Chapter 2.

Deformation
Deformation is the change in volume and shape of bodies when subjected to forces.

Deformation is noticeable for materials like rubber whereas it is hard to be seen for structural members like reinforced concrete — why?

Displacement: is a vector quantity which measures the movement of a particle or a point in a body from one position to another.

\[ \vec{u}_A \text{ is the displacement vector of point } A. \]

The best way to quantify deformation is to measure strains.

2.2 Strains

As there are two kinds of stresses, there are two kinds of strain. Normal strains caused by normal stresses and shear strains caused by shear stresses.

\[ \text{Normal strain} = \frac{\text{change in length}}{\text{original length}} \quad \left( \varepsilon = \frac{\Delta L}{L_0} = \frac{S}{L_0} \right) \]
Shear strain: the change in the $90^\circ$ angle in the original body.

\[ \gamma = (1 + \varepsilon) \theta_0 \]

Shear strain: $\gamma$

\[ \cos \gamma = 1 \]
\[ \sin \gamma = \gamma \]
\[ \tan \gamma = \frac{\sin \gamma}{\cos \gamma} = \gamma \]

Change of shape and volume

Original body

Deformed body
One of the most important tests to perform is the tension or compression test, which is meant to establish the relationship between normal stress \( \sigma \) and normal strain \( e \) using a standard specimen.

1. **Gage length** \( L_0 \) is the length along which strain is monitored.

2. **Extensometer** is a device to measure change in length.

The end product of the test is a stress-strain curve from which material properties can be calculated.
3.2 The Stress-Strain Diagram.

Using a standard specimen of known gauge length L₀ and of original cross section area A₀ and run a tension test according to ASTM (American Society for Testing and Materials) specifications. This required applying a force P with a specified rate then:

- Calculate normal stress \( \sigma = \frac{P}{A₀} \)
- Calculate normal strain \( \varepsilon = \frac{\Delta L}{L₀} \)

Plot stress versus strain, one may have a curve like this:

\[
\text{slope} = E \quad \text{(Modulus of elasticity or Young's modulus)}
\]

\( E \) elastic yield strain hardening necking

Stress-strain curve for ductile steel.
3.3 **Stress-Strain Behavior of Ductile and Brittle Materials**

Ductile materials are those materials which exhibit large deformation before failing.

Ductility is measured by one of two:

- \( \% \text{ elongation} = \frac{L - L_0}{L_0} \times 100 \)
- \( \% \text{ reduction in area} = \frac{A_0 - A_f}{A_0} \times 100 \)

For materials which do not possess definite yield point, the yield stress (strength) is obtained by the use of offset method, which is normally at 0.002 strain.

Brittle materials are those which exhibit little or no yielding before failing.

Examples: Concrete and cast iron?

See Fig. 3.9 and Fig 3.11 in your textbook pages (92 & 93).
3.4 Hooke's Law

The linear stress-strain relationship is called Hooke's law.

\[ \sigma = E \varepsilon \]

Remember, \( E \) is called the modulus of elasticity.

Remark: Although one may vary the yield strength of steel by varying the % of carbon, the modulus of elasticity remains the same as in steel.

- Spring steel (1% Carbon)
- Hard steel (0.6% Carbon) (heat treated)
- Structural steel (0.2% Carbon)
- Soft steel (0.1% Carbon)
- Natural rubber

For steel \( E = 29 \times 10^3 \) ksi (200 GPa)

For rubber \( E = 0.1 \times 10^3 \) ksi (0.7 MPa)

That is why deformation in rubber is noticeable.
There are two more important properties for materials. One is called the Modulus of resilience \( U_r \) and the other is the modulus of toughness \( U_f \).

\[ \sigma \quad \varepsilon \]

3.6 Poisson's Ratio \( v \)

\[ \varepsilon_x = \frac{L-L_0}{L} > 0 \]

\[ \varepsilon_y = \varepsilon_z = \frac{D-D_0}{D_0} < 0 \]

