Problem 2.2:

a) First order,
b) First order (differentiate once to get rid of the integral on y
c) Zero order,
d) First order,
e) Second order

Problem 2.4

Only (c) and (d) are nonlinear. Superposition will not hold in (e) because of the term +10. As an example to show linearity, consider (d):

\[
\frac{dy_1(t)}{dt} + t^2 y_1(t) = \int_{-\infty}^{t} x_1(\lambda) d\lambda
\]

\[
\frac{dy_2(t)}{dt} + t^2 y_2(t) = \int_{-\infty}^{t} x_2(\lambda) d\lambda
\]

Multiply the first equation by a constant, say \( a \), and the second equation by another constant, say \( b \); add to obtain:

\[
\frac{d[ay_1(t) + by_2(t)]}{dt} + t^2 [ay_1(t) + by_2(t)] = \int_{-\infty}^{t} [ax_1(\lambda) + bx_2(\lambda)] d\lambda
\]

This is of the same form as the original equation.

Problem 2.10

Using Kirchhoff's voltage equation and Ohm's law, the appropriate equations are

\[
x(t) = L \frac{di(t)}{dt} + y(t)
\]

\[
y(t) = R i(t)
\]

\[
\frac{di(t)}{dt} = \frac{1}{R} \frac{dy(t)}{dt}
\]

Substitute the last equation in the first and rearrange to obtain

\[
\frac{dy(t)}{dt} + \frac{R}{L} y(t) = \frac{R}{L} x(t)
\]
b) The system is linear

\[
\begin{align*}
\frac{dy_1}{dt} + \frac{R}{L} y_1 &= \frac{R}{L} x_1 + a \\
\frac{dy_2}{dt} + \frac{R}{L} y_2 &= \frac{R}{L} x_2 + b
\end{align*}
\]

and add

\[
\begin{align*}
a \frac{dy_1}{dt} + \frac{R}{L} y_1 &= a \frac{R}{L} x_1 \\
b \frac{dy_2}{dt} + \frac{R}{L} y_2 &= b \frac{R}{L} x_2
\end{align*}
\]

\[
\begin{align*}
y_1 &= \frac{R}{L} i_1 \\
y_1 &= x_1 - \frac{L}{R} \frac{dy_1}{dt}
\end{align*}
\]

\[
\begin{align*}
y_2 &= \frac{R}{L} i_2 \\
y_2 &= x_2 - \frac{L}{R} \frac{dy_2}{dt}
\end{align*}
\]

\[
\begin{align*}
a y_1 + b y_2 &= a x_1 - \frac{a}{R} \frac{dy_1}{dt} + b x_2 - \frac{b}{R} \frac{dy_2}{dt} \\
&= a x_1 + b x_2 - \frac{a}{R} \frac{dy_1}{dt} - \frac{b}{R} \frac{dy_2}{dt}
\end{align*}
\]

\[
\begin{align*}
a \frac{R}{L} x_1 + b \frac{R}{L} x_2 &= a \frac{R}{L} y_1 + b \frac{R}{L} y_2 + \frac{a}{R} \frac{dy_1}{dt} + \frac{b}{R} \frac{dy_2}{dt}
\end{align*}
\]

(c) Consider

\[
\frac{dy(t - \tau)}{dt} = \frac{dy(t')}{dt'} \frac{dt'}{dt} \quad \text{where} \quad t' = t - \tau
\]
Thus:

\[
\frac{dy(t-\tau)}{dt} + \frac{R}{L} y(t-\tau) = \frac{R}{L} x(t-\tau)
\]

which shows that the system is fixed.

