— R4. FACTORING. —

- ☐ Factoring of numbers
- ☐ Factoring of polynomials.
- ☐ Methods of Factoring.
  - ◇ GCF.
  - ◇ Special Products.
  - ◇ Trial & Error.
  - ◇ Grouping.

<u>Factoring a natural nbr</u> is writing it as a product of two or more natural nbrs.

Eg. $15 = 3.5$

$12 = \underbrace{2.2.3}_{\text{factors}}$

A nbr that cannot be factored is called prime.

Eg. 7 , 13.

Factoring of polynomials

Factoring of a polynomial $p(x)$ is writing it as a product of 2 or more polynomials.

$$2x^2 - x = x(2x - 1)$$

$$x^2 - 1 = (x-1)(x+1)$$

A factoring is complete if it cannot factored anymore.

A factoring is over the integers if all its factors have integer coefficients.

A factoring is over $\mathbb{R}$ if all factors have real coefficients.

Eg.

i) $x^2 - 4 = (x-2)(x+2)$ factoring over $\mathbb{Z}$ & $\mathbb{R}$.

ii) $x^2 - 2 = \left(x - (\sqrt{2})^2\right) = (x - \sqrt{2})(x + \sqrt{2})$

factoring over $\mathbb{R}$ but not a factoring over $\mathbb{Z}$

In this course, we are interested only in factoring over the integers.

A polynomial is prime if it cannot be factored into two polynomials of lower degree.

Eg. $x+2$ prime

$x^2 - 4$ not prime $\left(x^2 - 4 = (x-2)(x+2)\right)$

$x^2 - 2$ prime

$x^2 + 1$ prime

Techniques of Factoring.

1. Factoring the Greatest Common Factor (GCF)

Exp 1.

a) $24\,x^2y^3 + 36\,x^3y$

$= 2^3\cdot 3\,x^2y^3 + 2^2 3^2\,x^3y$

$GCF = 2^2\cdot 3\,x^2y$

$= 2^2\cdot 3\,(2y^2 + 3x)$

$$\begin{array}{r|l} 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

b)

2. Using Special Formulas

$a^2 - b^2 = (a-b)(a+b)$ (difference of squares)

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ (difference of 2 cubes)

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ (Sum of 2 cubes)

Perfect Squares

$a^2 + 2ab + b^2 = (a+b)^2$

$a^2 - 2ab + b^2 = (a-b)^2$

R4p4

Exp 2. Factor

i) $25x^2 - 49y^4$

ii) $81x^4 - y^4$

$$(9x^2 - y^2)(9x^2 + y^2) = (3x - y)(3x + y)(9x^2 + y^2)$$

iii) $16x^4 - (y - 2z)^2$

iv) $27a^3 + 64b^3$

$$(3a)^3 + (4b)^3 = (3a + 4b)\left((3a)^2 - (3a)(4b) + (4b)^2\right)$$

$$= (3a + 4b)(9a^2 - 12ab + 16b^2)$$

v) $8c^6 - 27d^9 = (2c^2)^3 - (3d^3)^3$

$$= (2c^2 - 3d^3)\left((2c^2)^2 + (2c^2)(3d^3) + (3d^3)^2\right)$$

$$= (2c^2 - 3d^3)(4c^4 + 6c^2d^3 + 9d^6)$$

IV) $36x^2y^2 + 84xy + 49$

R4p5

3) Trial & Error, (for trinomial of deg 2.)

a) Leading Coefficient 1,

$x^2 + 3x - 4$ $\qquad r_1 + r_2 = 3 \qquad r_1 r_2 = -4$

$(x + r_1)(x + r_2)$

$(x - 2)(x + 2)$ ×

$(x - 4)(x + 1)$ ×

$(x + 4)(x - 1)$ ✓

b) Leading coeff $a \neq 1$

i) $6x^2 - 7x - 3$

$(3x + 1)(2x - 3)$ ✓

ii) $4y^2 - 11y + 6$

Exp. of a prime polynomial.

$x^2 + 3x + 5$

$(x + 5)(x + 1)$ ×

$(x - 5)(x - 1)$ ×

$\Rightarrow x^2 + 3x + 5$ prime.

Grouping.

a) $4x^3 + 2x^2 - 2x - 1$

$(4x^3 + 2x^2) + (-2x - 1)$

$2x^2(2x+1) - (2x+1) = (2x+1)(2x^2-1)$

b) $y^2 - x^2 + 6x - 9$

$y^2 - (x^2 - 6x + 9) = y^2 - (x-3)^2$

$(y - (x-3))(y + (x-3)) = \boxed{(y - x + 3)(y + x - 3)}$

c) $x^2 - 6x + 9 - y^4$

Factoring by Subtitution.

i) $6x^4 - 13x^2 - 5$

ii) $6(4z-3)^2 + 7(4z-3) - 3$

Factoring over $\mathbb{Q}$ $\qquad \frac{4}{25}x^2 - 49y^2$

Factoring over $\mathbb{R}$ $\qquad x^2 - 7$