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Assigning Cardinal Weights in Multi-Criteria Decision Making Based on Ordinal Ranking

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ABSTRACT
A methodology is proposed to convert ordinal ranking of a number of criteria into numerical weights. Specifically, a simple mathematical expression is developed to provide the weight for each criterion as function of its rank and the total number of criteria. The proposed methodology is empirically developed, evaluated, and validated based on a set of experiments involving students and faculty at the authors’ university. Given only criteria ranks provided by a decision maker, this methodology can be used to determine relative weights for any set of criteria. Copyright © 2008 John Wiley & Sons, Ltd.

KEY WORDS: multiple-criteria decision making; decision analysis; ordinal ranking; criteria weights

1. INTRODUCTION
The relative importance of relevant criteria is a concept which is central to multiple-criteria decision making (MCDM) methods and is captured in many methods by some form of numerical weight parameter. However, it is recognized that the assessment of weights is a cognitively demanding task (Larichev, 1992) and in some circumstances it may be more appropriate to ask decision makers to provide information in the order of criteria weights, rather than a direct assessment of their numerical value.

This paper addresses the following important and practical decision problem: if only the criteria ranks are supplied by the decision maker, how do we determine relative criteria weights? A methodology is proposed to convert the ordinal ranking of a number of criteria into numerical weights. The relative criteria weights are determined based on the assumption that a universal weight–rank functional relationship exists between criteria ranks and average weight values. Empirical evidence from the literature supports this assumption. In this paper, a set of experiments is used to empirically determine this universal functional relationship which is applicable in all similar multiple-criteria decision problems. The results are validated by conducting another set of experiments which confirm the consistency of our functional relationship.

There are several schools of thought on modeling human mental processes, especially on the association between criteria ranks and weights. Therefore, we acknowledge that there are different opinions on the existence of a general rank–weight model. However, our assumption of a universal functional relationship between criteria ranks and the associated average weight values is based on several facts. First, this relationship has been empirically determined and validated, and shown to be consistent across different decision makers and different decision problems. Second, our rank–weight relationship is also consistent with the empirical findings of other researchers such as Doyle et al. (1997), Bottomley et al. (2000), and Bottomley and Doyle (2001). Third, different rank–weight functions have been proposed by many other authors, such as Stillwell et al. (1981), Solymosi and Dompi (1985), Barron (1992), and Lootsma (1999). Finally, criteria rank–weight models have been widely accepted (referenced and applied) in the literature. This literature is too extensive to completely enumerate in this paper, but a sample of 17 recent publications is surveyed in Section 2.3.
Several additional points could be raised in the above discussion on the relationship between ranks and weights. First, it is conceptually true that an infinite set of weights, described by Paelinck's (1975) theorem, exists for any specific ordinal ranking of criteria. However, only a particular form of these weights is justified on the basis of theoretical and empirical evidence. For example, Doyle et al. (1997), Bottomley et al. (2000) and Bottomley and Doyle (2001) find the relationship between ranks and average weights to be linear and valid for a broad range of contexts. Second, a general model is aimed to represent the actual and consistent average rank–weight relationship of a typical large group of people; it does not exclude individual differences among decision makers. Third, given only the criteria ranks by an individual decision maker, the average values obtained from this functional relationship are our best estimates of the relative criteria weights for this individual.

The specific objective of this paper is to calculate the relative weights for all the criteria based only on their provided ranks, thus enabling the conversion of an individually specified ranking of a set of criteria to best estimates of associated numeric values. The methodology is empirically developed through real-life experiments that involve (1) the factors that hinder students' learning, and (2) the factors that affect instructor evaluation. Each experiment consists of a two-part survey of university students and faculty. First, participants were asked to list the relevant factors in the order of importance, and then they were asked to give a numerical weight for each factor (the highest weight must be given to the first-ranked factor).

In the second part of the survey, participants were supplied with a list of 'standard' factors, to which they were asked to assign ranks and weights as before. The analysis in this paper is based on regression and statistical analysis methods in order to combine individual weights. The method that best fits the experimental data and minimizes the errors is recommended for general use in assigning weights to any set of ranked criteria. In order to validate the proposed methodology, a second set of experiments involving another sample of students and a different set of criteria was subsequently conducted.

