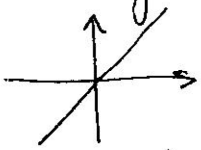
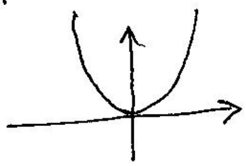
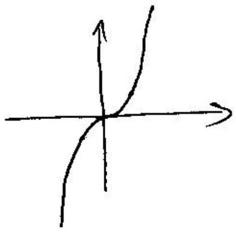
— 3.4. Polynomial Functions —

Graph of $f(x) = ax^n$

We know how to graph

1. $f(x) = x$ 

2. $f(x) = x^2$ 

3. $f(x) = x^3$ 

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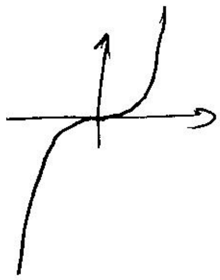
For n even, $f(x) = x^n$ has a graph 

For n : odd, $n > 1$, $f(x) = x^n$ is like 

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Exp 1. Graph

a) $f(x) = -x^5 + 2$ b) $g(x) = (x-1)^4$

c) $h(x) = -2(x+1)^5 + 3$

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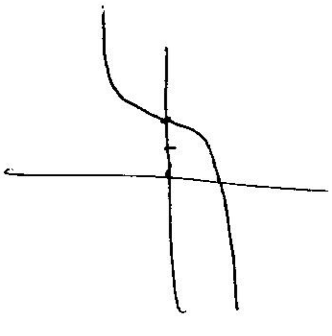
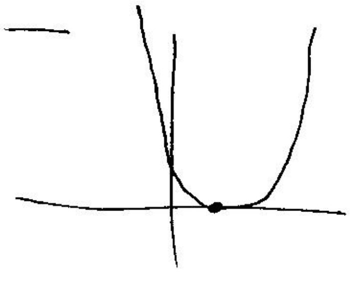
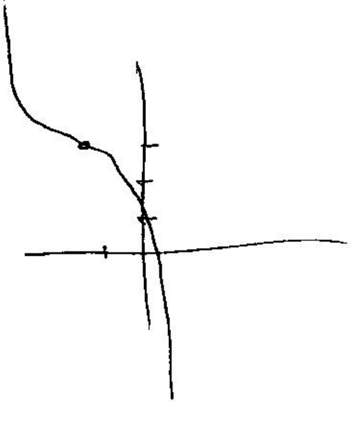
a) 

b) 



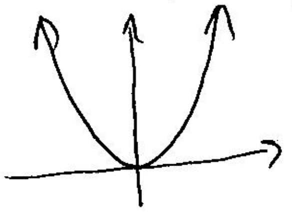
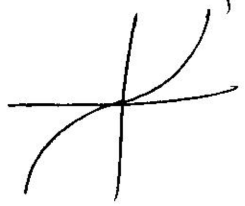
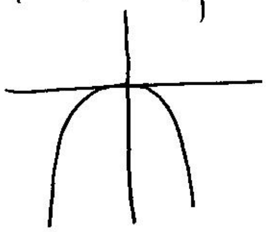
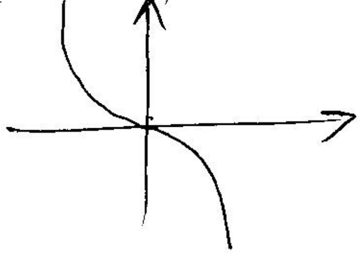
End Behaviour

We mean by this how the graph looks like on the far left ($x \to -\infty$) & on the far right ($x \to +\infty$

The graph can go up (i.e $f(x) \to +\infty$) or it can go down (ie. $f(x) \to -\infty$)

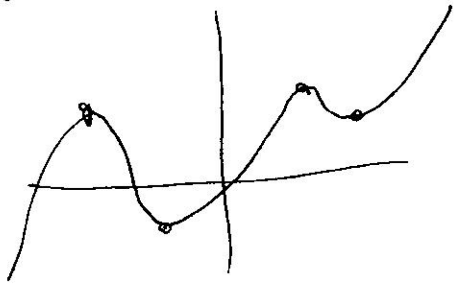
The graph of a poly. $f(x) = ax^n + a_n x^{n-1} + \dots + a_1 x + a_0$ behaves like its leading

Leading Coefficient Criterion.

a_n \ n	n: even	n odd
$a_n > 0$	up at the far left up at the far right Eg. 	down at the far left up at the far right Eg. 
$a_n < 0$	down at the far left down at the far right Eg 	up at the far right down at the far left 

Turning Points

A local maximum or a local minimum gives a turning point in the graph



A polynomial of degree n, has at most $(n-1)$ turning points

$$\#(t.p) \leq \deg(p(x)) - 1$$

or

$$\deg p(x) \geq \#(t.p) + 1$$

Exp1. Determine the end behaviour of

a) $f(x) = x^4 - 3x - 4$

b) $g(x) = -x^3 + 2x - 1$

c) $h(x) = 2(x+2)^2(1-2x)^3(x-1)^5$

d) $k(x) = -3(-x+2)^4(2-x)^5(2x^2-x+2)^2$

Sol^n. a) $a_n = 1 > 0$ $\quad n = 4$ even

up & up

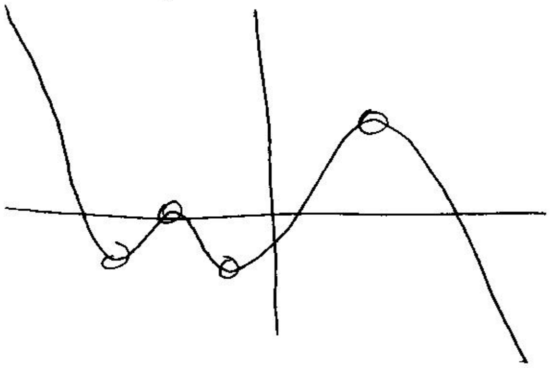
b) $a_n = 2(1)^2(-2)^3(1)^5 = 2(-8) = -16 < 0$

