Cell Loss Recovery Using Two-Dimensional Erasure Correction for ATM Networks

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Abstract:
In ATM networks, a fixed-length cell consisting of a header and a payload is transmitted. A cell may be dropped during the transmission due to buffer overflow or a detected error in the header. This paper proposes a cell loss recovery technique using two-dimensional erasure correction. The cells to be transmitted are arranged in a matrix form. A parity check cell is appended to each row and each column based on single parity encoding. Such an encoding scheme is capable of recovering many cell-loss patterns. It is shown through analysis and simulation that the proposed scheme, though simple, results in a very low post-decoding cell loss rate and outperforms the performance of one-dimensional recovery when both schemes are operating at the same redundancy rate.

1. Introduction
Asynchronous Transfer Mode (ATM) is a network protocol that is widely adopted for use on B-ISDN systems because of its many advantages over STM-based protocols [5]. The transmission unit in ATM-based networks is the cell. A cell is a fixed-length packet of 53 bytes, of which five are reserved for the cell header. The fifth byte of the header is a cyclic redundancy check (CRC) calculated from the other four bytes and used to detect the presence of error in the header. The cell format is shown in Figure 1.

At every intermediate node the header is checked, and if it is found in error the cell is discarded. A cell may also be discarded because of buffer overflow in multiplexing and cross-connecting equipment. Many proposals were studied to recover the lost cells [1]-[3],[4]. In the system presented in [5] and analyzed in detail in [2] the cells to be encoded are arranged in a matrix consisting of M-1 rows and N-1 columns of data cells. Each column is terminated by a parity check cell obtained by mod-2 addition of the M-1 cells in that column. In this scheme, one lost cell per column can be recovered.

The location of the lost cells in the sequence of received cells can be identified by examining the cell sequence number (SN) of the received sequence which is already allocated for as a part of the payload in standard cell format. In some applications only important data cells are protected against loss. It follows that data cells in a row may not be consecutive in their sequence numbers, thus necessitating the advent of another reference to discover the lost cells. In these applications a special cell, called the cell loss detection (CLD) cell is appended to each row. The CLD cell carries the sequence numbers of the M-1 data cells in the row. Lost cells are identified by checking the sequence numbers of the received data cells against the sequence numbers carried by the CLD cell. All CLD cells are mod-2 added to produce a CLD check cell, thus enabling the decoder to recover one lost CLD cell.

In this paper we extend the work in [2] by encoding the data cells rowwise as well as columnwise. This two-dimensional encoding scheme allows many patterns of lost cells to be recovered, resulting in a significant reduction in the post-decoding cell loss rate. The rest of the paper is organized as follows. In Section 2 the encoding and decoding schemes are explained. Since the locations of lost cells are determined first, the decoding process reduces to that of erasure correction. The erasure correction capability of the proposed scheme is discussed in Section 3. Section 4 presents some analytical and simulation results. The performance of the system, in terms of post-decoding cell loss rate and processing delay, is compared to the one-dimensional encoding system when both systems are utilizing the same redundancy rate (no. of check cells/(no. of check cells + data cells)). Conclusions are summarized in Section 5.

2. Encoding and Decoding Schemes
The encoder operates on a matrix of (M-1)×(N-1) data cells to produce an M×N matrix of cells. It is worth noting that only the payload segment of the cell is processed by the encoder and the decoder. The Mth row contains the parity check cells on the columns where each parity check cell is obtained by mod-2 addition
of the data cells in the same column. In the same way the $N^\text{th}$ column contains the parity check cells on the rows where each parity check cell is obtained by mod-2 addition of the data cells in the same row. All cells carry the same header. The matrix structure is illustrated in Figure 2.

![Figure 2: Two-dimensional encoding matrix](image)

At the destination node the receiver examines the sequence number of the arriving cells to determine the lost cells, if any. Lost cells can be identified by verifying the sequential sequence of the arriving cells if they have been transmitted in sequence, or by checking their sequence numbers against the sequence numbers carried by the CLD cells. Once a lost cell is identified, a payload of all-zeros will be generated and placed in the correct location in the matrix.

The decoder proceeds as follows. It scans the first row for missing cells. If a single missing cell is detected it is then recovered as the mod-2 addition of all other cells in that row. If more than one missing cell is detected they are left intact. The same process is carried out for the other rows in sequence. After scanning all rows the whole process is repeated for the columns. Once this step is completed there is a possibility that rowwise decoding can recover some more lost cells. Therefore, a third round of decoding along the rows will be carried out. It generally improves the correction by executing more rounds but at the expense of increased delay [4]. It is easily seen that these three rounds of decoding are sufficient to recover all recoverable patterns of five or less lost cells and most of the higher patterns. Therefore, we found three-round decoding a good compromise between correction capabilities and decoding delay, and hence is adopted here.

In principle, the decoding can as well be carried out in the reverse sequence, i.e. columnwise, rowwise then column-wise. However, by proceeding with rows first we eliminate some of the processing delay because rowwise decoding can start once a complete row is received and there is no need to wait until the whole matrix is constructed. By the time the matrix is fully received, the first round of decoding would have been completed.