Subsequent sections of this paper are organized as follows. Relevant literature is reviewed in Section 2. Problem definition and experimental design are introduced in Sections 3 and 4, respectively. Conversion of ranks into weights is discussed in Section 5. Finally, conclusions and comments are given in Section 6.

2. LITERATURE REVIEW

There are many different MCDM methods that use weights to describe the relative importance of different criteria. Lootsma (1999) and Belton and Stewart (2001) provide comprehensive and recent overviews of MCDM approaches. In this section, we specifically focus on the following MCDM aspects: (a) weight elicitation procedures, (b) using criteria ranks to rate alternatives, and (c) inferring criteria weights from ranks.

2.1. Weight elicitation procedures

Well-known traditional methods for determining criteria weights include the tradeoff method and the pricing-out method (Keeney and Raiffa, 1976), the ratio method and the swing method (Von Winterfeldt and Edwards, 1986), conjoint methods (Green and Srinivasan, 1990), and the analytic hierarchy process (AHP) (Saaty, 1994). Borcherding et al. (1991) compare the tradeoff, pricing out, ratio, and swing methods. More recent methods include habitual domains (Tzeng et al., 1998), multiobjective linear programming (Costa and Climaco, 1999), and linear programming (Mousseau et al., 2000).

Tzeng et al. (1998) classify weighting methods into objective or subjective, according to whether weights are indirectly computed from outcomes or directly obtained from decision makers. Weber and Borcherding (1993) classify weight-determining procedures according to whether they are statistical or algebraic, holistic or decomposed, and direct or indirect. According to Weber and Borcherding (1993), the concept of weight can be defined only in reference to one of the specific theories of preference. These theories may include AHP, ELECTRE-type methods (Roy, 1996), and multiattribute value theory (MAVT) (Dyer and Sarin, 1979). In the MAVT model, well-defined preferences are assumed to lead to consistent weights, regardless of the elicitation procedure.

2.2. Using criteria ranks as incomplete weight information

When cardinal criteria weights are not available, incomplete information (i.e. ordinal criteria ranks)
has been used to rate alternative decisions. Belton and Stewart (2001, Chapter 6) discuss several types of these approaches. Cook and Kress (1991) use criteria ranks as one of the three inputs to a data envelopment analysis model. Cook and Kress (1996) use an extreme-point approach, in which each alternative is allowed the maximum chance to have the highest possible rank. Recently, Cook and Kress (2002) extend their 1996 work by allowing criteria to be compared on either a ratio or an ordinal scale. Gomes et al. (1997) use ordinal ranking of both criteria and alternatives to determine the global weights of alternatives.

Marichal and Roubens (2000) propose a linear programming model to find the weights of interacting criteria, given a partial pre-order of both alternatives and criteria. Xu (2001) develops a procedure that uses the distance between the individual partial pre-orders of criteria to aggregate into a global ranking of alternatives. Salo and Punkka (2004) present a method called rank inclusion in criteria hierarchies, in which the decision maker ranks a given set of criteria. The decision maker's input defines a feasible set of criteria weights, which is used with dominance rules to recommend the appropriate decisions.

2.3. Inferring criteria weights from ranks

The set of weights that satisfy a particular criteria ranking is described by Paelinck's (1975) theorem whose proof is provided by Claessens et al. (1991). Specific functions for assigning weights \( w_r \) to \( n \) criteria with ranks \( r = 1, \ldots, n \), have been suggested by few authors. Stillwell et al. (1981) propose three functions: rank reciprocal (inverse), rank sum (linear), and rank exponent weights. Solymosi and Dompi (1985) and Barron (1992) propose rank order centroid weights. Lootsma (1999) and Lootsma and Bots (1999) suggest two types of geometric weights. All of these rank–weight functions will be further discussed in Section 5, except the exponent weights and Lootsma and Bots (1999) geometric weights because they require extra input in addition to criteria ranks.

