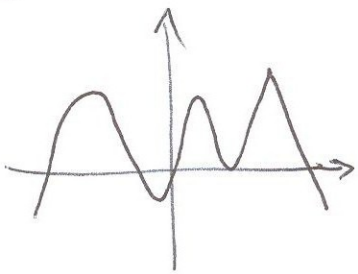
2.6 Graphs of Basic Functions

Objectives To learn how to graph and get the main features of

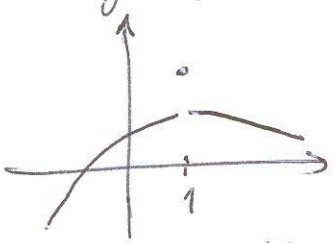
- basic functions: x, x^2, x^3, $\sqrt{x}$, $\sqrt[3]{x}$
- Piecewise defined functions
- Greatest Integer functions.

Continuity.

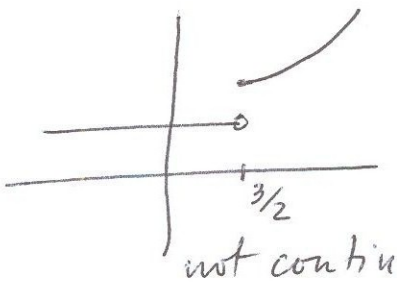
A function is continuous over an interval if its graph has no cuts or jump



Continuous



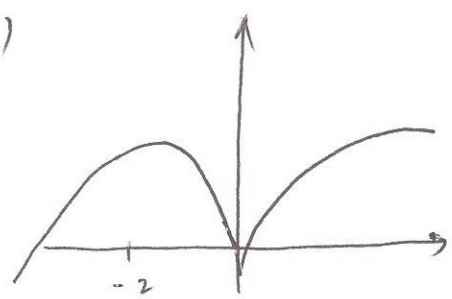
not continuous at 1



not continuous at 3/2

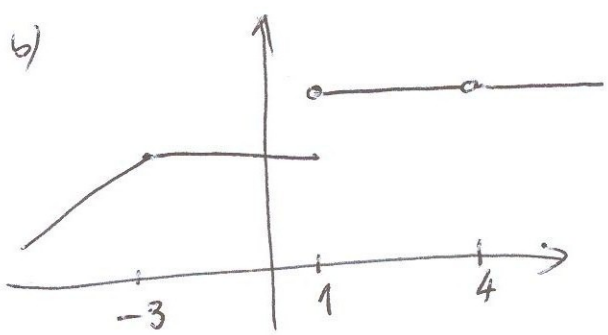
Exp 1. Give the intervals where each function is continuous.

a)



a) continuous over $\mathbb{R}$

b)