3. Performance Analysis

The performance of the proposed scheme is measured in terms of the post-decoding cell loss probability ($P_L$). We assume that cells are lost at random with probability $p$. This assumption is also valid for bursts of lost cells with proper interleaver/deinterleaver [1]. The post-decoding cell loss probability $P_L$ is evaluated as:

$$P_L = \frac{1}{MN} \sum_{i=1}^{MN} i A_i p^i (1-p)^{MN-i}$$  \hspace{1cm} (1)

Where $A_i$ is the number of unrecoverable $i$-erasure patterns (i.e. the number of unrecoverable patterns with $i$ lost cells in the matrix). The three-round decoder explained above can recover all single-, double- and triple-erasure patterns. That is:

$$A_1 = A_2 = A_3 = 0.$$  \hspace{1cm} (2)

The only unrecoverable 4-erasure pattern is the rectangular shape, i.e. when the four missing cells are located in the same two rows and the same two columns.

One such pattern is shown in Figure 3. There are $\binom{N}{2}$ ways for selecting two
columns out of $N$ columns, and each way is associated with $\binom{M}{2}$ ways of selecting two rows out of $M$ rows. Therefore,

$$A_4 = \binom{N}{2} \binom{M}{2}$$

Now we move to evaluate $A_5$. The only unrecoverable 5-erasure patterns are those where four of the five erasures form a rectangular shape. For each of the $A_4$ patterns there are $MN-4$ locations for the fifth erasure. Therefore:

$$A_5 = \binom{N}{2} \binom{M}{2} (MN - 4)$$

The above derivations were verified by exhaustive search for moderate values of $M$ and $N$.

Let’s illustrate the above discussion numerically. Consider the case $M=N=4$. For this matrix size there are a total of $\binom{MN}{4} = 1820$ 4-erasure patterns of which only $A_4=36$ are unrecoverable, and there are $\binom{MN}{5} = 4368$ 5-erasure patterns of which $A_5=432$ are unrecoverable.

For more than five erasures the problem becomes more complicated. We will make a worst-case analysis by assuming that all six and higher erasure patterns are unrecoverable. That is

$$A_i = \binom{MN}{i} \quad , \quad i \geq 6$$

The above analysis is valid provided that all lost cells can be located. For the case where lost cells are identified by examining the sequential sequence of the transmitted cells (when all cells are protected), locating lost cells is always possible. However when CLD cells are used to identify the missing cells (when only important cells are protected), the probability in (1) is valid under the condition that all CLD cells are or can be made available. This is equivalent to the event that no or only one CLD cell is missing, because the decoder has the capability of recovering one lost CLD cell. Denote the probability of this event by $E$. Then:

$$E = (1 - p)^M + p(1 - p)^{M-1}$$

We will assume that when more than one CLD is missing the decoder will not attempt any decoding. Therefore, for systems relying on CLD to find lost cells, Equation (1) should be modified to:

$$P'_L = P_L \times E + p(1 - E)$$

### 4. Results and Discussion

The proposed two-dimensional recovery scheme was applied to a matrix of size $M=N=16$. The redundancy ratio is $31/(16 \times 16) \approx 1/8$. The proposed scheme was compared with the one-dimensional correction scheme in [3]. For fair comparison the redundancy ratio has to be equal. To satisfy this requirement we took $M=N=8$ for the one-dimensional scheme. This is equivalent to a redundancy ratio of $8/(8 \times 8) = 1/8$.

The performance curves for the two schemes are shown in Figures 4 and 5. Figure 4 refers to the case when missing cells are identified from the sequential sequence while Figure 5 refers to the case when lost cells are identified using
CLD cells. The superiority of the proposed scheme over the other scheme is evident, particularly at low loss rates. The apparent inferiority of the proposed system to the other system at larger loss rates ($p>10^{-2}$) is due mainly to the looseness of the bound in (1) at such large values.

We ought to compare the two schemes in terms of the processing delay. In what follows we assume that the dominant delay is that caused by scanning the matrix in search for the missing cells, and that the actual encoding and decoding time (simple mod-2 addition) is relatively insignificant. We also assume that the matrix is square ($M=N$).

It is worth noting that for the one-dimensional scheme to be effective in recovering the bursts of lost cells, encoding and decoding must be carried out column-wise (opposite to the direction of transmission). The one-dimensional scheme performs one round of encoding (columnwise) while the two-dimensional scheme performs two round of encoding (rowwise then columnwise). However the rowwise encoding of the two-dimensional scheme can be carried out while the matrix is being constructed. As a result there is no extra delay added by the two-dimensional encoding scheme.

At the other end, the one-dimensional scheme requires one round of decoding (columnwise) whereas the two-dimensional scheme requires three round of decoding: rowwise, columnwise then rowwise. Once again the first round rowwise decoding is carried out while the matrix is being reconstructed. As a result the two-dimensional scheme requires twice the processing time of the one-dimensional scheme. For a moderate matrix size, like the one used here, such an increase in delay is not crucial.

5. Conclusions

A two-dimensional cell recovery scheme was proposed and analyzed. It requires two rounds for encoding (rowwise and columnwise) and three rounds for decoding (rowwise, columnwise, rowwise). The scheme's capability to recover lost cells was discussed, and the performance in terms of post-decoding cell loss probability was derived. Though simple, the proposed scheme offers significant reduction in cell loss probability over the one-dimensional scheme at a cost of slight increase in processing delay.

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References