Several studies, mentioned below, have compared the above rank–weight functions and found centroid weights to be superior in terms of accuracy and ease of use. Olson and Dorai (1992) compare centroid weights to AHP on a student job selection problem, concluding centroid weights provide almost the same accuracy while requiring much less input and mental effort from decision makers. Edwards and Barron (1994) extend SMART into SMARTER (SMART Exploiting Ranks) using centroid weights. Barron and Barrett (1996a) analyse the effectiveness of centroid weights in SMARTER. Srivastava et al. (1995) compare five weight elicitation methods, including rank sum and centroid weights, finding centroid weights to be modestly superior to other methods. On the basis of simulation experiments, Barron and Barrett (1996b) find centroid weights superior to rank sum and reciprocal (inverse) weights. Jia et al. (1998) also use simulation to compare centroid and rank sum weights with equal weighting and ratio weights, again favouring centroid weights. Noh and Lee (2003) find that the simplicity and ease of use of centroid weights make it a practical method for determining criteria weights compared with AHP and fuzzy method.

In a series of three papers, Bottomley, Doyle, and Green report on empirical experiments conducted to compare different weight elicitation procedures. Doyle et al. (1997) and Bottomley et al. (2000) compare two methods of assigning numerical values to weights: direct rating (DR), in which people directly rate each criterion, and point allocation (PA), in which a total budget of 100 points is divided among criteria. The empirical results indicate the following: (1) people prefer DR, (2) DR gives more consistent and reliable weights, and (3) the rank–weight relationship is basically linear. In addition, Doyle et al. (1997) conduct a series of experiments that reveal a theoretical straight-line relationship between rank and average weight. In the empirical experiments of Doyle et al. (1997) with 6, 9, and 12 criteria, the slope of the linear model depends on the number of criteria being ranked. Bottomley and Doyle (2001) compare DR with two other weight elicitation procedures: Max100, in which the most important criterion is first given a weight of 100, and Min10, in which the least important criterion is first given a weight of 10. Max100 exhibits the highest reliability, rank–weight linearity, and subject preference.

Although Bottomley, Doyle, and Green empirically confirm the linear rank–weight relationship, they do not propose a generally applicable specific function. This paper is the first to empirically develop such a rank–weight function, and also to empirically aggregate rank inputs from several decision makers into group weights. As will be presented later, our experiments use the Max100 procedure, and produce an explicit linear function.
which can be used to relate the ranks and weights for any number of criteria.

3. PROBLEM DEFINITION

This paper considers a deterministic MCDM problem with $m$ alternatives and $n$ decision criteria. Weights reflect the relative importance of each decision criterion, and are usually normalized by making their sum equal to 1 ($\sum_{j=1}^{n} w_j = 1$). Given the specific performance value $a_{i,j,k}$ of each alternative $k$ $(k = 1, 2, ..., m)$ in terms of each criterion $j$ $(j = 1, 2, ..., n)$, the overall performance of each alternative $k$ can be calculated as follows:

$$P_k = \sum_{j=1}^{n} w_j a_{i,j,k}, \quad k = 1, 2, ..., m \tag{1}$$

We assume that input can be obtained from several individuals, where each individual $i$ may list and rank only a subset of the criteria. Weights reflect the relative importance of each alternative criterion, and are usually normalized by making their sum equal to 1 ($\sum_{j=1}^{n} w_j = 1$). Given the specific performance value $a_{i,j,k}$ of each alternative $k$ $(k = 1, 2, ..., m)$ in terms of each criterion $j$ $(j = 1, 2, ..., n)$, the overall performance of each alternative $k$ can be calculated as follows:

$$P_k = \sum_{j=1}^{n} w_j a_{i,j,k}, \quad k = 1, 2, ..., m \tag{1}$$

4. EXPERIMENT DESIGN AND DATA COLLECTION

The methodology suggested in this paper is empirical in nature; a set of experiments has been conducted in order to develop and evaluate the methodology. These experiments were conducted within the MAVT framework, which assumes that decision maker preferences correspond to consistent weights under any experimental set-up. The experimental design aims to test whether rank-weight relationships change according to the particular set of criteria or the particular group of decision makers. Therefore, the experiment involves two groups of participants (students and faculty) and two sets of criteria (factors that hinder students’ learning, and factors affecting course instructor evaluation). The determination of these factors and their weights will help the university in designing ways to improve students’ learning. However, the main objective of the experiments is to develop a general methodology to convert ordinal data into cardinal weights for any set of criteria. The experiment consisted of a survey distributed among a sample of university students and faculty, and conducted in two consecutive parts as follows:

(I) In the first part of the survey, the participants were asked to answer two questions: Question 1. List the factors that hinder students learning and retaining course materials. Question 2. List the factors that affect the evaluation of course instructors.

For each list, the participants were asked to arrange the factors in order of priority (most important to least important). After they listed them, they were required to give weights to all factors in each prioritized list, starting with a weight of 100% for the most important (first) factor. This is the Max100 weight elicitation method suggested by Bottomley and Doyle (2001).

(II) In the second part of the survey, the participants were provided with two ready-made lists of ‘standard’ criteria: 12 factors hindering students’ learning (Question 1), and 16 factors affecting instructor evaluation (Question 2). The participants were asked to rank each set of factors based on their importance from 1 (most important) to 12 or 16 (least important). After ranking the factors in each list, they were requested to assign weights to each one, starting with a weight of 100% for the most important (rank 1) factor. This part of the survey was administered only after finishing the first part. The reason for this is to avoid influencing participants during Part I by the factors listed in the Part II.

This survey was distributed among a sample of university students and faculty members. The student sample was composed of six university classes given in different academic departments, and the students were in different fields of study. From the six classes 111 students completed the
survey. On the other hand, it was difficult to get a large number of faculty members to participate in survey because of their busy schedules. As a result, only 23 faculty members participated in our experiments.

5. ASSIGNING CRITERIA WEIGHTS BASED ON ORDINAL RANKS

The main objective of this paper is to empirically develop, on the basis of the data obtained from the experiments, a general methodology for assigning average criteria weights based on criteria ranks. In order to achieve this objective, the following steps had to be taken.

5.1. Step 1

First, we separated the data obtained from the survey into four categories based on the two sets of criteria (two questions) and the two sets of decision makers (students and faculty). We then separated each category into distinct groups according to the number of criteria, n, given by each participant in the Part I of the survey. Since the number of criteria in Part II of the survey is 12 for Question 1 and 16 for Question 2, the groups corresponding to $n = 12$ or $n = 16$ included responses to both parts of the survey. In reality, the number of responses in the range $n > 12$ was almost zero and therefore this range was excluded from further analysis.

5.2. Step 2

For each of the four categories and each value of $n$, $n = 1, \ldots, 12$, we calculated the average weight for each rank. As an example, the average weights of each rank for the four categories are shown in Table I for the case $n = 3$.

5.3. Step 3

For each value of $n$, the Wilcoxon signed-rank test for paired observations was used to test whether the differences between the two sets of criteria (two questions) are significant. The same test was used on the differences between the two sets of decision makers (students and faculty). At significance level $\alpha = 0.05$, the effects of both the different criteria and the different decision makers on the weights were found to be insignificant. Therefore, all inputs from the four categories (the two questions and the two sets of participants) were combined for each value of $n$, producing the average weights for each rank shown in Table II.

5.4. Step 4

Five different models were applied to estimate the weight of each rank for each value of $n$. All these models are based on the assumption of an average

<table>
<thead>
<tr>
<th>Rank $r$</th>
<th>Q1. Students</th>
<th>Q2. Students</th>
<th>Q1. Faculty</th>
<th>Q2. Faculty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>78.89</td>
<td>85</td>
<td>85</td>
<td>95</td>
</tr>
<tr>
<td>3</td>
<td>65.74</td>
<td>65.32</td>
<td>72.5</td>
<td>67.5</td>
</tr>
</tbody>
</table>