\( \varepsilon_y \) or \( \varepsilon_z \) = \(-v\) \( \varepsilon_x \)

Poisson's ratio \( v \) is nothing but the relation between lateral strain to the longitudinal strain.

\[ \nabla = \varepsilon \]

\[ \nabla = \frac{\Delta L}{L} \]

\[ \mu \]

Modulus of rigidity
## Average Mechanical Properties of Typical Engineering Materials

(U.S. Customary Units)

<table>
<thead>
<tr>
<th>Materials</th>
<th>Specific Weight ( \gamma ) (lb/ft^3)</th>
<th>Modulus of Elasticity ( E ) (10^6) ksi</th>
<th>Modulus of Rigidity ( G ) (10^3) ksf</th>
<th>Yield Strength (ksi) ( \sigma_y ) Tens. Comp. a Shear</th>
<th>Ultimate Strength (ksi) ( \sigma_u ) Tens. Comp. a Shear</th>
<th>% Elongation in 2 in. specimen</th>
<th>Poisson's Ratio ( \nu )</th>
<th>Cost of Thermal Expansion ( \alpha ) (10^-6)/°F</th>
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A Specific values may vary for a particular material due to alloy or mineral composition, mechanical working of the specimen, or heat treatment. For a more exact value reference books for the material should be consulted.

b The yield and ultimate strengths for ductile materials can be assumed equal for both tension and compression.

c Measured perpendicular to the grain.

d Measured parallel to the grain.

* Deformation measured perpendicular to the grain when the load is applied along the grain.
## Average Mechanical Properties of Typical Engineering Materials
(U.S. Customary Units)

| Material          | Specific Weight (lb/ft³) | Modulus of Elasticity (ksi) | Modulus of Rupture (ksi) | Yield Stress (ksi) | % Elongation in 1 in. specimen | Ultimate Stress (ksi) | % Elongation in 1 in. specimen | Poisson's Ratio | Cost of Thermo-Expansion or \( \alpha \) (in)
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* The yield and ultimate strengths for ductile materials can be assumed equal for both tension and compression.
* Measured parallel to the grain.
* Measured parallel to the grain.
* Deformation measured perpendicular to the grain when the load is applied along the grain.
2-3. Bar \( ABC \) is originally in a horizontal position. If loads cause the end \( A \) to be displaced downwards \( \Delta_a = 0.002 \text{ in.} \) and the bar rotates \( \theta = 0.2^\circ \), determine the average normal strain in the ends \( AD, BE, \) and \( CF \).

\[
\begin{align*}
\delta_{EA} &= \delta_{EB} - 0.002 \frac{E}{6} = \tan 0.2^\circ \\
&= 0.02294 \text{ in.} \\
\delta_{FC} &= \delta_{EF} - 0.002 \frac{E}{12} = \tan 0.2^\circ \\
&= 0.04388 \text{ in.}
\end{align*}
\]

\[
\begin{align*}
\varepsilon_{AD} &= \frac{\delta_{AD}}{L_{AD}} = 0.002 \frac{E}{8} = 2.5 \times 10^{-4} \text{ in./in.} \\
\varepsilon_{BE} &= \frac{\delta_{BE}}{L_{BE}} = 0.02294 \frac{E}{8} = 2.8675 \times 10^{-3} \text{ in./in.} \\
\varepsilon_{FC} &= \frac{\delta_{FC}}{L_{CF}} = 0.04388 \frac{E}{8} = 5.485 \times 10^{-3} \text{ in./in.}
\end{align*}
\]

2-22. A square piece of material is deformed into the skewed position shown. Determine the shear strain \( \gamma \) at points \( B \) and \( C \).

\[
\begin{align*}
\gamma_{xy} &= \frac{0.3 \pi}{180} \text{ at } B \\
\gamma_{xy} &= \frac{0.3 \pi}{180} \text{ at } C
\end{align*}
\]

Both \( \gamma \) which is \( 5.24 \times 10^{-3} \text{ rad.} \)

\[
\theta = \tan 0.3^\circ = 5.24 \times 10^{-3} \text{ rad.}
\]
Saint-Venant Principle: The stress tends to be large and non-uniform near or at support and where the load is applied and the stress tends to flatten and becomes uniform at a distance equal to the largest dimension away from support and load application.

