— 3.2. Synthetic Division —

Objectives

- We present a method which allows to divide a polynomial by $(x - c)$ quickly & efficiently.
- We give a new method to compute the value $p(c)$.

We have seen in chapter R, the long division which allows to divide a polynomial $p(x)$ by a polynomial $q(x)$.

The synthetic Division is a quick method of division of a polynomial $p(x)$ by a polynomial $q(x)$ of the form $(x - c)$.

Example 1. Let's illustrate this method in dividing $p(x) = 2x^3 - 7x^2 + 5$ by $x - 3$

Sol^n. $(x - c) = (x - 3) \Rightarrow c = 3$

$$\begin{array}{r|rrrr} 3 & 2 & -7 & 0 & 5 \\ & & 6 & -3 & -9 \\ \hline & \underbrace{2 \quad -1 \quad -3}_{\text{coeff of quotient}} & & & \underbrace{-4}_{\text{remainder}} \end{array}$$

Don't forget the missing powers.

$\Rightarrow$ quotient : $2x^2 - x - 3$

Remainder: -4

So

$$\frac{2x^3-7x^2+5}{x-3} = 2x^2-x-3 + \frac{-4}{x-3}$$

or $2x^3-7x^2+5 = (x-3)(2x^2-x-3) - 4$

Exp. Use the synthetic division to divide

a) $p(x)=3x^5+5x^4-4x^3+7x+3$ by $(x+2)$

b) $p(x)=x^3-9x^2+27x-27$ by $(x-3)$

c) $\frac{x^3-27}{x-3}$ d) $\frac{x^4-16}{x-2}$

Sol^n a)

$$\begin{array}{r|rrrrr} -2 & 3 & 5 & -4 & 7 & 3 \\ & & -6 & 2 & 4 & -22 \\ \hline & \underbrace{3 \quad -1 \quad -2 \quad 11}_{x^3 \;\; x^2 \;\; x \;\; \text{const}} & & & & \underbrace{-19}_{\text{remainder}} \end{array}$$

$$\frac{p(x)}{x+2} = 3x^3-x^2-2x+11 + \frac{-19}{x+2}$$

or

$$\boxed{p(x) = (x+2)(3x^3-x^2-2x+11) - 19}$$

b)

$$\begin{array}{r|rrrr} 3 & 1 & 0 & 0 & -27 \\ & & 3 & 9 & 27 \\ \hline & 3 & 3 & 9 & 0 \\ & \downarrow & \downarrow & \downarrow & \\ & x^2 & x & \text{cons} & \end{array} \Rightarrow \frac{(x^3-27)}{x-3} = 3x^2+3x+9$$

Remainder Theorem.

The remainder of the division of $p(x)$ by $(x-c)$ is equal to $\boxed{p(c)}$.

Exp. Let $P(x) = 3x^5 + 5x^4 + 4x^3 - 7x + 9$

Use the remainder theorem, to find the value $P(-2)$.

Sol^n. $P(-2) = \text{Remainder}\left(\dfrac{P(x)}{x-(-2)} = \dfrac{P(x)}{x+2}\right)$

$$\begin{array}{r|rrrrr} -2 & 3 & 5 & +4 & -7 & 9 \\ & & -6 & 2 & -12 & 38 \\ \hline & 3 & -1 & 6 & -19 & \boxed{47} \end{array} \longrightarrow \text{remainder}$$

$$\Rightarrow \boxed{P(-2) = 47}$$

Exp. Find the remainder of the division of $p(x) = x^{101} - x^{96} + 1$ by $x - i$.

$$\text{Rem} = p(i) = i^{101} - i^{96} + 1 = i - i^0 + 1 = i - 1 + 1$$
$$= \boxed{i}$$

A <u>zero</u> of a polynomial $p(x)$ is a number c such that $p(c)=0$

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c is a zero of $p(x) \iff$ Remainder of $\frac{p(x)}{x-c} = 0$

<u>Exp.</u> Decide whether the given number k is a zero of $p(x)$

a) $p(x) = x^3 - 4x^2 + 9x - 6$ $\quad k = 1$

b) $p(x) = x^4 + x^2 - 3x + 1$ $\quad k = -1$

c) $p(x) = x^4 - 2x^3 + 4x^2 + 2x - 5$ $\quad k = 1+2i$

<u>Sol^n.</u>

$$\begin{array}{c|ccccc} 1+2i & 1 & -2 & 4 & 2 & -5 \\ & & 1+2i & -5 & -1-2i & 5 \\ \hline & & -1+2i & -1 & 1-2i & \boxed{0} \end{array}$$

Remainder

$(1+2i)(-1+2i) = (2i+1)(2i-1) = (2i)^2 - 1 = -4 - 1 = -5$

$(1+2i)(1-2i) = 1 + 4 = 5$

$\Rightarrow$ $1+2i$ is a zero of $p(x)$.