Table I. Average actual weights of each rank for each category for $n = 3$

<table>
<thead>
<tr>
<th>$n$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>79.26</td>
<td>81.94</td>
<td>86.42</td>
<td>92.85</td>
<td>93.39</td>
<td>90</td>
<td>93.75</td>
<td>90</td>
<td>99</td>
<td>87.56</td>
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<tr>
<td>3</td>
<td>65.53</td>
<td>72.3</td>
<td>79.35</td>
<td>85.56</td>
<td>90</td>
<td>84.38</td>
<td>88</td>
<td>80</td>
<td>84.94</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>58.55</td>
<td>67.36</td>
<td>71.67</td>
<td>80</td>
<td>76.25</td>
<td>85</td>
<td>75</td>
<td>85</td>
<td>71.64</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>58</td>
<td>62.5</td>
<td>70</td>
<td>72.63</td>
<td>85</td>
<td>75</td>
<td>71.64</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>54.44</td>
<td>50</td>
<td>63.88</td>
<td>70</td>
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<tr>
<td>7</td>
<td>40</td>
<td>47.63</td>
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<td>8</td>
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<tr>
<td>10</td>
<td>30</td>
<td>45</td>
<td>48.93</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>11</td>
<td>40</td>
<td>41.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>35.09</td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

Table II. Actual average weights for each rank according to the number of criteria $n$
association between rank and weight which is consistent across many decision makers and many decision contexts. After plotting the data in Table II, we proposed the first model given below. Our model assumes that the relationship between the weight and rank is linear, in which the slope is itself a function of the number of criteria \( n \). Stillwell et al. (1981) propose models 2 and 3, Solymosi and Dompi (1985) and Barron (1992) propose model 4, whereas Lootsma (1999) proposes model 5. According to our survey instructions, the weight assigned by participants to the first rank must be 100%. Therefore, all models were adjusted for our data by making \( w_1 = 100 \). The five models used to calculate the weight of each rank are:

1. Linear weights with variable slope:
   \[ w_r = 100 - s_n(r - 1) \]
   where \( w \) is the weight, \( r \) is the rank, and \( s_n \) is the absolute value of the slope when the number of criteria (ranks) is equal to \( n \). \( s_n \) is obtained by least-squares linear regression.

2. Rank sum linear weights with fixed slope:
   \[ w_r = 100(n + 1 - r)/n \]

3. Inverse or reciprocal weights:
   \[ w_r = 100/r \]

4. Centroid weights:
   \[ w_r = 100 \sum_{i=1}^{n} i/\sum_{i=1}^{n} i^2 \]

5. Geometric weights:
   \[ w_r = 100/(\sqrt{2})^{r-1} \]

As an example, actual average weights and weights calculated by all models for the case \( n = 5 \) are shown in Figure 1.

**5.5. Step 5**

In order to compare how closely each model approximates actual weights, the mean absolute percentage errors (MAPE) were calculated for all five models and all values of \( n \). MAPE values are calculated by comparing the actual average weights with the theoretical weights derived from each model. Since \( w_1 = 100 \), only \( n - 1 \) points are used in this comparison. The results shown in Table III clearly show that model 1 consistently outperforms all other models. Therefore, model 1 is chosen to represent the relationship between the rank and the weight as a straight line whose negative slope \( (-s_n) \) varies according to the number of criteria \( n \). The inverse but linear relationship between rank and weight seems to be quite natural to people, as illustrated by the familiar relationship between course letter grades and corresponding grade points (\( A = 4, B = 3 \), and so on). Moreover, this linear relationship is confirmed by Doyle et al. (1997), Bottomley et al. (2000), and Bottomley and Doyle (2001). Table IV shows the slopes calculated for model 1 as a function of \( n \) using least-squares regression.

**5.6. Step 6**

In order to determine the relationship in model 1 between the slope \( (-s_n) \) and the number of criteria \( n \), we plotted the values of \( s_n \) versus \( n \) as displayed in Figure 2. Figure 2 shows a decreasing nonlinear curve with a clear pattern that suggested three possible models to estimate absolute slope \( s_n \) as a function of the number of criteria \( n \):

(I.) Quadratic model: \( s_n = a + bn + cn^2 \).