Just like stress, strain is defined at a point which may vary and change from point to point.

\[ \sigma_{xy} = \frac{P}{A} \]

strain of element \( dx \)
\[ \varepsilon(x) = \frac{dS}{dx} \]

for constant stress and strain \( \varepsilon(x) = \varepsilon = \text{constant} \) and the above become
\[ S = \varepsilon L \]
for a number made of material where Modulus of elasticity $E$:

\[ \sigma(x) = E \epsilon(x) \quad \text{--- Moen's law} \]

\[ \frac{N(x)}{A(x)} = E \epsilon(x) \quad \text{--- for variable load and area} \]

\[ \epsilon(x) = \frac{N(x)}{EA(x)} \]

which upon substitution, one has:

\[ \sigma = \int \frac{N(x)}{EA(x)} \, dx \]

For constant $N(x)$ and $A(x)$:

\[ \sigma = \frac{NL}{EA} \]

for beam composed of different materials or different segments of different area, then

\[ \sigma = \sum \frac{N_i L_i}{E_i A_i} \]

where $n$ is number of segment.

\[ \delta = \frac{N L}{E A} \]

**Problem 4-1**

Determine displacement of $B$ and the end $A$.

\[ \sigma_B = \frac{N_{BA} L_{BA}}{E_{BA} A_{BA}} = \frac{(12 \times 10^3)^3}{(250 \times 10^9)} \frac{(1 \times 10^2)^3}{4} \]

\[ \sigma_B = 0.00159 \text{ m} = 1.59 \text{ mm} \]

\[ \delta_A = \delta_{BA} + \delta_{BA} = 0.0059 + \frac{N_{BA} L_{BA}}{E_{BA} A_{BA}} = 0.0059 + \frac{(12 \times 10^3)^3 \left(1 \times 10^2 \right)^3}{(250 \times 10^9) \left(2 \times 10^9 \right)} \]

\[ = 0.00614 \text{ m} = 6.14 \text{ mm} \]
A-36 steel \( \Rightarrow E = 200 \times 10^9 \)

\[ \sigma_{A/D} = \frac{N_{AB}}{E_{AB}} + \frac{N_{CD}}{E_{CD}} + \frac{N_{DC}}{E_{DC}} \]

\[ = \frac{50 \times 10^3}{(0.005)(0.1)} + \frac{(50 \times 10^3)(0.1)}{3(0.005)(0.1)} + \frac{(50 \times 10^3)(0.8)}{2(0.006)(0.1)} \]

\[ = \frac{(50 \times 10^3)(0.6 + 0.23 + 0.82)}{(0.006)(0.1)} \times 10^4 \quad = 4.4 \times 10^4 \text{ MPa} \]

\[ = 4.4 \times 10^4 \text{ mm}^2 \]

AB & CD are steel wires

\( d_{AB} = 0.5 \text{ in}, \quad d_{CD} = 0.3 \text{ in} \)

\( T_{AB} = 16.2 \text{ kips} \)

Determine \( W \) & \( x \) such that \( AC \) remains horizontal.

\[ IF_B = 0: \quad N_{AB} + N_{CD} = W x \quad \text{--- (1)} \]

\[ IM_{D_{DC}} = 8 N_{CD} = \frac{W x^2}{2} \quad \text{--- (2)} \]

\( AC \) remains horizontal: \( \sigma_{AB} = \sigma_{CD} \)

\[ \sigma_{CD} = \frac{N_{CD}}{T''(0.3)^2} \quad = 16.2 \times 10^3 \quad \text{--- (3)} \]

\[ W = \frac{16.2}{10^3} \text{ kips/ft} \quad \text{--- (4)} \]

