

- 2.2. Circles -

p1

Objectives.

- Give the geometric definition & equation
- Learn how to find the standard equation, the center & radius.

Definition. Given a point C & a positive number $r > 0$, the circle of center C & radius r is the set of all points whose distance to C is r .

Center-Radius Equation.

If $C = (h, k)$ & $P(x, y)$ is a point of the circle

$$\Rightarrow d(P, C) = r \quad (\Leftrightarrow) \quad \sqrt{(x-h)^2 + (y-k)^2} = r$$

$$\Leftrightarrow \boxed{(x-h)^2 + (y-k)^2 = r^2}$$

Exp 1. Find the center-Radius form of the equation of the circle with

(a) $C = (-3, 4)$, radius = 6

(b) Center $(0, 0)$ radius = 4

a) $(x - (-3))^2 + (y - 4)^2 = 6^2$

$$(x + 3)^2 + (y - 4)^2 = 36$$

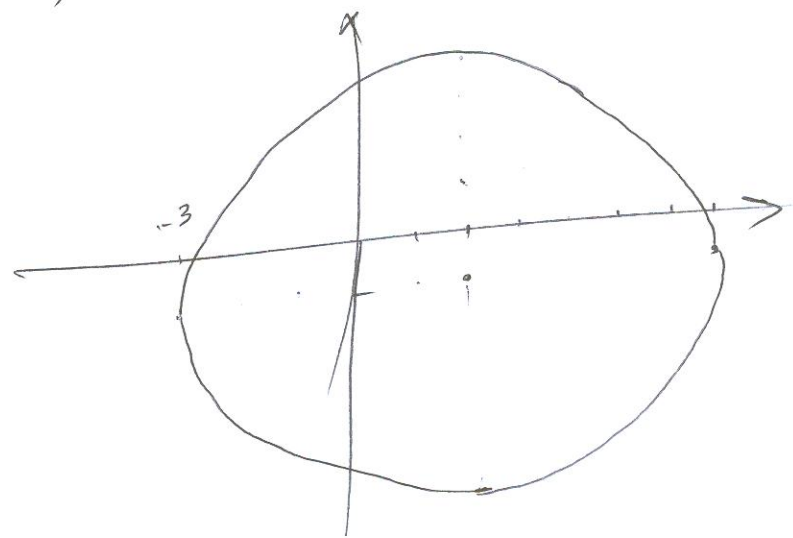
b) $(x-0)^2 + (y-0)^2 = 4^2$
 $x^2 + y^2 = 16$

Exp 2. Find the center, radius & graph the circle

a) $(x-2)^2 + (y+1)^2 = 25$

b) $x^2 + y^2 = 12$

a) $C = (2, -1)$ $r^2 = 25 \Rightarrow r = 5$



General Form.

The graph of $x^2 + y^2 + cx + dy + e = 0$ is

- a) a circle
- or b) a point
- or c) empty (there is no graph)

Exp 3. Show that

$$x^2 - 6x + y^2 + 10y + 25 = 0$$

is a circle & find the center & radius.

Solⁿ (group terms with x & the terms with y

$$(x^2 - 6x) + (y^2 + 10y) = -25$$

Complete the square in x & in y

$$(x^2 - 6x + 9) + (y^2 + 10y + 25) = -25 + 9 + 25$$

$$(x-3)^2 + (y+5)^2 = 16 = 4^2 \rightarrow \text{circle.}$$

$$C = (3, -5), \quad r = 4$$

Exp 4. Same question

$$2x^2 + 2y^2 - 6x + 10y = 1$$

Exp 5. Find the graph of
 $x^2 + 10x + y^2 - 4y + 33 = 0$

$$(x^2 + 10x) + (y^2 - 4y) = -33$$

$$(x^2 + 10x + 25) + (y^2 - 4y + 4) = -33 + 25 + 4$$

$$\underbrace{(x+5)^2 + (y-2)^2}_{\text{positive or zero}} = \underbrace{-4}_{\text{neg}} \Rightarrow \text{No sol}^n$$

\Rightarrow No graph.

Exp 6. Same question

$$x^2 - 10x + y^2 + 4\sqrt{2}y = -33$$

$$(x^2 - 10x + 25) + (y^2 + 4\sqrt{2}y + 8) = -33 + 25 + 8$$

$$(x-5)^2 + (y+2\sqrt{2})^2 = 0$$

\Rightarrow Graph = $\{(5, -2\sqrt{2})\}$ 1 point.

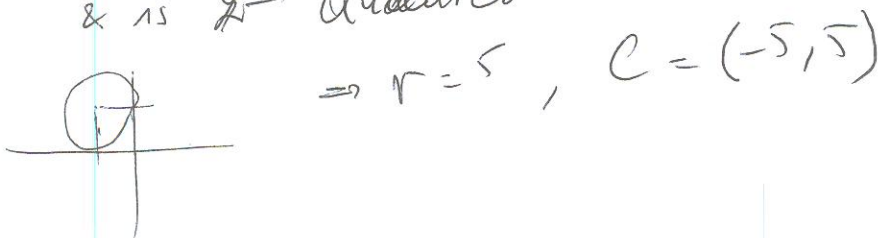
Finding Equation from geometric information. p5

Exp 7. A circle has center $(-4, 3)$ & passes through $(5, 8)$. Find its equation.

Exp 8. Find the eqⁿ of circle with one diameter with endpoints $(-1, 1)$, $(5, 9)$

Exp 9. Same question with center $(7, -3)$ & circle tangent to x -axis

Exp 10. Circle tangent to x -axis & y -axis & is 2nd Quadrant & radius 5.



Exp 11. Do the two circles intersect.

$$(x-2)^2 + (y-1)^2 = 9$$

$$(x-6)^2 + (y-4)^2 = 16$$

$$C_1 = (2, 1), \quad C_2 = (6, 4)$$

$d(C_1, C_2)$ should be $\leq \sqrt{6} + \sqrt{4} = 5$

$$d(C_1, C_2) = \sqrt{(6-2)^2 + (4-1)^2} = \sqrt{4^2 + 3^2} = 5$$

\Rightarrow intersect at 2 pt.

Exp. 11. Find eqⁿ of circle of least radius that contains $(1, 4), (-3, 2)$