Definitions

Stress Range
\[ \sigma_r = \sigma_{\text{max}} - \sigma_{\text{min}} \]

Alternating Stress
\[ \sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \]

Mean Stress
\[ \sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \]

Stress Ratio
\[ R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \]

Amplitude Ratio
\[ \Lambda = \frac{\sigma_a}{\sigma_m} \]

Note that \( R = -1 \) for a completely reversed stress state with zero mean stress.

Fluctuating Stress Failure Data

This plot shows the fatigue strength of several steels as a function of mean stress for a constant number of cycles to failure.

Note that a tensile mean stress results in a significantly lower fatigue strength for a given number of cycles to failure.

Note that a curved line passes through the mean of the data.
Master Fatigue Plot

Fluctuating Stress Failure Interaction Curves
**Soderberg Interaction Line**

\[ \frac{S_a}{S_e} + \frac{S_m}{S_{yt}} = 1 \]

Any combination of mean and alternating stress that lies on or below the Soderberg line will have infinite life.

**Factor of Safety Format**

\[ \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{yt}} = \frac{1}{n_f} \]

Note that the fatigue stress concentration factor is applied only to the alternating component.

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**Goodman Interaction Line**

\[ \frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \]

Any combination of mean and alternating stress that lies on or below the Goodman line will have infinite life.

**Factor of Safety Format**

\[ \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n_f} \]

Note that the fatigue stress concentration factor is applied only to the alternating component.
Gerber Interaction Line

Any combination of mean and alternating stress that lies on or below the Gerber line will have infinite life.

Factor of Safety Format

\[
\frac{n_f \sigma_a}{S_e} + \left( \frac{n_f \sigma_m}{S_{yt}} \right)^2 = 1
\]

Note that the fatigue stress concentration factor is applied only to the alternating component.

ASME-elliptic (distortion-energy)

\[
\left( \frac{S_a}{S_e} \right)^2 + \left( \frac{S_m}{S_{yt}} \right)^2 = 1
\]

Factor of Safety Format

\[
\left( \frac{n_f \sigma_a}{S_e} \right)^2 + \left( \frac{n_f \sigma_m}{S_{yt}} \right)^2 = 1
\]
### Table 7-15
Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for DE-Gerber and Longer Failure Loci

<table>
<thead>
<tr>
<th>Intersecting Loci</th>
<th>Intersection Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_a$ + $\left(\frac{S_b}{S_a}\right)^2 = 1$</td>
<td>$S_a = \frac{\frac{S_d}{S_b}^{\frac{2}{S_a}}}{\left[1 + \left(\frac{2S_d}{S_a}\right)^2\right]}$</td>
</tr>
<tr>
<td>Load line $r = \frac{S_b}{S_a}$</td>
<td>$S_a = \frac{S_a}{1 + r}$ (Point B)</td>
</tr>
<tr>
<td>$S_a$ + $\frac{S_b}{S_a} = 1$</td>
<td>$S_a = \frac{S_a}{1 + r}$ (Point C)</td>
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</tr>
<tr>
<td>$S_a$ + $\frac{S_b}{S_a} = 1$</td>
<td>$S_a = S_a - S_a \cdot r_{crit} = S_a/S_n$ (Point D)</td>
</tr>
</tbody>
</table>

Fatigue factor of safety

$$n_f = \frac{1}{2 \left(\frac{S_a}{S_{crit}}\right)^2} \left[-1 + \sqrt{1 + \left(\frac{2S_a}{S_{crit}\sigma_a}\right)^2}\right]$$

### Table 7-16
Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for DE-Elliptic and Longer Failure Loci

<table>
<thead>
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<tr>
<td>$\left(\frac{S_a}{S_b}\right)^2 + \left(\frac{S_b}{S_a}\right)^2 = 1$</td>
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<td>$S_a$ + $\frac{S_b}{S_a} = 1$</td>
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</tr>
<tr>
<td>Load line $r = S_b/S_a$</td>
<td>$S_a = \frac{S_a}{1 + r}$</td>
</tr>
<tr>
<td>$\left(\frac{S_a}{S_b}\right)^2 + \left(\frac{S_b}{S_a}\right)^2 = 1$</td>
<td>$S_a = \frac{S_a}{1 + \left(\frac{2S_a}{S_{crit}\sigma_a}\right)^2}$</td>
</tr>
<tr>
<td>$S_a$ + $\frac{S_b}{S_a} = 1$</td>
<td>$S_a = S_a - S_a \cdot r_{crit} = S_a/S_n$ (Point D)</td>
</tr>
</tbody>
</table>

Fatigue factor of safety

$$n_f = \frac{1}{\sqrt{(\sigma_a/S_a)^2 + (\sigma_a/S_n)^2}}$$