From (1): \( N_{CD} = 1145 \text{ kips} \)

From (3): \( N_{AB} = 3172 \text{ kips} \)

From (1) & (2): \( 16 N_{CD} = (N_{AB} + N_{CD}) x \quad \Rightarrow \quad x = 4.24 \text{ ft} \)

Check that \( \sigma_{AB} \leq \sigma_{DC} \):

\[ \frac{N_{AB}}{T''(0.3)^2} \leq 16.2 \times 10^3 \quad \text{ok} \]
When the number of unknowns exceeded the number of equations of equilibrium, we say that the structure is statically indeterminate and thus the unknowns (reactions or internal normal forces) cannot be obtained from equations of equilibrium alone and one needs additional equations. These equations can be obtained from the deformation of the structure. These extra equations are known as compatibility equations.

Compatibility equations are nothing but relationships between displacement of members which will be more useful if converted in terms of internal unknown forces which are used with eq. (7) to solve for unknowns.
Examples of statically indeterminate problems (P and L are given)

\[ \delta_{AB} + \delta_{BC} = 0 \]

\[ \frac{\delta_{AB}}{2L} = \frac{\delta_{CD}}{L} \]

\[ \frac{\delta_{AB}}{3L} = \frac{\delta_{CD}}{4L} \]

\[ \delta_{BL} = \delta_{CD} + \Delta \]

\[ \delta_{BL} = \frac{\delta_{CD}}{2L} \]

\[ H = PL \]

\[ \frac{\delta_{1}}{3L} = \frac{\delta_{C}}{2L} \]
Equilibrium Equations:

1. \[ \sum M_{Q} = 0 \]
   \[ 1600 \, \text{N} \times \frac{2}{3} - F_{c} - F_{a} = 0 \]
   \[ \Rightarrow F_{a} = F_{c} = F \]

2. \[ \sum F_{y} = 0 \]
   \[ F_{a} + F_{b} + F_{c} = 160 \times 10^{3} \]
   \[ 2F + F_{b} = 4 \times 10^{3} \]

3. Compatibility Equation:
   \[ S_{a} = S_{c} = \left( S_{b} + 0.0003 \right) \]

   Compatibility equation should be expressed in terms of forces (unknowns).

   \[ \frac{F_{a}(16)}{F_{a} \cdot E_{a}} = \frac{F_{b} \cdot L_{b}}{A_{b} \cdot E_{b}} + 0.0003 \]

   \[ \frac{F(0.125)}{(400 \times 10^{-6})(70 \times 10^{-3})} = \frac{F_{b}(0.1247)}{(400 \times 10^{-6})(70 \times 10^{-3})} + 0.0003 \]

   \[ 125F - 124.7F_{b} = 8400 \]

Solving (2) & (3) \[ \Rightarrow \]
\[ F = 75762 \, \text{N} \]
\[ F_{b} = 8547 \, \text{N} \]

\[ \sigma_{a} = \frac{F}{A} = \frac{75762}{400 \times 10^{-6}} = 189 \times 10^{6} \, \text{Pa} = 189 \, \text{MPa} \]

\[ \sigma_{b} = \frac{F_{b}}{A} = \frac{8547}{400 \times 10^{-6}} = 21.4 \times 10^{6} \, \text{Pa} = 21.4 \, \text{MPa} \]
4.6 Thermal Stress

There are two types of stress: one is called mechanical stress (due to physically applied load or reaction) and the other is thermal stress for restrained members (due to increase or decrease in temperature).

