Q1: If the supply and demand functions are given respectively by:

\[ p = -q + 30 \]

and

\[ p = 2q + 8 \]

Then:

a) Find the equilibrium point.

b) If a tax of $5 per unit has been imposed on the product, then find the new equilibrium point

**Solution**

a) \[ -q + 30 = 2q + 8 \]

\[ 3q = 22 \]

\[ q = \frac{22}{3} \Rightarrow p = \frac{68}{3} \]

Thus the equilibrium point is \( (\frac{22}{3}, \frac{68}{3}) \)

b) The new supply function will be:

\[ p = 2q + 8 + 5 = 2q + 13 \]

then \( 2q + 13 = -q + 30 \)

\[ 3q = 17 \Rightarrow q = \frac{17}{3} \Rightarrow p = \frac{73}{3} \]

Thus the new equilibrium point is \( (\frac{17}{3}, \frac{73}{3}) \)

Q2: A producer has two different locations to make two different types of sewing machines. The following table contains the weekly production capacity, the minimum number of each type needed, and the weekly operating costs for each location. **Find the number of days that each location should be operated to produce the required number of machines at minimum cost.**

<table>
<thead>
<tr>
<th></th>
<th>Location 1</th>
<th>Location 2</th>
<th>Minimum Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td>60</td>
<td>120</td>
<td>5,400</td>
</tr>
<tr>
<td>Type II</td>
<td>90</td>
<td>100</td>
<td>6,300</td>
</tr>
<tr>
<td>Weekly Cost</td>
<td>$4,000</td>
<td>$4,500</td>
<td></td>
</tr>
</tbody>
</table>

**Solution**

Let \( x \) and \( y \) be the number of weeks that locations 1 & 2 are operated respectively, then, want to minimize: \( z = 4,000x + 4,500y \)

\[ 60x + 120y \geq 5,400 \]

Subject to: \( 90x + 100y \geq 6,300 \)

\[ x \geq 0 \text{ and } y \geq 0 \]

Then the feasible region of the constraints is given by the shaded area of the following figure:
The corner points of the feasible region are A, B and C. The coordinates of B are found by finding the intersection of the lines $60x + 120y = 5400$ and $180x + 200y = 12600$. Then the values $Z$ at these points are 360,000, 281,250 and 283,500 respectively. This implies that $x = 45$ and $y = 22.5$ minimizes $Z$. This means that we have to operate location 1: $7*45 = 315$ days and location 2: $7*22.5 = 157.5$ days to achieve the required need of machines at minimum cost.