b) continuous over $(-\infty, 1)$, $(1, 4)$, $(4, \infty)$

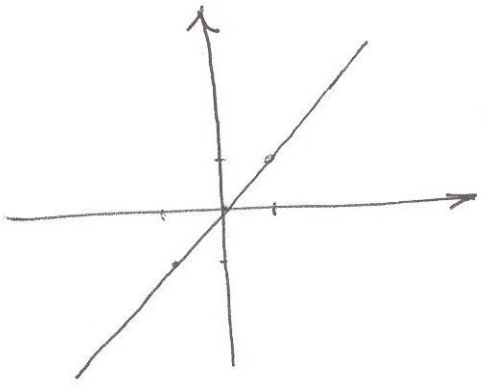
Basic functions

1) $f(x) = x$

$= 1x + 0$

$\Rightarrow$ slope $m = 1$

y-int $= 0$



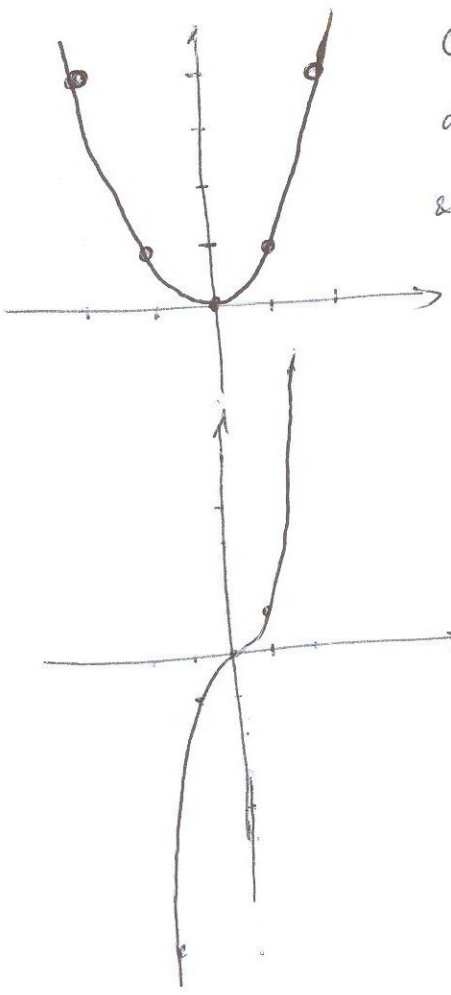
cont on $(-\infty, \infty)$

$\nearrow$ on $(-\infty, \infty)$

2) (Squaring) $f(x) = x^2$

Dom: $\mathbb{R}$

Range: $[0, \infty)$



cont on $(-\infty, \infty)$

decr on $(-\infty, 0]$

& incr on $[0, \infty)$

3) (Cubing) $f(x) = x^3$

$\mathcal{D} = \mathbb{R}$

Range: $\mathbb{R}$

Cont on $\mathbb{R}$.

increasing on $\mathbb{R}$.

4) Square Root $f(x) = \sqrt{x}$

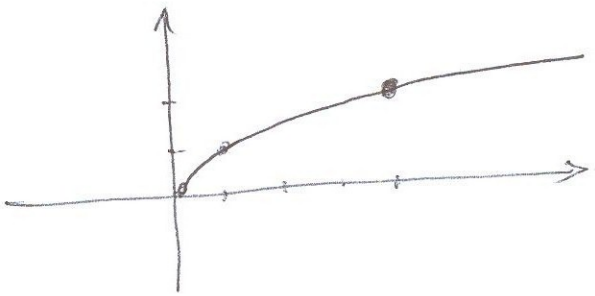
$\mathcal{D} = [0, \infty)$

$R = [0, \infty)$

continuous

increasing

0	0
1	1
4	2

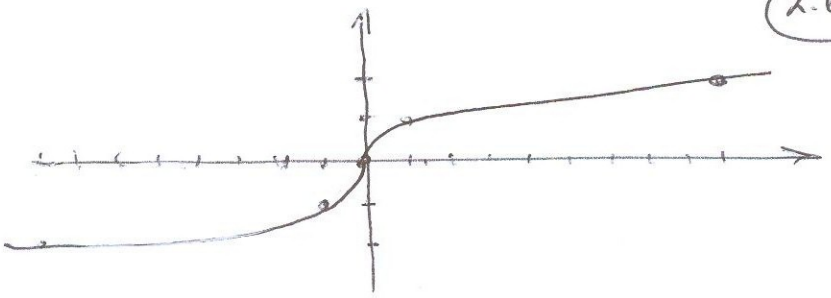


5) $f(x) = \sqrt[3]{x}$

$\mathcal{D} = \mathbb{R}$

Range: $\mathbb{R}$

increasing.

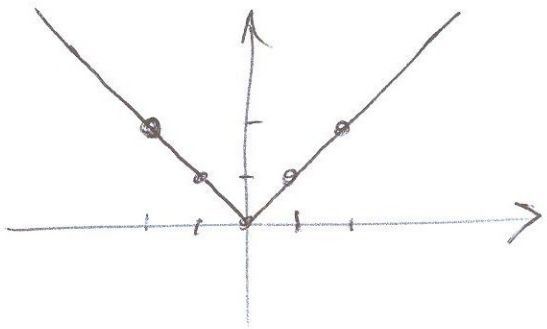


x	-8	-1	0	1	8
y	-2	-1	0	1	2

6) $f(x) = |x|$

$\mathcal{D} = \mathbb{R}$

Range: $[0, \infty)$



x	-2	-1	0	1	2
y	2	1	0	1	2

Piecewise Defined Functions.