\[
\alpha = \text{the change of unit length due to one degree of temp.}
\]

\[
\varepsilon = \text{the strain due to one unit degree.}
\]

\[
\varepsilon = \alpha \Delta T
\]

\[
S = (\alpha \Delta T) L
\]

\[
\varepsilon_{\text{total}} = \varepsilon_{\text{elastic}} + \varepsilon_{\text{thermal}}
\]

\[
E_{\text{total}} = E_{\text{elastic}} E_{\text{thermal}}
\]

\[
\sigma = E (\varepsilon_{\text{elastic}})
\]

\[
\varepsilon_{\text{total}} = \varepsilon_{\text{thermal}}
\]

\[
\varepsilon_{\text{elastic}} = 0
\]

\[
\nabla = 0
\]

\[
\varepsilon = x \Delta T
\]

Strain without stress

Stress without strain
4.74. The electrical switch closes when the linkage rods CD and AB heat up, causing the rigid arm BDE both to translate and rotate until contact is made at E. Originally, BDE is vertical, and the temperature is 20°C. If AB is made of bronze CS6100 and CD is made of aluminum 6061-T6, determine the gap $s$ required so that the switch will close when the temperature becomes 110°C.

\[
\begin{align*}
S &= S_{Br} + (S_{Al} - S_{Br}) (15) \\
&= 0.5 S_{Br} + 1.5 S_{Al} \\
&= 0.5 \alpha_{Br} \Delta T \ L + 1.5 \alpha_{Al} \Delta T \ L \\
&= (0.5 \alpha_{Br} + 1.5 \alpha_{Al}) \Delta T \ L \\
&= (-0.5 \times 17 \times 10^{-6} + 15 \times 24 \times 10^{-6}) (90) (300) \\
&= 0.7425 \text{ mm}
\end{align*}
\]
4-91. The steel bolt has a diameter of 7 mm and fits through an aluminum sleeve as shown. The sleeve has an inner diameter of 8 mm and an outer diameter of 10 mm. The nut at A is adjusted so that it just presses up against the sleeve. If the assembly is originally at a temperature of \( T_1 = 20^\circ C \) and then is heated to a temperature of \( T_2 = 100^\circ C \), determine the average normal stress in the bolt and the sleeve. \( E_s = 200 \text{ GPa} \), \( E_a = 70 \text{ GPa} \). \( \alpha_s = 14 \times (10^{-6})/^\circ C, \alpha_a = 23 \times (10^{-6})/^\circ C. \)

**Prob. 4-91**

\[
\Delta T = 80^\circ C
\]

\[
A_d = 28.3 \text{ mm}^2
\]

\[
A_s = 38.5 \text{ mm}^2
\]

\[
E_d = 7 \times 10^3 \text{ MPa}
\]

\[
E_s = 7 \times 10^3 \text{ MPa}
\]

\[
\Sigma F_x = 0 \quad N_{st} + N_{sd} = 0 \quad \Rightarrow \quad N_{st} = -N_{sd} = N
\]

As \( \alpha_s > \alpha_{sd} \), then aluminum will not elongate freely as it will without presence of steel. Therefore aluminum will be in compression and steel will be in tension.

**Compatibility:**

\[
\frac{N_{sd}}{E_d A_d} + \frac{N_{st}}{E_s A_s} = \frac{N_{st}}{E_s A_s} + \frac{N_{sd}}{E_d A_d} \quad \Delta T \quad (N_{sd} = -N)
\]

\[
\frac{N}{E_d A_d} - \frac{N}{E_s A_s} = \Delta T \left( \alpha_{sd} - \alpha_s \right) \quad \Rightarrow \quad N = \frac{\Delta T \left( \alpha_{sd} - \alpha_s \right)}{\left( \frac{1}{E_d A_d} + \frac{1}{E_s A_s} \right)}
\]

Substituting \( \Rightarrow \quad N = 1134.4 \text{ N} \)

**Stress:**

\[
\sigma_{sd} = \frac{N_{sd}}{A_d} = \frac{1134.4}{\pi \left( \frac{8}{2} \right)^2/4} = -29.5 \text{ MPa}
\]

\[
\sigma_{st} = \frac{N_{st}}{A_s} = \frac{-1134.4}{\pi \left( \frac{10}{2} \right)^2/4} = 40.1 \text{ MPa}
\]