Minimizing and maximizing compressor and turbine work respectively
Reversible steady-flow work

- In Chapter 3, **Work Done during a Process** was found to be
  \[ W_b = \int_{1}^{2} P \, dv \]

- It depends on the path of the process as well as the properties at the end states.
Work Done During a steady state process

In a steady state process, usually there are no moving boundaries

- It would be useful to be able to express the work done during a steady flow process, in terms of system properties
- Recall that steady flow systems work best when they have no irreversibilities
Consider general form of the Energy Balance for steady flow steady state processes.

\[ \dot{Q} - \dot{W} + m_i \left[ h_i + \frac{V_i^2}{2} + g(z_i) \right] = \dot{m}_e \left[ h_e + \frac{V_e^2}{2} + g(z_e) \right] \]

\[ \delta q_{rev} - \delta w_{rev} = dh + dke + dpe \]

\[ w_{rev} = -\int_{1}^{2} v dP - \Delta ke - \Delta pe \]
For devices dealing with compressible fluids, like turbines and compressors, \( v \) is not constant, but the KE and PE are negligible. Hence

\[
W_{\text{rev}} = -\int_{1}^{2} vdP - \Delta ke - \Delta pe
\]

In order to integrate, we need to know the relationship between \( v \) and \( P \).
Important observation

Note that the work term is smallest when $v$ is small, so for a pump (which uses work) you want $v$ to be small. For a turbine (which produces work) you want $v$ to be large.

$$w_{rev} = - \int_{1}^{2} v \, dv \, dP$$
Minimizing the Compressor Work

- The best way, is to keep the specific volume as low as possible during the compression process, by **cooling** it.

Maximizing the turbine Work

- The best way, is to keep the specific volume as high as possible during the expansion process, by **heating** it.
Effect of cooling the compressor

- To understand how the cooling affects the work, let us consider three processes:
  - Isentropic process (No cooling)
  - Polytropic process (some cooling)
  - Isothermal process (maximum cooling)

- Assume also that all three processes
  - Have the same inlet and exit pressures.
  - Are internally reversible
  - The gas behaves as an ideal gas
  - Specific heats are constants.
1- Isothermal process

Consider an ideal gas, at constant $T$

\[ w_{rev, in} = \int_{1}^{2} v dP \]

\[ v = \frac{RT}{P} \]

Remember, this is only true for the isothermal case, for an ideal gas.

\[ w_{rev, in} = RT \ln \left( \frac{P_2}{P_1} \right) \]
2- Isentropic process

Isentropic means reversible and adiabatic (Q=0) i.e. No cooling is allowed

Recall from isentropic relations for an ideal gas

\[ P v^k = C \]
\[ v = C^{\frac{1}{k}} P^{-\frac{1}{k}} \]

plug in and integrate

\[ w_{rev,in} = \int_{1}^{2} v dP \]

\[ w_{rev,in} = \frac{kRT_1}{k-1} \left[ \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right] \]

Remember, this equation only applies to the isentropic case, for an ideal gas, assuming constant specific heats
3- Polytropic process

\[ w_{\text{rev, in}} = \int_1^2 v dP \]

\[ P v^n = C \]

Back in Chapter 3 we said that in a polytropic process \( P v^n \) is a constant

This is exactly the same as the isentropic case, but with \( n \) instead of \( k \)!!

\[ w_{\text{rev, in}} = \frac{v_2 P_2 - v_1 P_1}{1 - \frac{1}{n}} = \frac{R(T_2 - T_1)}{1 - \frac{1}{n}} = \frac{nR(T_2 - T_1)}{n - 1} \]

\[ w_{\text{rev, in}} = \frac{nRT_1}{n-1} \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \]
Let us plot the three processes on a P-v Diagram for the same final and initial pressures.

The area to the left of each line represents the work, $vdP$.

Note, that it takes the maximum work in isentropic compression while it takes minimum work for an isothermal compression.

$$w_{rev,in} = \int_{1}^{2} vdP$$
So as an engineer, you should **compress gas isothermally**, in order to consume minimum work.

However, for a turbine, we need to produce the maximum work. So, a turbine should expand **isentropically** (adiabatically and reversibly). That is why we assume $Q = 0$ in the 1st low analysis of a turbine.
Multistage compression with intercooling

- One common way is to use cooling jackets around the casing of the compressor.
- However, this is not sufficient in some cases.
- Instead, multistage compression is more common, with cooling between steps.
- The gas is compressed in stages and cooled to the initial temperature after each stage.
- This is done by passing it a heat exchanger called “intercooler”.
- Multistage cooling is attractive in high pressure ratio compression.
Two stage Compressor

The *colored area on the P-\(\nu\) diagram* represents the work saved as a result of two-stage compression with intercooling.
Minimizing the work input for a two stage Compressor

The size of the colored area (the saved work input) on previous slide varies with the value of the intermediate pressure $P_x$.

The total work input for a two-stage compressor is the sum of the work inputs for each stage of compression.

$$W_{\text{comp, in}} = W_{\text{comp I, in}} + W_{\text{comp II, in}}$$

$$= \frac{nRT_1}{n-1} \left[ \left( \frac{P_x}{P_1} \right)^{(n-1)/n} - 1 \right] + \frac{nRT_1}{n-1} \left[ \left( \frac{P_2}{P_x} \right)^{(n-1)/n} - 1 \right]$$
The only variable is $P_x$.

The $P_x$ value that will minimize the total work is determined by differentiating the above expression with respect to $P_x$. And setting the result to zero.

This gives

$$\left(\frac{P_x}{P_1}\right) = \left(\frac{P_2}{P_x}\right)$$

That is to minimize the compression work during two stage compression, the pressure ratio across each stage of the compressor must be the same.

$$W_{\text{comp I,in}} = W_{\text{comp II,in}}$$