MATH 102
QUIZ # 6

NAME: .................................................................  SEC. #: ........

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Q1. Find the Taylor polynomial of order 4 for \( \sin x \) about \( \frac{\pi}{2} \).

\[
P_4(x) = f\left(\frac{\pi}{2}\right) + f'\left(\frac{\pi}{2}\right)(x - \frac{\pi}{2}) + \frac{f''\left(\frac{\pi}{2}\right)}{2!}(x - \frac{\pi}{2})^2 + \frac{f'''\left(\frac{\pi}{2}\right)}{3!}(x - \frac{\pi}{2})^3 + \frac{f^{(4)}\left(\frac{\pi}{2}\right)}{4!}(x - \frac{\pi}{2})^4
\]

\[
P_4(x) = 1 - \frac{(x - \frac{\pi}{2})^2}{2} + \frac{(x - \frac{\pi}{2})^4}{24}
\]

Q2. Use any method to show that

\[
\left\{ \frac{n^2}{n!} \right\}_{n=1}^{+\infty}
\]

is eventually strictly increasing or eventually strictly decreasing.

\[
a_{n+1} = \frac{(n+1)^2}{(n+1)!}
\]

\[
\frac{a_{n+1}}{a_n} = \frac{n+1}{n^2} < 1 \text{ for } n \geq 2
\]

Eventually strictly decreasing.

Q3. Determine whether the sequence in Q2 converges, and if so find its limit.

Converges, since \( M = 0 \) is a lower bound.

\[
a_{n+1} = \frac{n+1}{n(n-1)!} = \frac{a_n}{n} + \frac{1}{n!}
\]

As \( n \to +\infty \), \( a_n \to L \) and \( a_{n+1} \to L \)

\( L = 0 \)