

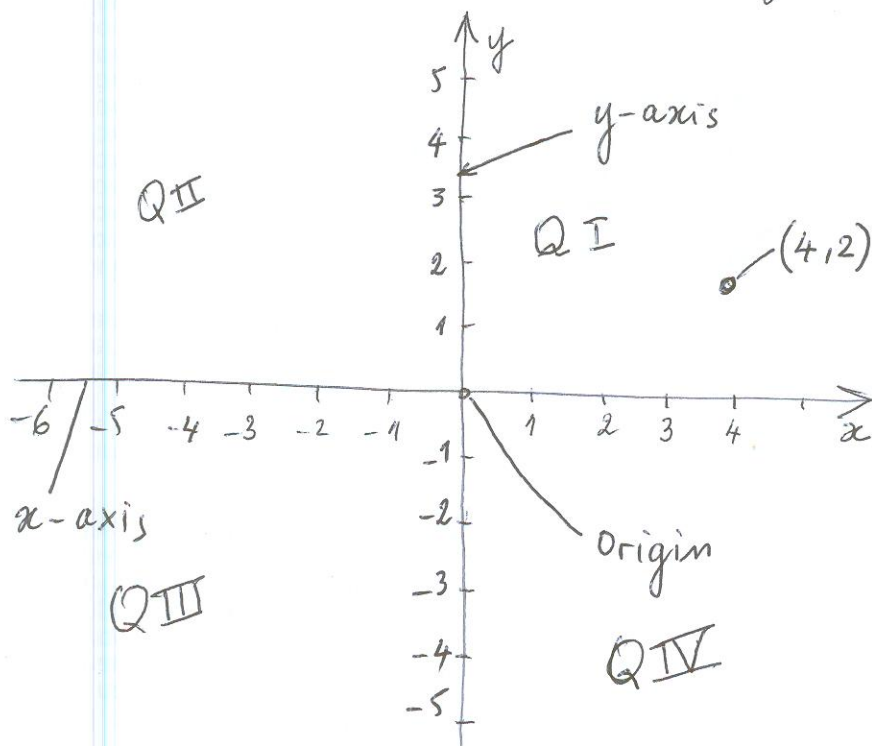
2.1. Rectangular Coordinates & Graphs

2.1

Objectives.

- Ordered pairs & the rectangular Coordinate System.
- Distance formula
- Midpoint Formula
- Graphing of equations in two variables.

As each point on the real number line is associated with a number, each point on a plane can be associated to an ordered pair (a, b) as follows



$$P = (a, b)$$

(a, b) coordinates of P

a : x-coordinate of P

b : y-coordinate of P

Coordinate System
or Cartesian System.

(a, b) is called an ordered pair

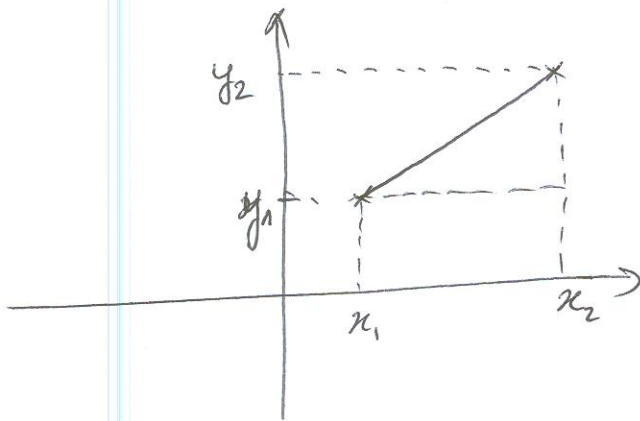
$$(a, b) = (c, d) \quad (\Rightarrow) \quad a = c \quad \& \quad b = d$$

Order in ordered pairs is important,

Eg. $(2, 3) \neq (3, 2)$.

The distance Formula.

If $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, the distance between
between P_1 & P_2 is



$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{or} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

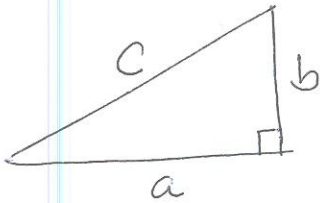
Exp 1. Find the distance between $P_1 = (2, 5)$
 $P_2 (4, 1)$

Exp 2. Which of the points $P(1, -2)$, $Q(8, 9)$
is closer to $S(5, 3)$

Application of the distance formula.Pythagoras Theorem.