Exp 2.

$$f(x) = \begin{cases} 4 & \text{if } x \leq -1 \\ x^2 - 2 & \text{if } -1 < x \leq 2 \\ -x + 5 & \text{if } x > 2 \end{cases}$$

a) Find value of $f(3)$, $f(-1)$, $f(1)$, $f(2)$, $f(-4)$

b) Graph $f(x)$; & find the domain & Range

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a) $f(3) = -(3) + 5 = 2$ $\because 3 > 2$

$f(-1) = 4$ $\because -1 \leq -1$

$f(1) = 1^2 - 2 = -1$ $-1 < 1 < 2$

$f(2) = 2^2 - 2 = 2$

$f(-4) = 4$ $\because (-4) \leq -1$

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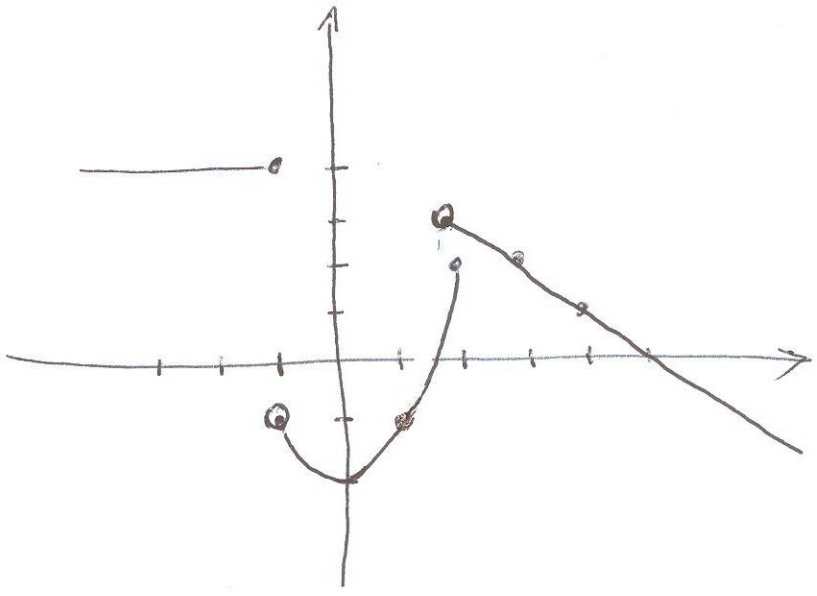
b)

x	$y = f(x)$	
-3	4	
-1	4	
$x \to -1^-$	①	→ empty pt
0	-2	
1	1	
2	2	
$x \to 2^+$	③	empty circle
3	2	
4	1	



$D = (-\infty, \infty)$

$R = (-\infty, 3] \cup \{4\}$

c) f is above the x-axis on $(-\infty, -1] \cup (\sqrt{2}, 5)$

d) f is increasing on $[0, 2]$,

Exp 3. $f(x) = \begin{cases} x^3 + 5 & \text{if } x \leq 0 \\ -x^2 & \text{if } x > 0 \end{cases}$

a) Find $f(-2)$, $f(0)$, $f(3)$

b) Graph $f(x)$. Find Range.

Exp 4. $f(x) = \begin{cases} 2+x & \text{if } x < -4 \\ -x & \text{if } -4 \leq x \leq 5 \\ 3x & \text{if } x > 5 \end{cases}$

a) Find $f(-5)$, $f(-4)$, $f(5)$, $f(6)$, $f(x^2+6)$

$f(-x^4-5)$

b) Graph, Find domain, range.

Greatest Integer Function.

For any real nbr x, the greatest integer function $[\![x]\!]$ of x is the greatest integer less or equal to x

