

# 1.8 Absolute Value Equations

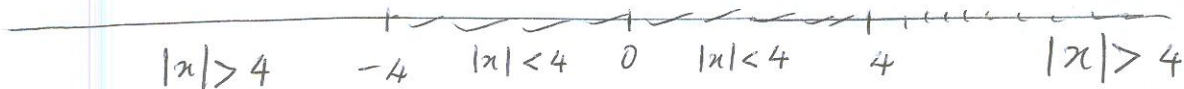
(1.8 p.1)

## & Inequalities

Objectives: To learn how to solve

- Equations with absolute value(s).
- Inequality with absolute value(s).

$$|x| = |x - 0| = d(x, 0)$$



So

If  $E(x)$  is an expression &  $k \geq 0$ . Then

- 1)  $|E(x)| < k \iff -k < E(x) < k$
- 2)  $|E(x)| > k \implies E(x) < -k \text{ or } E(x) > k$
- 3)  $|E(x)| = k \implies E(x) = -k \text{ or } E(x) = k$

Exp 1 - Solve

a)  $|x + 2| = 7$

b)  $\left| \frac{6x+1}{x-1} \right| = 3$

c)  $|2x - 3| = |5x + 4|$

a)  $x + 2 = \pm 7 \iff x + 2 = -7 \text{ or } x + 2 = 7$   
 $\boxed{-x = -9} \quad \boxed{x = 5}$

c)  $\frac{6x+1}{x-1} = \pm 3 \iff$

$$6x + 1 = 3(x - 1) \\ = 3x - 3$$

$$\& \quad 6x + 1 = -3(x - 1) \\ = -3x + 3$$

$$6x - 3x = -4 \quad \text{or} \quad 6x + 3x = 3 - 1$$

$$3x = -4$$

$$x = -\frac{4}{3}$$

$$9x = 2$$

$$x = \frac{2}{9}$$

Check. that  $x = -\frac{4}{3}$  &  $x = \frac{2}{9}$  belong to the domain

$$6) \quad |2x - 3| = |5x + 4|$$

$$(2x - 3) = 5x + 4 \quad \text{or} \quad 2x - 3 = -(5x + 4)$$

!

!

Check in original equation.

Inequalities

Exp 2. Solve

$$a) \quad |2x + 5| \geq 3$$

$$b) \quad \left| \frac{2}{3}x + \frac{1}{2} \right| \leq \frac{1}{6}$$

$$a) \quad |2x + 5| \geq 3 \quad \Leftrightarrow$$

$$2x + 5 \geq 3$$

$$2x \geq 3 - 5 = -2$$

$$x \geq -1$$

$$[-1, \infty)$$

$$\text{or} \quad 2x + 5 \leq -3$$

$$2x \leq -3 - 5$$

$$x \leq -8$$

$$(-\infty, -8]$$

$$SS = (-\infty, -8] \cup [-1, \infty)$$

b)  $|\frac{2}{3}x + \frac{1}{2}| \leq \frac{1}{6}$

x 6

$6|\frac{2}{3}x + \frac{1}{2}| \leq 6(\frac{1}{6})$

$|4x + 3| \leq 1$

(=)

$-1 \leq 4x + 3 \leq 1$

$-4 \leq 4x \leq -2$

$-1 \leq x \leq \frac{1}{2}$

SS =  $[-1, \frac{1}{2}]$

Special Cases:

Case  $k \leq 0$  in  $|E(x)| \leq k$  or  $|E(x)| \geq k$

Exp 3. Solve

a)  $|2x - 3| < 0$       b)  $|2x - 3| \leq 0$  ,

c)  $|2x - 3| > 0$       d)  $|2x - 3| \geq 0$

e)  $|2x - 3| < -2$       f)  $|2x - 3| \leq -2$

g)  $|2x - 3| \geq -2$       h)  $|2x - 3| \geq -2$

a) No sol      b)  $|2x - 3| \leq 0 \Leftrightarrow 2x - 3 \geq 0 \Leftrightarrow x \geq \frac{3}{2}$

d)  $|2x - 3| \geq 0 \Leftrightarrow x \in \mathbb{R}$  , c)  $|2x - 3| > 0$

e)  $\emptyset$       f)  $\emptyset$       g)  $\mathbb{R}$  ,      h)  $\mathbb{R}$

