R3. Polynomials

Objectives

- ☐ Rules of exponents
- ☐ Def. of polynomials
- ☐ Addition & Subtraction
- ☐ Multiplication
- ☐ Division

Rules of Exponents: $a \in \mathbb{R}$, $m, n \in \mathbb{N}$, $b \in \mathbb{R}$.

1. $a^m \cdot a^n = a^{m+n}$

2. $(a^m)^n = a^{mn}$

3. $(ab)^m = a^m b^m$

4. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$

Convention. $b \neq 0$, $b^0 = 1$

But 0^0 is undefined

Example 1:

Simplify $b \neq 0$, $m-$

a) $\left(\frac{2^5}{b^4}\right)^3$

b) $\left(\frac{-2m^6}{t^2 z}\right)^5$

Algebraic Expressions

$3x^2 - 2x + 1$, $\sqrt{m^2 - 4}$, $\frac{2x}{x^2 + 1}$

Monomial. ax^n , $a \in \mathbb{R}$, x: variable, $n \in \mathbb{N}$

$2x^3$, $-3x^4$

A Polynomial in one variable is a sum of monomials.

$7x^4 - 3x^3 + 4x + 3$,

7, −3, 4, 3 Coefficients

The degree of a monomial is the exponent of the variable.

The degree of a polynomial is the greatest degree of any monomial.

A polynomial of 2 terms is a binomial.

" " " 3 terms is a trinomial.

Examples

$2x^3 - \sqrt{2}x + 1$ poly.

4 poly of degree 0

$x^3 + 2x^{-1} + 2$ not poly

$\sqrt{x} + 4x^5$ not poly

Polynomials of several variables.

$3x^2y^3$ monomial of deg $= 2+3 = 7$

$p(x,y) = 4x^4y^5 - 2x^2y^2 - 4x^2y + 5$

Leading term: is term with highest degree.

Leading coefficient: is the coefficient of of the leading term.

A polynomial is in Standard Form or descending powers is the same polynomial put in decreasing powers.

Eg.

$3x^4 - 3 + 4x^3 - 6x^6 = -6x^6 + 3x^4 + 4x^3 - 3$

Standard Form

$-6x^6$: leading coeff

-6 : leading coeff

Addition & Subtraction.

Exp Perform the addition

a) $(3x^3 - 2x^2 - 6) + (4x^2 - 6x - 7)$

b) $4(x^2 - 3x + 7) - 5(2x^2 - 8x - 4)$

Multiplication.

Exp. Simplify a) $(3x-4)(2x^2-5x+1)$

b) $r^2(3r+2)(2r-3)$

FOIL Method.

$$(a+b)(c+d) = ac + ad + bc + bd$$

F First — O Outer — I Inner — L Last

Exp. $(6m+1)(4m-3)$

Special Products.

$$(x+y)(x-y) = x^2 - y^2$$

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

Exp. Find the products

$(3p+11)(3p-11)$

$(5m^3-3)(5m^3+3)$

c) $(2m+5)^2$

d) $(3x-7y^4)^2$

Exp. Find the products

$$[(3x-1)+2y]\cdot[(3x-1)-2y]$$

Division of Polynomials.

Divide $-5x^2-8x+x^4+3$ by $x-3$

First arrange in standard form

$$\begin{array}{r} x^3+3x^2+4x+4 \\ x-3 \;\overline{)\; x^4+0x^3-5x^2-8x+3} \\ \underline{x^4-3x^3} \\ 3x^3-5x^2 \\ \underline{3x^3-9x^2} \\ 4x^2-8x \\ \underline{4x^2-12x} \\ 4x+3 \\ \underline{4x-12} \\ 15 \end{array}$$

$$\frac{-5x^2-8x+x^4+3}{x-3} = (x^3+3x^2+4x+4)+\frac{15}{x-3}$$

R3 p6

$$\frac{x^4 + 5x^2 + 5x + 27 - 2x^3}{x^2 + 3}$$

$$x^2 - 2x + 2 \longrightarrow \text{quotient}$$

$$x^2 + 3 \;\big)\; x^4 - 2x^3 + 5x^2 + 5x + 27 \longrightarrow$$

divisor

$$x^4 \qquad + 3x^2$$

$$-2x^3 + 2x^2 + 5x$$

$$-2x^3 \qquad - 6x$$

$$2x^2 + 11x + 27$$

$$2x^2 \qquad + 6$$

$$0 + \underbrace{11x + 21}_{\text{Remainder}}$$