$[\![5.2]\!] = 5$, $[\![5]\!] = 5$

$[\![-3.8]\!] = -4$ $[\![-3.0]\!] = -3$

Properties. 1) $[\![x]\!]$ is an integer

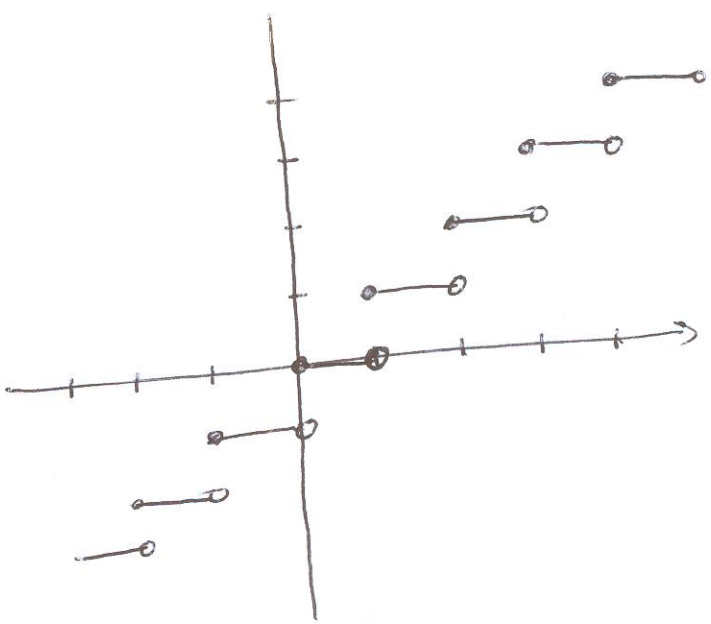
2) $[\![x]\!] \leq x$.

3) $x-1 < [\![x]\!] \leq x$.

4) $[\![x]\!] \leq x < [\![x]\!] + 1$

Exp 5. Graph $f(x) = [\![x]\!]$, Find domain & range. & x-int, y-int.

$$f(x) = \begin{cases} -1 \\ 0 \\ 1 \\ 2 \end{cases} \quad \text{if} \quad \begin{array}{l} -1 \leq x < 0 \\ 0 \leq x < 1 \\ 1 \leq x < 2 \\ 2 \leq x < 3 \end{array}$$



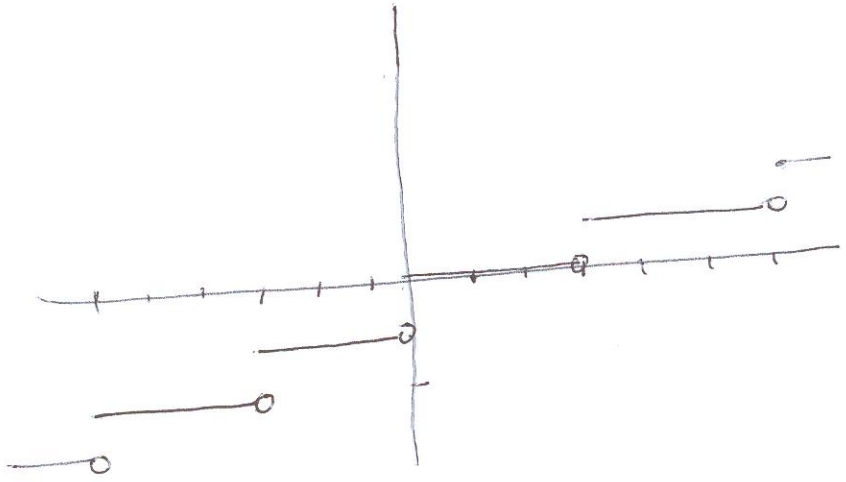
$D = (-\infty, \infty)$

$R = \mathbb{Z} = \{\text{set of integers}\}$

x-int $= [0, 1)$

y-int 0

Exp. Graph $f(x) = [\![\tfrac{1}{3}x]\!]$



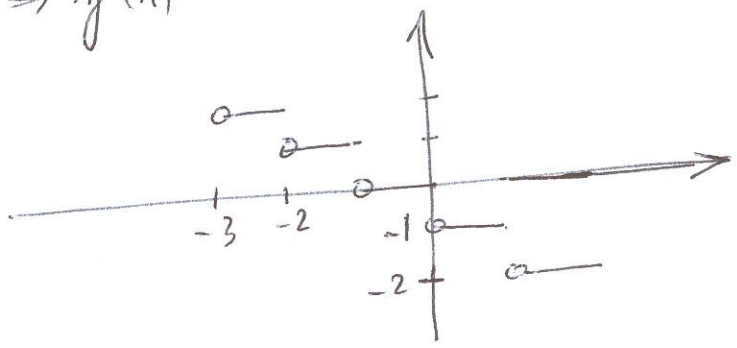
Exp. $g(x) = [\![-x]\!]$

$[\![-x]\!] = n \iff n \leq -x < n+1 \iff -n-1 < x \leq -n$

$n=0 \quad -1 < x \leq 0 \Rightarrow g(x) = 0$

$n=1 \quad -2 < x \leq -1$

$n=-1 \quad 0 < x \leq 1$



Exp. $h(x) = [\![2x-1]\!]$