s})^2}{2} \right) = 66 \text{ W} .$$
54. (a) Since Sample Problem 15-9 deals with a similar situation, we use the final equation (labeled “Answer”) from it:

\[ v = \sqrt{2gh} \quad \Rightarrow \quad v = v_0 \text{ for the projectile motion.} \]

The stream of water emerges horizontally (\( \theta_0 = 0^\circ \) in the notation of Chapter 4), and setting \( y - y_0 = -(H - h) \) in Eq. 4-22, we obtain the “time-of-flight”

\[
t = \sqrt{-\frac{2(H - h)}{-g}} = \sqrt{\frac{2}{g}(H - h)}.
\]

Using this in Eq. 4-21, where \( x_0 = 0 \) by choice of coordinate origin, we find

\[ x = v_0 t = \sqrt{2gh} \sqrt{\frac{2}{g}(H - h)} = 2\sqrt{h(H - h)}. \]

(b) The result of part (a) (which, when squared, reads \( x^2 = 4h(H - h) \)) is a quadratic equation for \( h \) once \( x \) and \( H \) are specified. Two solutions for \( h \) are therefore mathematically possible, but are they both physically possible? For instance, are both solutions positive and less than \( H \)? We employ the quadratic formula:

\[
h^2 - Hh + \frac{x^2}{4} = 0 \quad \Rightarrow \quad h = \frac{H \pm \sqrt{H^2 - x^2}}{2},
\]

which permits us to see that both roots are physically possible, so long as \( x < H \). Labeling the larger root \( h_1 \) (where the plus sign is chosen) and the smaller root as \( h_2 \) (where the minus sign is chosen), then we note that their sum is simply

\[
h_1 + h_2 = \frac{H + \sqrt{H^2 - x^2}}{2} + \frac{H - \sqrt{H^2 - x^2}}{2} = H.
\]

Thus, one root is related to the other (generically labeled \( h' \) and \( h \)) by \( h' = H - h \).

(c) We wish to maximize the function \( f = x^2 = 4h(H - h) \). We differentiate with respect to \( h \) and set equal to zero to obtain

\[
\frac{df}{dh} = 4H - 8h = 0 \quad \Rightarrow \quad h = \frac{H}{2}
\]

as the depth from which an emerging stream of water will travel the maximum horizontal distance.