(d) Note that the solution to the homogeneous equation is

\[y_h(t) = Ae^{-\alpha t}, \quad t > 0\]

Assume a complete solution of this form where \(A\) is time varying. Substitute into the differential equation of part (a) to obtain

\[A(t) = \int_0^t \frac{R}{L} x(\lambda)e^{R\alpha \lambda} \, d\lambda + A_0\]

Since the inductor current is assumed 0 at \(t = 0\), this gives \(A_0 = 0\), so the solution to the differential equation is

\[y(t) = \int_0^t \frac{R}{L} x(\lambda) \exp \left[-\frac{R}{L}(t-\lambda)\right] d\lambda\]

**Problem 2.11**

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<tr>
<th>Property</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
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</tr>
</tbody>
</table>
Problem 2-23

By KVL around the loop,

\[ x(t) = R_1 i(t) + \frac{1}{C} \int_0^t i(\lambda) d\lambda + R_2 i(t) \]

But \( i(t) = y(t)/R_2 \). Substitute this into the integro-differential equation and differentiate once to get

\[ \frac{R_1 + R_2}{R_2} \frac{dy(t)}{dt} + \frac{y(t)}{R_2 C} = \frac{dx(t)}{dt} \]

One way to find the impulse response is to find the step response and differentiate it. The solution to the homogeneous equation is

\[ a(t) = A e^{-\frac{(R_1 + R_2)}{C} t}, \quad t > 0 \]

With a step input, the right-hand side of the differential equation is an impulse. To get the required initial condition, we integrate the differential equation through \( t = 0 \):

\[ \frac{R_1 + R_2}{R_2} \int_0^0 da(t) dt + \frac{1}{R_2 C} \int_0^0 a(t) dt = \int_0^0 \delta(t) dt = 1 \]

To match the right-hand side, the integrand of the first term on the left-hand side must contain a unit impulse and, therefore, the second term on the left-hand side is proportional to a unit step. Hence the integral on the second term through \( t = 0 \) is 0 (a step discontinuity). The first term is a perfect differential. Thus, we obtain \( a(0+) = \frac{R_2}{(R_1 + R_2)} \) as the required initial condition, and the step response is

\[ a(t) = \frac{R_2}{R_1 + R_2} e^{-\frac{(R_1 + R_2)}{C} u(t)} \text{ and } h(t) = \frac{da(t)}{dt} = \frac{R_2}{R_1 + R_2} \left[ \delta(t) - \frac{1}{(R_1 + R_2) C} e^{-\frac{(R_1 + R_2)}{C} u(t)} \right] \]
Problem 2-32

(a) Use KCL at the output node to obtain

\[ x(t) - y(t) = \frac{R_1}{R_2} y(t) + C \frac{dy(t)}{dt} \]

When rearranged, this gives the answer given in the problem statement.

(b) The homogeneous equation for the impulse response is

\[ R_1 C \frac{dh(t)}{dt} + \left( 1 + \frac{R_1}{R_2} \right) y(t) = 0 \]

Assume a solution of the form

\[ h(t) = Ae^{pt}, \quad t > 0 \]

Substitute the assumed solution into the homogeneous differential equation to get the characteristic equation

\[ R_1 C p + \left( 1 + \frac{R_1}{R_2} \right) = 0 \quad \text{or} \quad p = -\frac{R_1 R_2 C}{R_1 + R_2} \]

To get the required initial condition, integrate the differential equation, with impulse forcing function, through \( t = 0 \):

\[ R_1 C \int_{0^-}^{0^+} \frac{dh(t)}{dt} \, dt + \left( 1 + \frac{R_1}{R_2} \right) \int_{0^-}^{0^+} h(t) \, dt = \int_{0^-}^{0^+} \delta(t) \, dt = 1 \]

The second term on the left-hand side is discontinuous at \( t = 0 \), but contains no impulse function; the first term on the left-hand side must contain an impulse function to balance the impulse function on the right-hand side. The first integral has an integrand that is a perfect differential, so

\[ R_1 C [h(0^+) - h(0^-)] = 1 \quad \text{or} \quad h(0^+) = -\frac{1}{R_1 C} \]

Substituting for \( h(0^+) = A \) and \( p \), we obtain the result for the impulse response given in the problem statement.