(II.) Inverse model: \( s_n = a + b/n \).

(III.) Exponential model: \( s_n = a e^{bn} \).
After applying least-squares regression to these three models, we obtained the parameter values shown below. We also calculated corresponding MAPE values to decide which model gives us the best fit for the slope.

**Model I**

\[ s_n = 27.81545 - 4.21143n + 0.203177n^2 \]

MAPE = 7.496247

**Model II**

\[ s_n = 3.19514 + \frac{37.75756}{n} \]

MAPE = 6.2862

**Model III**

\[ s_n = 22.596544 + e^{-0.12291n} \]

MAPE = 9.011

Based on error values, Model II is identified as the best method of estimating slope values. The fitted slopes for this model are compared with the actual slope in Figure 2. Therefore, for any set of \( n \) ranked factors, assuming a weight of 100% for the first-ranked (most important) factor, the percentage weight of a factor ranked as \( r \) is given by

\[
\frac{w_{r,n}}{100} = 100 - s_n(r - 1)
\]

or

\[
w_{r,n} = 100 - \left( 3.19514 + \frac{37.75756}{n} \right)(r - 1)
\]

\( 1 \leq r \leq n, \quad 1 \leq n \leq 21, \quad n \) and \( n \) are integer

It should be obvious that the upper limit of 21 on \( n \) is sufficient for all practical purposes. First, real-life multi-criteria decision problems do not have so many relevant criteria. Second, no decision maker has the patience to assign individual ranks to so many different criteria.

### 5.7. Step 7

Equation (2) represents our proposed universal functional relationship between criteria ranks and weights. In order to validate this relationship, a second set of empirical experiments has been conducted. The additional experiments involved 59 new students, which were given three questions relating to the requirements for the optimal design of a numerical analysis course. The first question listed seven faculty requirements, the second six textbook requirements, and the third six grading policy requirements. Students were required to give the proper weightage to each criterion (requirement) on a scale from 1 to 9, with 1 = least important and 9 = most important.

Although the students did not explicitly supply criteria ranks, the ranks are implied in the descending order of the weights. Since both questions 2 and 3 involved 6 criteria (\( n = 6 \)), their data were combined together. For the two new sets of data, the average weight was determined for each rank, and then normalized to correspond with \( w_{1,n} = 100 \). For the two new sets of data, the normalized actual weights were compared with the theoretical values obtained from models 1 through 5. The results, summarized in Table V, confirm the overwhelming superiority of our model I specified by Equation (2) over other models. Not only are the errors much lower for model 1, but also the theoretical slope estimates are amazingly close to the actual values obtained from the new sets of data.

### 6. CONCLUSIONS

This paper presented an empirical methodology to determine cardinal criteria weights on the basis of

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Table IV. Model 1 (linear model) slope and coefficient of determination for each value of \( n \)

<table>
<thead>
<tr>
<th>No. of criteria ( n )</th>
<th>2</th>
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DOI: 10.1002/mcda
only individual ordinal ranking of these criteria. In order to collect data necessary for developing the methodology, experiments were conducted involving university students and faculty. For each individual decision maker, a model has been proposed in which the weight of each criterion is a function of both its rank and the total number of criteria. This linear model, defined by (2), outperforms other functional forms reported in the literature. It is also compatible with the empirical findings of Doyle et al. (1997), Bottomley et al. (2000) and Bottomley and Doyle (2001). As rank–weights have been shown to be consistent across different sets of criteria and different groups of decision makers, this model can be used to assign weights to any set of ranked criteria.

A natural extension of this work is to develop aggregation methods for group decision making, i.e. combining criteria rankings obtained from several individuals into overall group criteria weights. Other possible extensions include considering partial or incomplete (fuzzy) ordinal rankings, and group decision making in which the inputs of different individuals are not equal (i.e. weighted voting).

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The authors would like to express gratitude to King Fahd University of Petroleum and Minerals for supporting this research effort, and also to Mr. Saleh Al-Duwais for assistance in data collection and analysis. Thanks are also due to the editor and two anonymous referees for helpful suggestions that significantly improved this paper.

REFERENCES


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