If the sides of a right triangle are a, b, c , then

$$c^2 = a^2 + b^2 \quad \text{where } c \text{ is the hypotenuse}$$



The converse is also true

If a triangle with sides a, b, c satisfies

$c^2 = a^2 + b^2$ then the triangle is right and c is the hypotenuse.

Exp 3. Determine if the 3 given points are the vertices of a right triangle.

a) $P_1(-2, -8), P_2(0, -4), P_3(-4, 7)$

b) $(-4, 3), (2, 5), (-1, -6)$

a) $d(P_1, P_2) = \sqrt{(0+2)^2 + (-4+8)^2} = \sqrt{4+16} = 2\sqrt{5}$

$$d(P_1, P_3) = \sqrt{(-4+2)^2 + (7+8)^2} = \sqrt{4+225} = \sqrt{229}$$

$$d(P_2, P_3) = \sqrt{(-4-0)^2 + (7+4)^2} = \sqrt{16+121} = \sqrt{135}$$

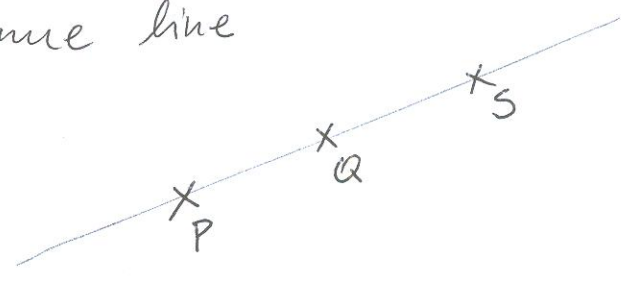
$$\Rightarrow c = \sqrt{229}, \quad a = 2\sqrt{5}, \quad b = \sqrt{135}$$

$$c^2 = a^2 + b^2$$

$$229 = 20 + 135$$

False \Rightarrow Not a right triangle.

Definition. 3 points P, Q, S are collinear if they lie on the same line



Fact. P, Q and S are collinear if the sum of the distances between two pairs of pts is equal to the distance between the remaining pair of points.

Ex 4. Which set of 3 points are collinear?

a) P(-1, 5), Q(2, 4), R(4, -10)

b) P'(-1, 4), Q'(-2, -1), R'(1, 14)

$a = d(P, Q) = 3\sqrt{10}$

$b = d(Q, R) = 2\sqrt{10}$

$c = d(P, R) = 5\sqrt{10}$

Choose the biggest distance, $c = 5\sqrt{10}$.

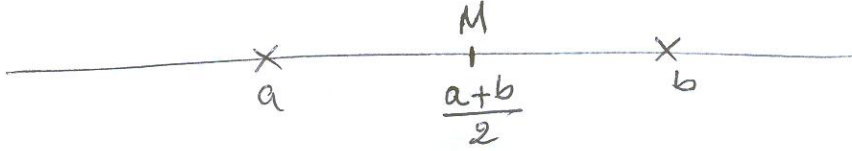
Check if $c = a + b$.

$5\sqrt{10} = 3\sqrt{10} + 2\sqrt{10}$ ✓

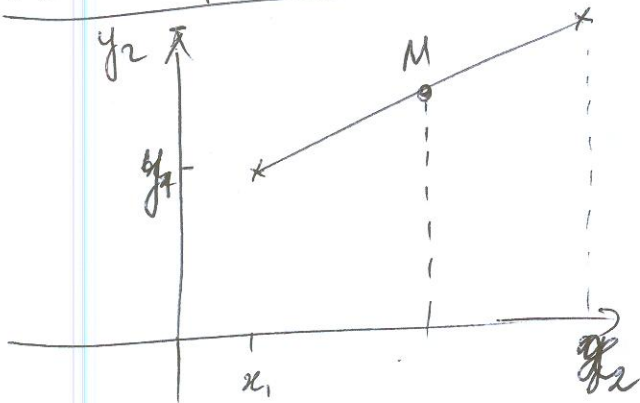
→ P, Q, S are collinear.

Midpoint Formula.

On the real line, the midpoint of a line segment with endpoints a & b is $\frac{a+b}{2}$



On the plane



$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Exp 5 Find the midpoint of the segment with endpoints $(8, -4)$, $(-6, 1)$

Exp 6. If a segment has one endpoint $(-9, 9)$ & the midpoint $(-7, 6)$. Find the other endpoint