5. The gravitational force between the two parts is

\[ F = \frac{Gm(M - m)}{r^2} = \frac{G}{r^2} (mM - m^2) \]

which we differentiate with respect to \( m \) and set equal to zero:

\[ \frac{dF}{dm} = 0 = \frac{G}{r^2} (M - 2m) \implies M = 2m \]

which leads to the result \( m/M = 1/2 \).
8. Using $F = \frac{GmM}{r^2}$, we find that the topmost mass pulls upward on the one at the origin with $1.9 \times 10^{-8}$ N, and the rightmost mass pulls rightward on the one at the origin with $1.0 \times 10^{-8}$ N. Thus, the $(x,y)$ components of the net force, which can be converted to polar components (here we use magnitude-angle notation), are

$$
\vec{F}_{net} = (1.0 \times 10^{-8}, 1.9 \times 10^{-8}) \implies (2.1 \times 10^{-8} \angle 61^\circ).
$$

The magnitude of the force is $2.1 \times 10^{-8}$ N.
21. From Eq. 14-13, we see the extreme case is when “g” becomes zero, and plugging in Eq. 14-14 leads to

\[ 0 = \frac{GM}{R^2} - R\omega^2 \implies M = \frac{R^3\omega^2}{G}. \]

Thus, with \( R = 20000 \) m and \( \omega = 2\pi \) rad/s, we find \( M = 4.7 \times 10^{24} \) kg.
33. (a) We use the principle of conservation of energy. Initially the rocket is at Earth’s surface and the potential energy is \( U_i = -GMm/R_e = -mgR_e \), where \( M \) is the mass of Earth, \( m \) is the mass of the rocket, and \( R_e \) is the radius of Earth. The relationship \( g = GM/R_e^2 \) was used. The initial kinetic energy is \( \frac{1}{2}mv^2 = 2mgR_e \), where the substitution \( v = 2\sqrt{gR_e} \) was made. If the rocket can escape then conservation of energy must lead to a positive kinetic energy no matter how far from Earth it gets. We take the final potential energy to be zero and let \( K_f \) be the final kinetic energy. Then, \( U_i + K_i = U_f + K_f \) leads to \( K_f = U_i + K_i = -mgR_e + 2mgR_e = mgR_e \). The result is positive and the rocket has enough kinetic energy to escape the gravitational pull of Earth.

(b) We write \( \frac{1}{2}mv_f^2 \) for the final kinetic energy. Then, \( \frac{1}{2}mv_f^2 = mgR_e \) and \( v_f = \sqrt{2gR_e} \).
45. (a) If \( r \) is the radius of the orbit then the magnitude of the gravitational force acting on the satellite is given by \( GMm/r^2 \), where \( M \) is the mass of Earth and \( m \) is the mass of the satellite. The magnitude of the acceleration of the satellite is given by \( v^2/r \), where \( v \) is its speed. Newton’s second law yields \( GMm/r^2 = mv^2/r \). Since the radius of Earth is \( 6.37 \times 10^6 \) m the orbit radius is \( r = 6.37 \times 10^6 \) m + \( 160 \times 10^3 \) m = \( 6.53 \times 10^6 \) m. The solution for \( v \) is

\[
v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2/\text{kg})(5.98 \times 10^{24} \text{ kg})}{6.53 \times 10^6 \text{ m}}} = 7.82 \times 10^3 \text{ m/s}.
\]

(b) Since the circumference of the circular orbit is \( 2\pi r \), the period is

\[
T = \frac{2\pi r}{v} = \frac{2\pi (6.53 \times 10^6 \text{ m})}{7.82 \times 10^3 \text{ m/s}} = 5.25 \times 10^3 \text{ s}.
\]

This is equivalent to 87.4 min.
61. The energy required to raise a satellite of mass $m$ to an altitude $h$ (at rest) is given by

$$E_1 = \Delta U = GM_E m \left( \frac{1}{R_E} - \frac{1}{R_E + h} \right),$$

and the energy required to put it in circular orbit once it is there is

$$E_2 = \frac{1}{2} mv^2_{orb} = \frac{GM_E m}{2(R_E + h)}.$$

Consequently, the energy difference is

$$\Delta E = E_1 - E_2 = GM_E m \left[ \frac{1}{R_E} - \frac{3}{2(R_E + h)} \right].$$

(a) Since

$$\frac{1}{R_E} - \frac{3}{2(R_E + h)} = \frac{1}{6370 \text{ km}} - \frac{3}{2(6370 \text{ km} + 1500 \text{ km})} < 0$$

the answer is no ($E_1 < E_2$).

(b) Since

$$\frac{1}{R_E} - \frac{3}{2(R_E + h)} = \frac{1}{6370 \text{ km}} - \frac{3}{2(6370 \text{ km} + 3185 \text{ km})} = 0$$

we have $E_1 = E_2$.

(c) Since

$$\frac{1}{R_E} - \frac{3}{2(R_E + h)} = \frac{1}{6370 \text{ km}} - \frac{3}{2(6370 \text{ km} + 4500 \text{ km})} > 0$$

the answer is yes ($E_1 > E_2$).