$n = 2 + 3 + 5 = 10$ even

down at far left

down at far right

Ex 2 Use the end behaviour & the number of turning points to give the possible degree of the poly with graph.



behaviour: up at far left
down at far right $\Rightarrow n$: odd

4 ~~turning~~ point $\Rightarrow$ deg $\geq 4 + 1 = 5$

$\Rightarrow$ deg $= 5$ or 7 or 9 ...

Fact:

The x-intercepts of a polynomial function are the zeros of the polynomial

At an x-intercept, the graph touches the x-axis in different ways according to the multiplicity of the zero as follows.

k		The graph of f(x)	
k: even		touches the x-axis without crossing	
odd	k=1	crosses x-axis with angle	
	k≥3	crosses x-axis with horizontal tangent	

Exp.

Intermediate Value theorem

$p(x)$ a polynomial, $a < b$ &
$p(a)$ & $p(b)$ have opposite signs, then
there is $a < c < b$ such that

$$p(c) = 0$$

Exp: Show that there is a zero between 2 & 3
for $p(x) = x^3 - 2x^2 - x + 1$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -1 & 1 \\ & & 2 & 0 & -2 \\ \hline & 1 & 0 & -1 & -1 \end{array} = p(2) < 0$$

$$\begin{array}{r|rrrr} 3 & 1 & -2 & -1 & 1 \\ & & 3 & 3 & 6 \\ \hline & 1 & 1 & 2 & 7 \end{array} = p(b) > 0$$

So there a zero c of $p(x)$ between 2 & 3

Graphing Polynomial Functions

1. Find all zeros with their multiplicities
2. Find the y-int.
3. Determine the end behaviour.
4. Put small segments for end. behaviours put the x-int & the way the graph crosses at this pts. Put y-int.

Join them.

Exp. Graph $f(x) = (-x+1)^3(x-3)(x+2)^2$

1) Zeros | k

Zeros	k	
1	3	. crosses with hor. tg.
-2	2	touches.
3	1	crosses

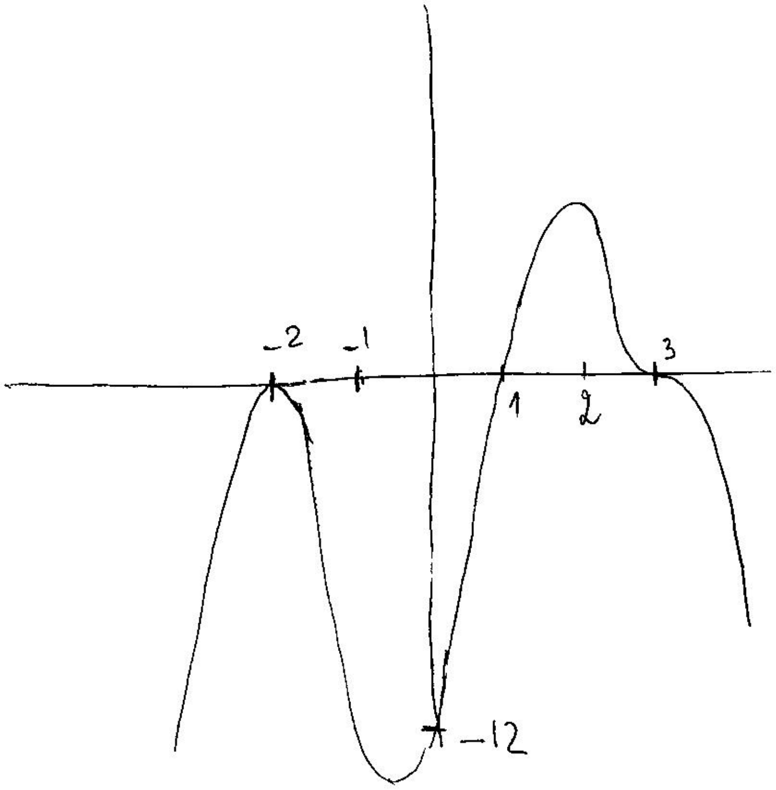
2) y-int, $f(0) = (1)^3(-3)(2)^2 = -(-3)4 = -12 < 0$

~~n=3+1+2=6 ... up, up~~

3) End behaviour

$a_n = (-1)^3(-3)(2)^2 = -12$, n: 6 even

down down



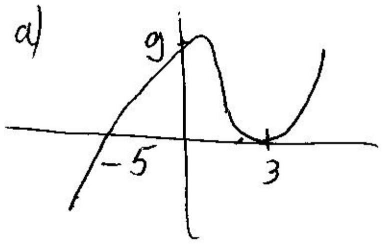
Exp Graph

a) $f(x) = 2x^3 - 5x^2 - x + 6$

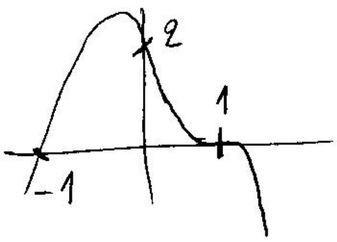
b) $g(x) = 2x^4 + x^3 - 6x^2 - 7x - 2$

Exp Find a lowest degree polynomial which is the equation for

a)



b)



c)